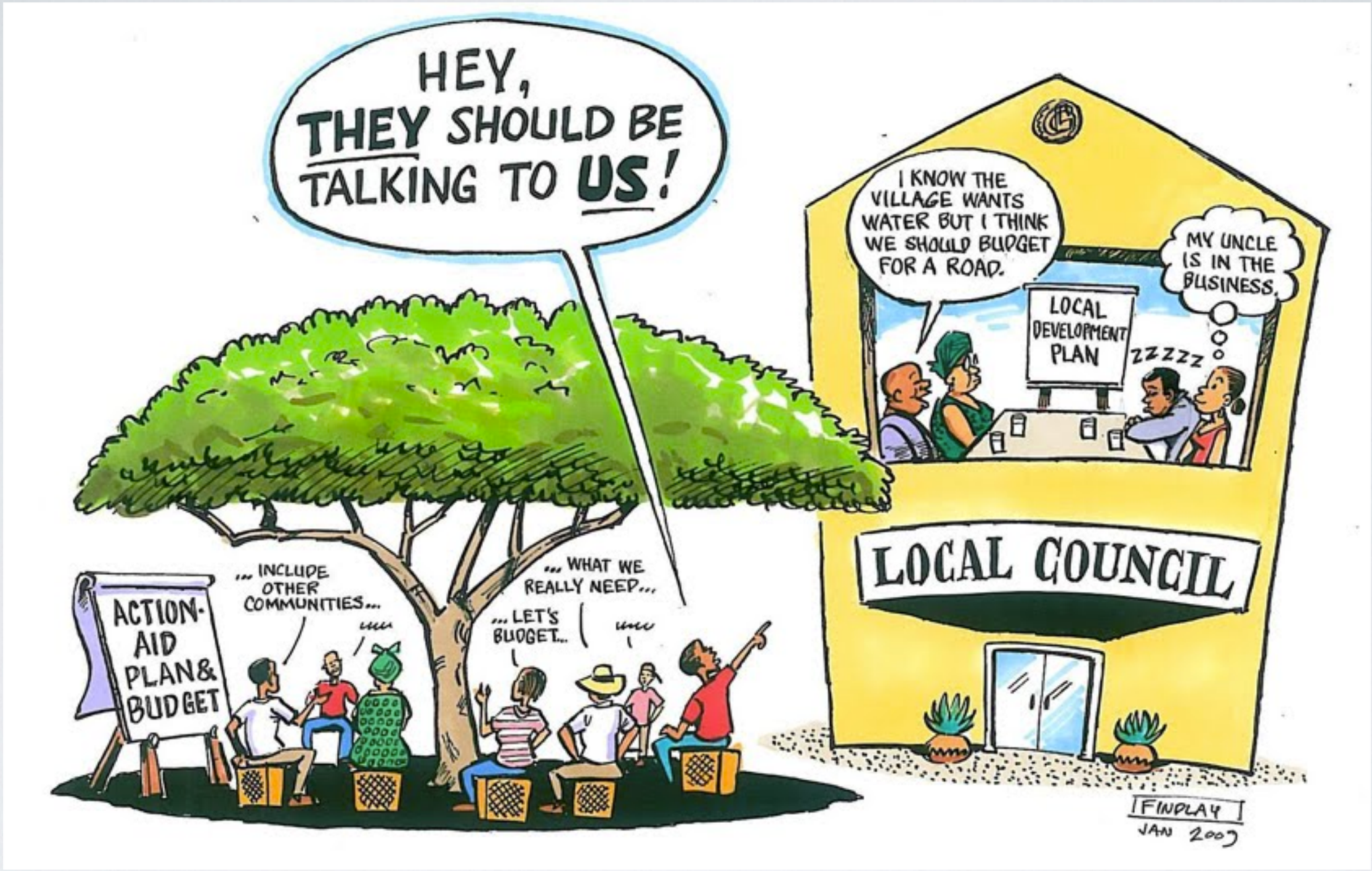
DECISION MAKING AT SCALE: ALGORITHMS, MECHANISMS, AND PLATFORMS

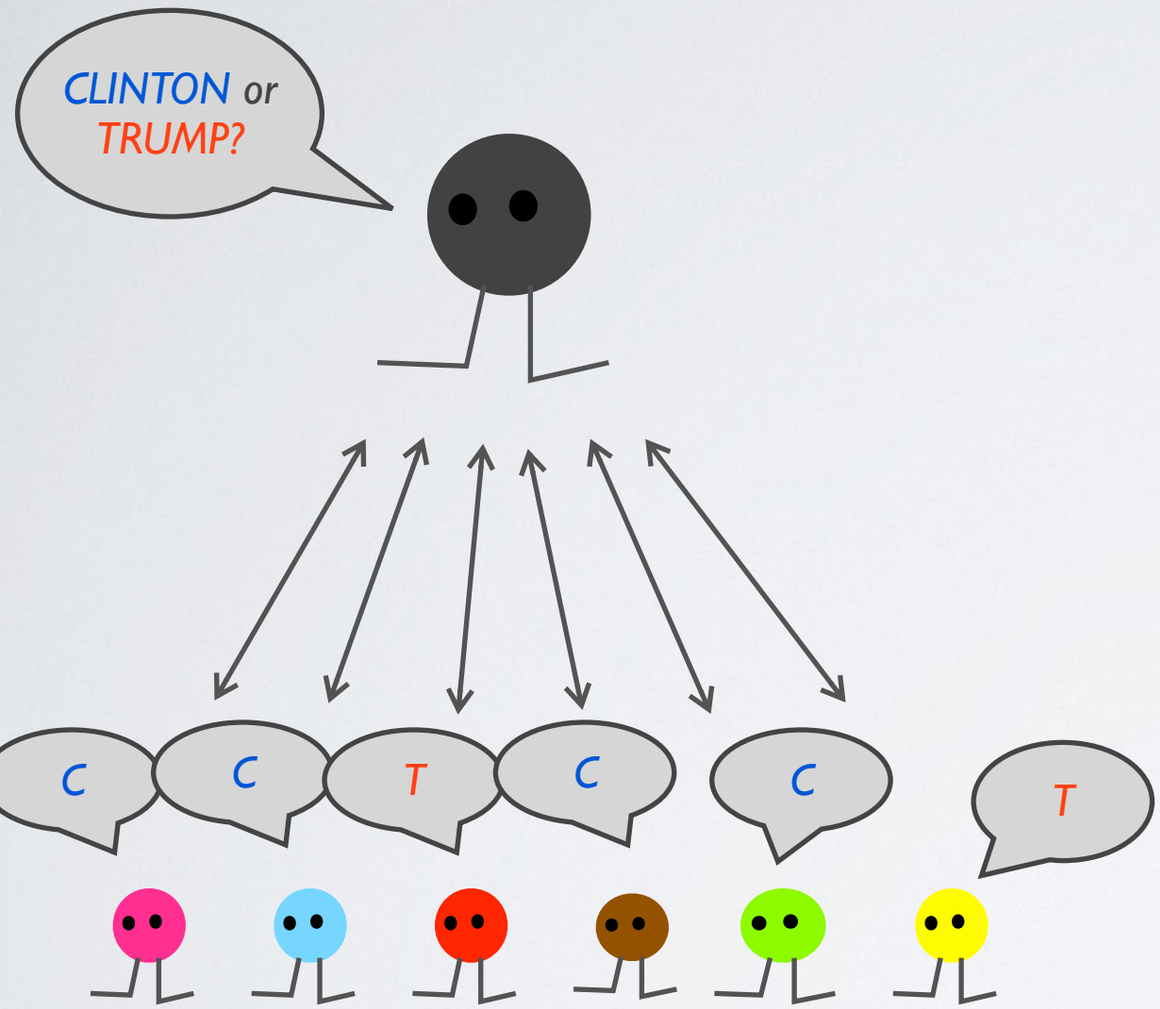
ASHISH GOEL

STANFORD

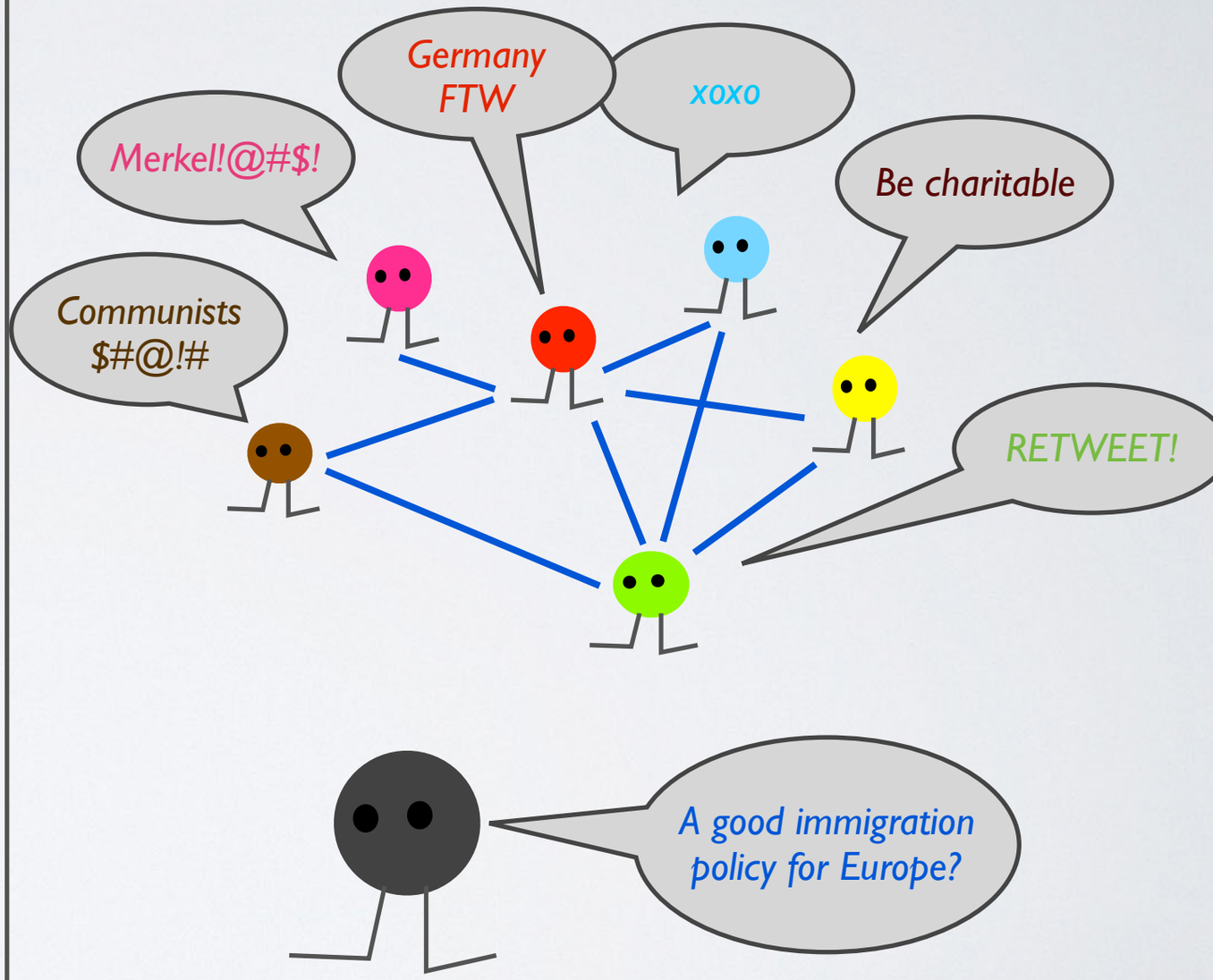


Participatory Democracy: A process emphasizing the role of citizen involvement in political decision-making.

Decision Making at Scale



Simple Polls



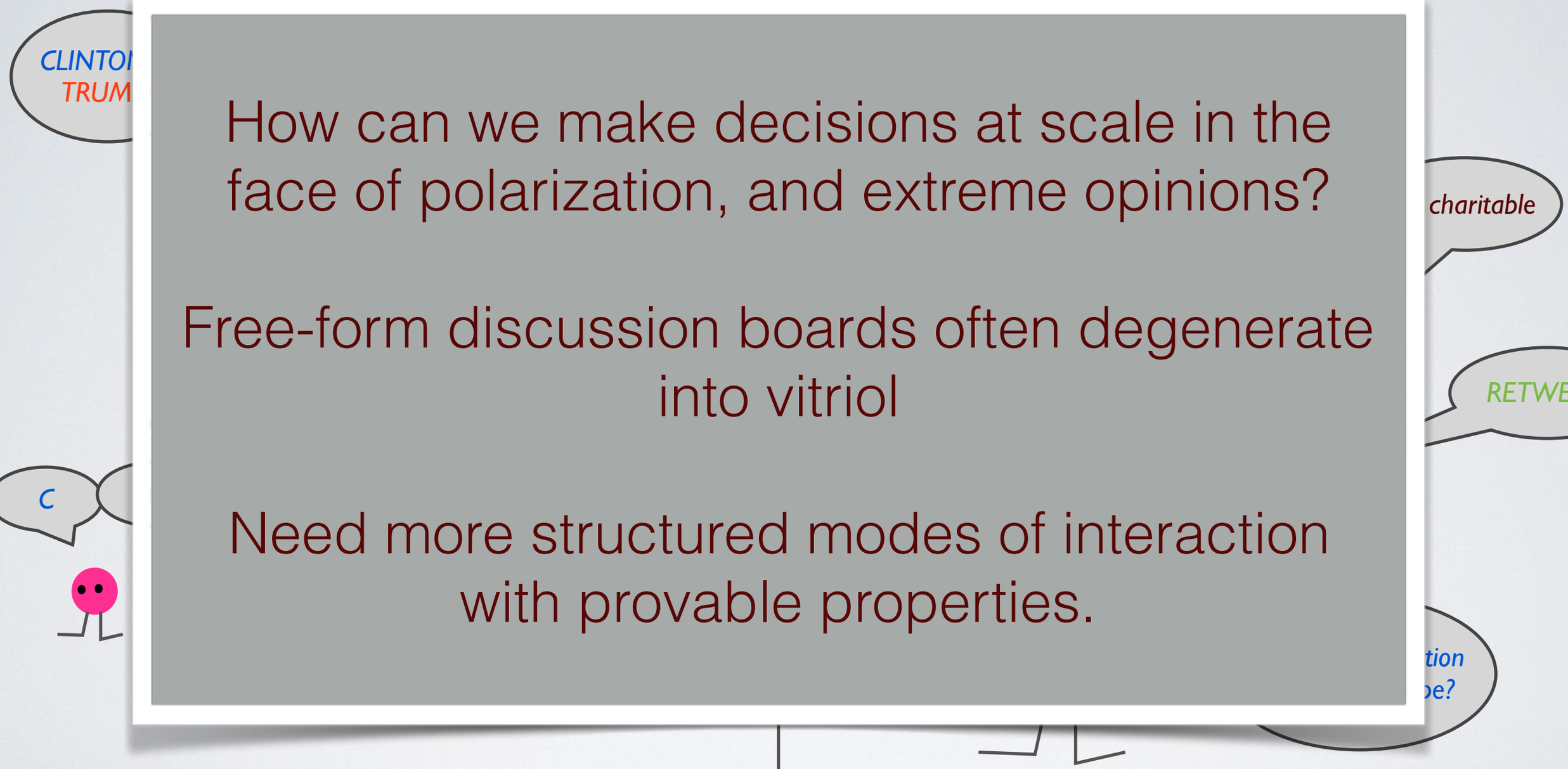
Complex Decisions

Decision Making at Scale

How can we make decisions at scale in the face of polarization, and extreme opinions?

Free-form discussion boards often degenerate into vitriol

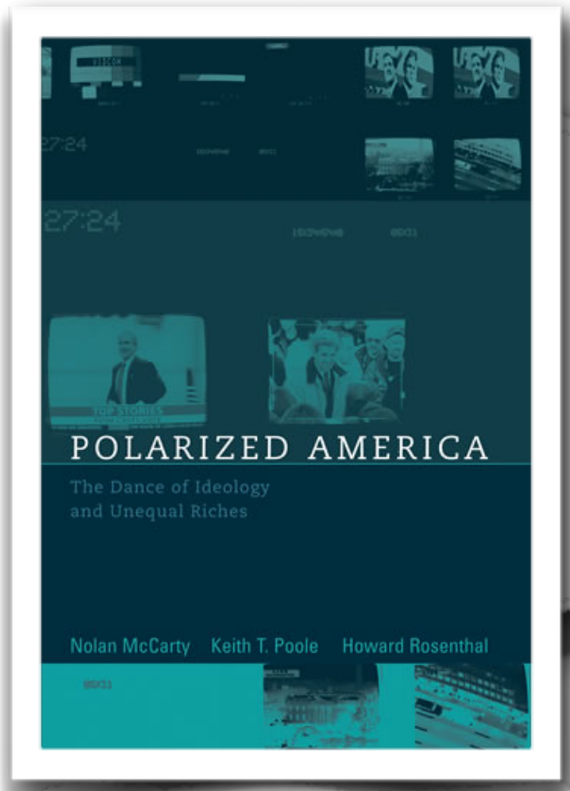
Need more structured modes of interaction with provable properties.



Simple Polls | Complex Decisions

Blogs Are Echo Chambers: Blogs Are Echo Chambers

Eric Gilbert, Tony Bergstrom and Karrie Karahalios
University of Illinois at Urbana-Champaign
[egilber2, abergst2, kkarahal]@cs.uiuc.edu



The New York Times

April 27, 2009

With Rivals Ahead, Doubts for CNN's Middle Road

By BILL CARTER

Divided We Stand:
The Polarizing of American Politics

BY STEVEN HILL

A recent Zogby poll found that 70 percent of

In combination, these two are greatly contributing to

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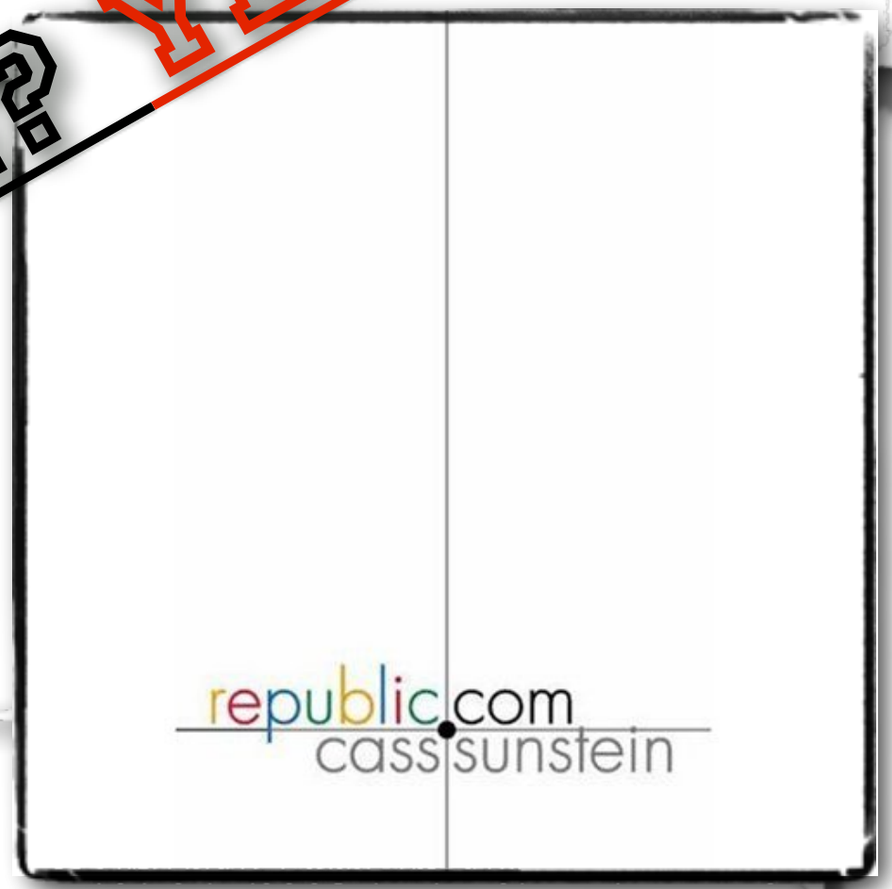
POLARIZATION? YES



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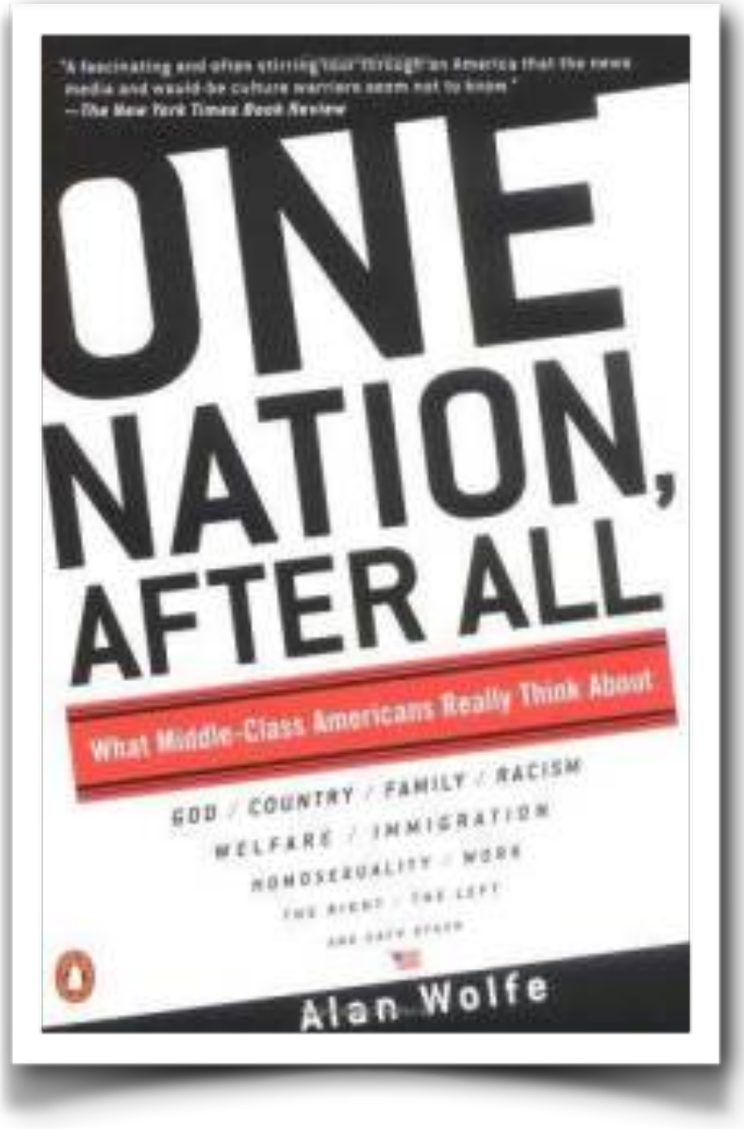
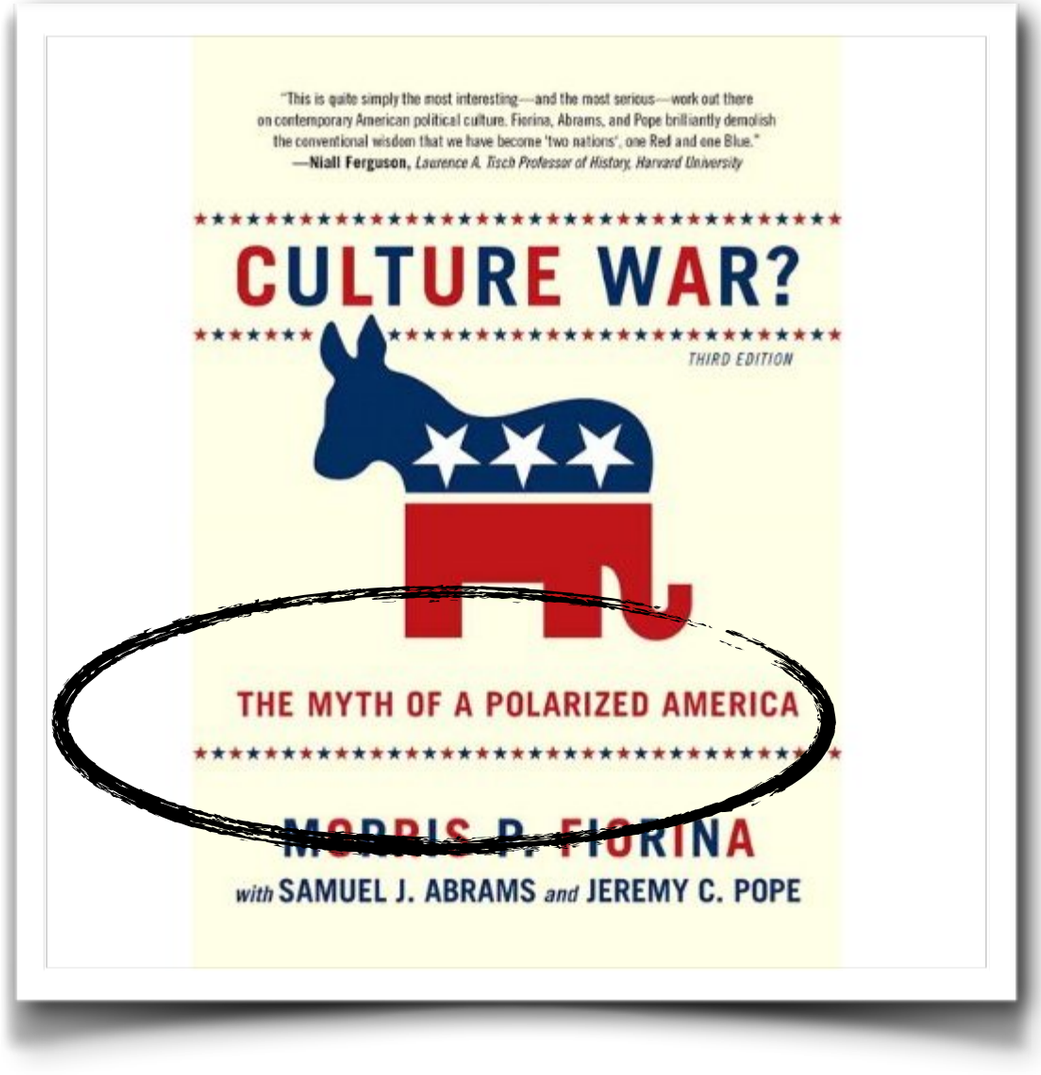
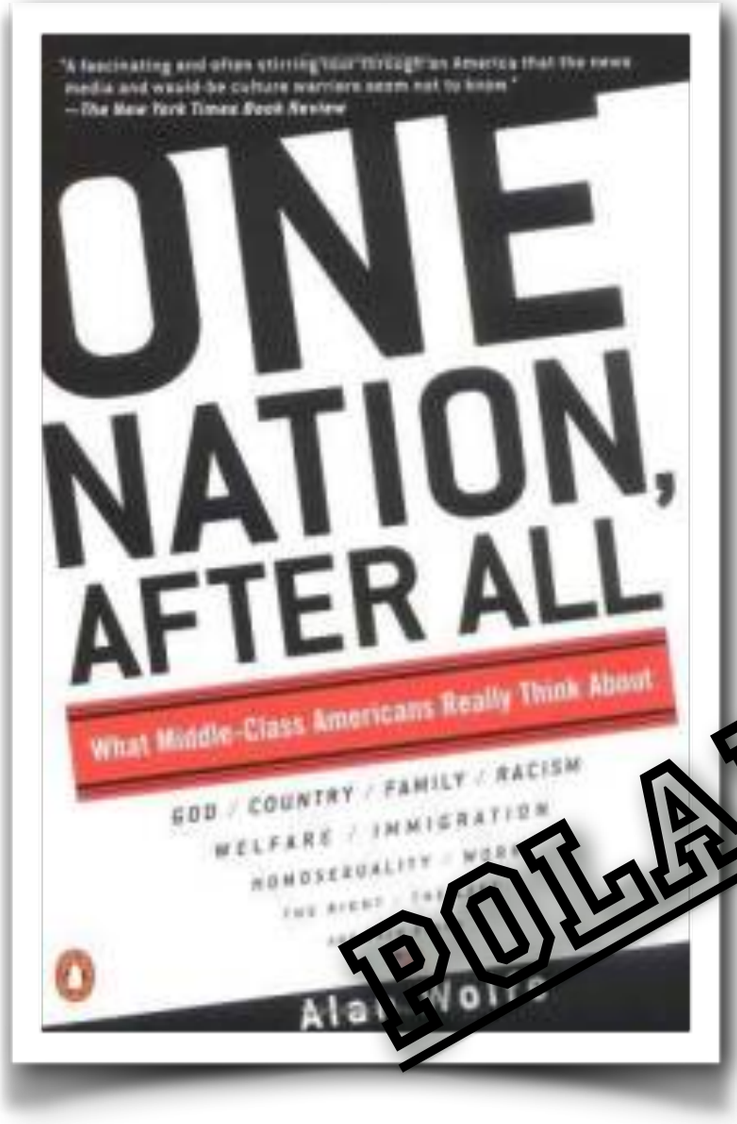
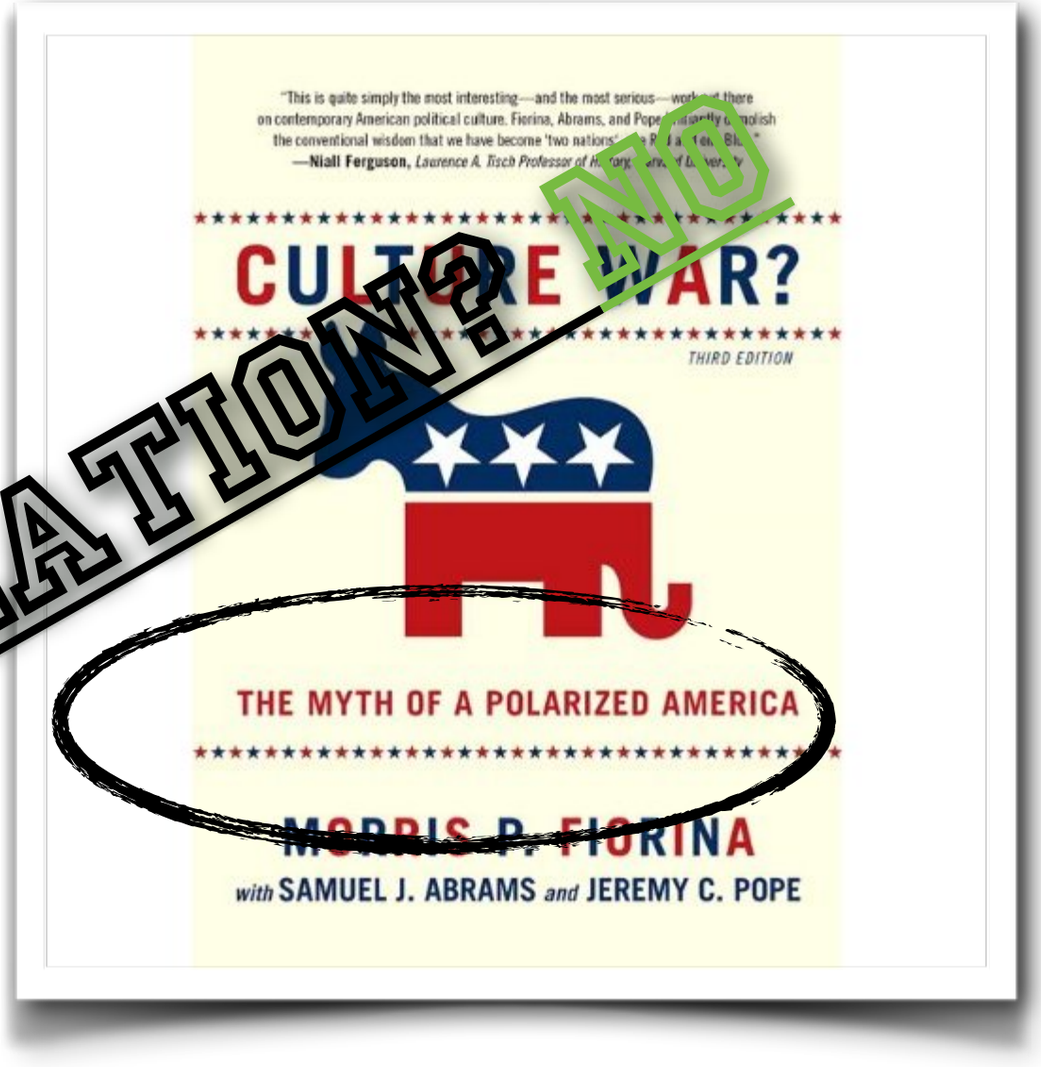

"A fascinating and often stirring tour through an America that the news media and would-be culture warriors seem not to know."
—The New York Times Book Review
ONE NATION, AFTER ALL
What Middle-Class Americans Really Think About
GOD / COUNTRY / FAMILY / RACISM
WELFARE / IMMIGRATION
HOMOSEXUALITY / WORK
Alan Wolfe


"This is quite simply the most interesting—and the most serious—work out there on contemporary American political culture. Fiorina, Abrams, and Pope brilliantly demolish the conventional wisdom that we have become 'two nations', one Red and one Blue."
—Niall Ferguson, Laurence A. Tisch Professor of History, Harvard University
CULTURE WAR?
THIRD EDITION
THE MYTH OF A POLARIZED AMERICA
MORRIS P. FIORINA
with SAMUEL J. ABRAMS and JEREMY C. POPE






ONE
NATION
AFTER AL
What Middle-Class Americans Really Think
GOD / COUNTRY / FAMILY / RACISM
WELFARE / IMMIGRATION
HOMOSEXUALITY / WORK
Alan Wolfe
I WORRY ABOUT THE DECREASED POLARIZATION OF OUR SOCIETY.
WAR?
THIRD EDITION
RIZED AMERICA
FIORINA
JEREMY C. POPE

High Level Design Goals

"Number of Candidates = Number of Voters"

- No voter can look at all candidates
- Need small group interactions, limit cognitive load

Support not just voting but also interaction, collaboration, and discussion

Align incentives. Promote fairness. Recognize minority opinion

Support unstructured decision spaces

Participatory Budgeting

Ashish Goel
Anilesh Krishnaswamy
Sukolsak Sakshuwong
Tanja Aitamurto

Stanford University

US-wide push for participatory budgeting; multiple workshops at the White House

Participatory Budgeting via K-Approval Voting

- A set of proposed spending projects get put to the ballot for residents directly
- Every voter chooses K projects, for a fixed K
- The projects that get the most votes get implemented, subject to a budget cap
- Our digital voting platform used in Chicago, NYC, Long Beach, Vallejo, Boston, Cambridge, among others

Our platform as used in Chicago

Participatory Budgeting 49th Ward

Español

You have chosen

4 / 4

projects.

Once you are done, click the **'Submit My Choices'** button at the bottom of the screen.

Parks & Environment

2. Touhy Park Cherry Blossom Trees and Water Fountain

Add a new water fountain in the northwest section of the park and plant six cherry blossom trees

$70,000

Deselect

3. New Water Feature at Pottawattomie Park

Replace the old, often non-functioning feature with a new feature with more sprays and a more durable infrastructure.

$75,000

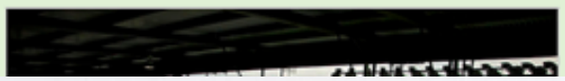
Can't choose more than 4 projects

4. Mini AstroTurf Soccer Field at Langdon Park

Apply striping demarcating a shared lane for bicycles in both directions.

$100,000

Deselect



Our platform as used in Chicago: Map elements

Participatory Budgeting 49th Ward

Help Español

You have selected

2 / 4

projects.

- Beach Path Extension at Hartigan Beach
- New Bus Stop Benches

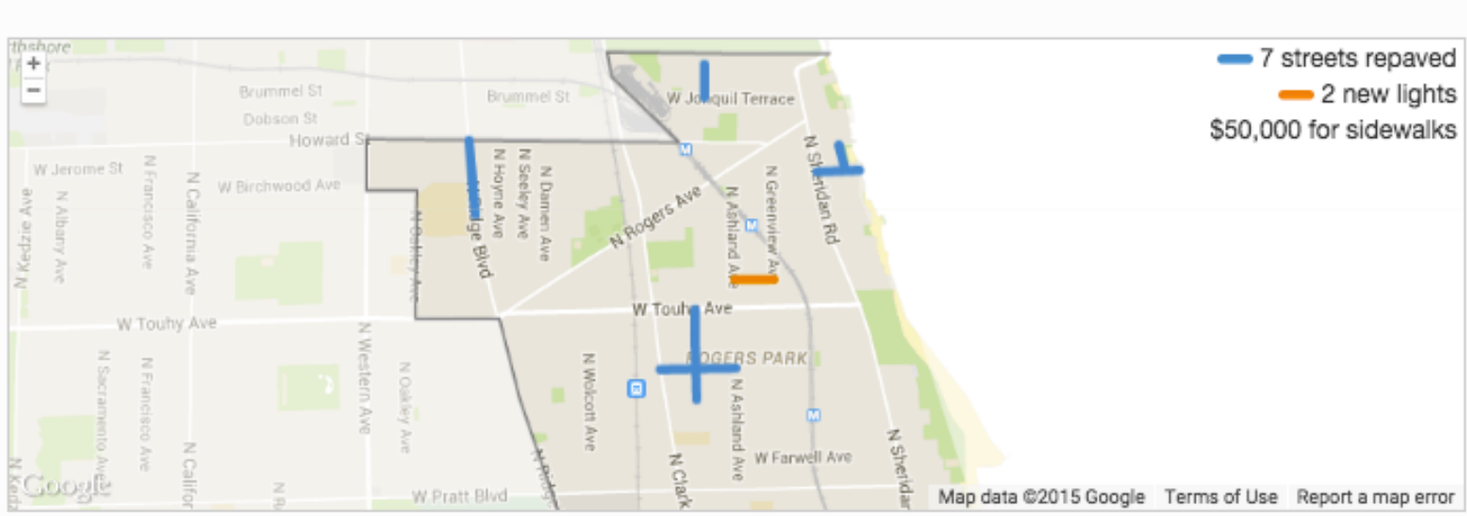
Street Resurfacing, Sidewalk Repairs, & Street Lighting

What percentage of the total budget should go toward street resurfacing, sidewalk repairs, and street lighting? (Select one box on the right)

Note: The approved percentage will be apportioned as follows: 10% of the total approved will be dedicated to sidewalk repairs and the balance will be apportioned at a 3:1 ratio, street resurfacing to lights. The average cost for the first five blocks of resurfaced streets is $38,000 per block. Due to federal requirements for accesibility ramps, the average cost for any street resurfaced after the first five blocks is $66,500 per block. The average cost of residential street lighting is $64,000 per block and the average cost to replace a block of sidewalk is $75,000.

Please select from the options below:

- 100% ($1,000,000)
- 90% ($900,000)
- 80% ($800,000)
- 70% ($700,000)
- 60% ($600,000)
- 50% ($500,000)
- 40% ($400,000)
- 30% ($300,000)
- 20% ($200,000)
- 10% ($100,000)
- 0% ($0)



7 streets repaved
2 new lights
$50,000 for sidewalks

Hover your mouse over the highlighted streets to see street names on the map.

Bikes & Transit

New Bus Stop Benches

Install new black metal bus benches at 15 stops currently without benches on Clark, Howard, Rogers, and Sheridan.

$36,000

Deselect

Three Problems with K-Approval Voting

- No recognition of trade-offs:
  - Wrong question: Is it better to build a fountain in a park or fill a pothole on a street?
  - Right question: Is it better to build a fountain for $10K or fill a pothole for $1K?
- Strange incentives: There exist cases where a voter should vote for A over B even if she prefers B and B is winning, i.e. K-Approval voting is not even weakly strategy proof
- Aggregation? What is the rationale for choosing projects?

Knapsack voting for divisible projects

$1 million

50 $200K

$600K 130

$150K 40

35 $70K

60 $250K

$300K 70

Our interface for asking users to submit a full budget

PB Vallejo

Selected **$1,017,000** of **$1,100,000** total budget.
You still have **$83,000** left.

Profile Log Out Español

You have selected

5

projects.

Parks & Environment

- ✔ 3 in 1! Bike Lanes for Safety and Convenience!
- ✔ Community Branding of Vallejo
- Dual-Purpose Recycling/Trash Cans on Waterfront
- A.B.C. Arts, Beautification, Community Development
- Empower Vallejo Students via Boating

Arts & Innovation

- Improve Safety & Comfort of River Park
- ✔ Improve Safety of Blue Rock Springs Park
- Equip an Electronics Lab at Vallejo High School
- Improving School Meals
- Improve Safety of Hanns Park

Bikes & Transit

- ✔ Pedestrian Crosswalk Safety and Traffic Calming
- ✔ Restore the Mare Island logo on the Blue Bridge
- Murals for Youth
- Prioritizing Street and Pothole Repair-Repaving
- Omega Boys & Girls Club Kitchen Project

Sidewalks, Streets and Alleys

- Special Fire Rescue Vehicle Replacement
- Workforce Skills Development for Adults
- Vallejo Museum: Preserving Vallejo's Past
- Sidewalk Repair and Street Tree Preservation
- Strike Up the Bands

Bikes & Transit

Pedestrian Crosswalk Safety and Traffic Calming

Increase pedestrian safety in dangerous crosswalks by enhancing the capacity of Public Works to deploy traffic calming and crosswalk safety devices.

Estimated Cost: $525,000

✔ Selected Remove

Restore the Mare Island logo on the Blue Bridge

We propose to fully restore the faded Mare Island Shipyard logo on the blue Causeway Bridge that goes onto Mare Island from Tennessee St.

Estimated Cost: $22,000

✔ Selected Remove

Murals for Youth

The project aims to inspire the youth by placing four murals on Vallejo City

Prioritizing Street and Pothole Repair-Repaving

Improve traffic safety by enhancing the capacity of Public Works to repair and

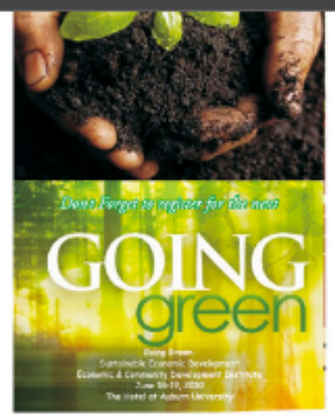
Our interface for asking users to submit a full budget

PB Boston

Selected **$6,300** of **$10,000** total budget.
You still have **$3,700** left.

Exit Profile Log Ou

- Incentives for Helping to get out the vote
- Incentives for participants
- ✔ Neighborhood Parties for Targeted Outreach
- ✔ YLC Bracelets
- Facebook Ad: Raise RSVPS to YLC Vote Event
- Youtube Ad
- YLC Door Hangers
- ✔ Designer
- Twitter Advertising: Pay Per Website Click
- Radio Ad
- YLC Banners
- Printing for MTBA Advertising
- YLC Hoodies
- ✔ YLC T-Shirts
- YLC Tote Bags
- Facebook Ad: Boost



Designer

Let's hold a design competition to make our merchandize and advertising wicked awesome!
The competition would be hosted by the design company, 99 design.
More money gets us access to more designs and more experienced designers.
Minimum: $300 = access to 30 different designs
Maximum: $1,000= Access to 90 designs from expert designers
Incremental Amount: $50

Estimated Cost: $300 - $1,000

Currently Selected Cost: $600

✔ Selected Remove

Knapsack Voting: Aggregation rule

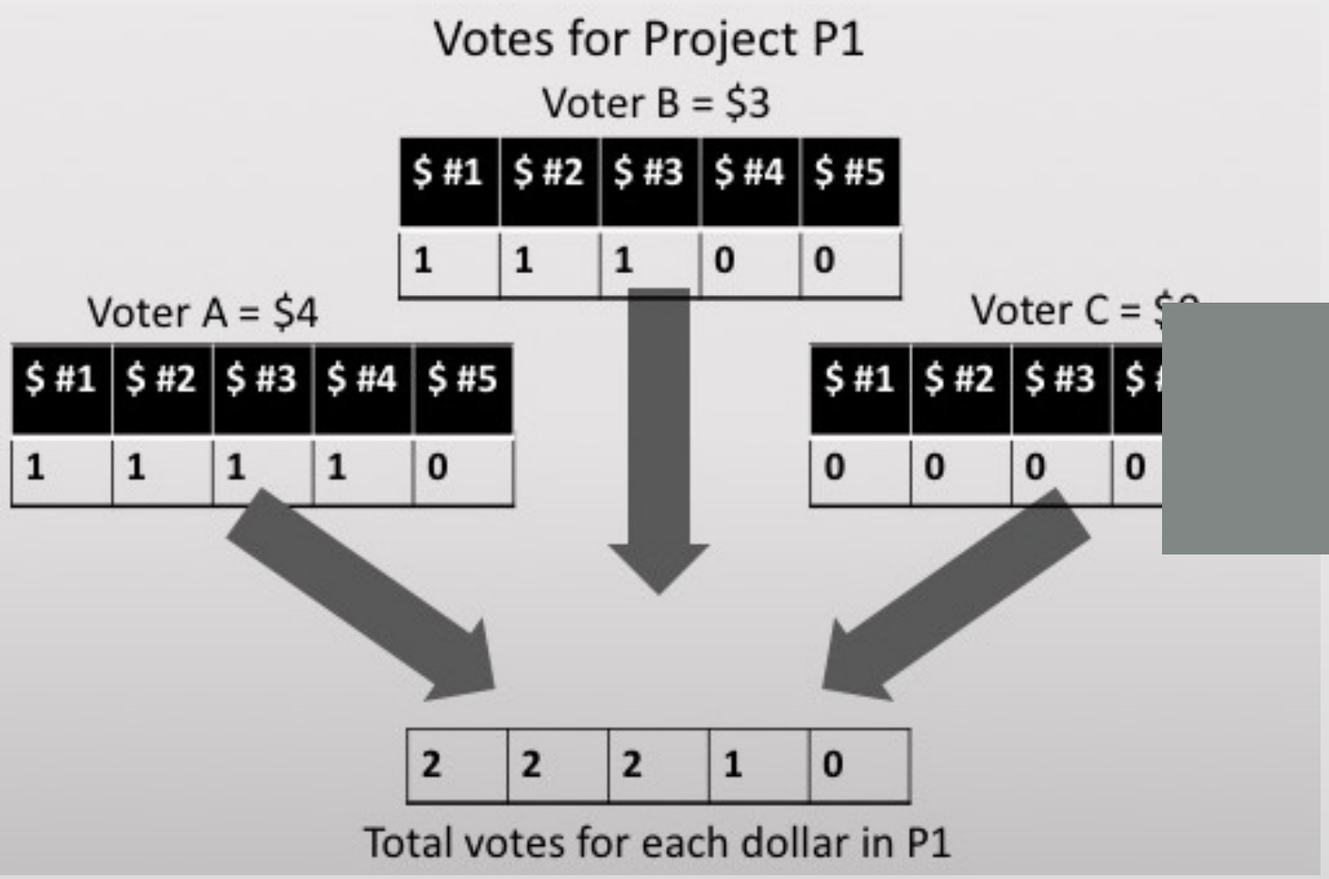
- Given budget B, the set of projects *P,* each voter *v gives*

$$\{w_p^v\}_{p \in P} \text{ such that } \sum_{p \in P} w_p^v = B$$

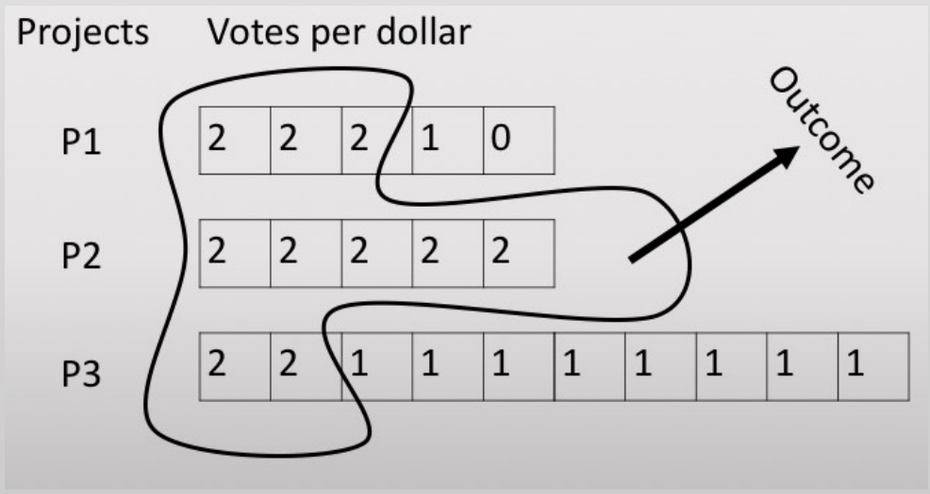
v's budget allocation

Voters	Projects		
	P1	P2	P3
A	$4	$5	$1
B	$3	$5	$2
C	$0	$0	$10

Budget = $10



- Imagine that each project is broken down into many sub-projects of $1 each.
  If a person puts $\$x$ on a project, pretend that is she is casting one vote each for the first x sub-projects.



- Calculate the total number of votes for each dollar sub-project of every project.
- Pick the B sub-projects that get the most votes.

Strategy-Proofness with overlap utilities

- Basic idea: make voters recognize trade-offs. Ask users to submit an entire budget, i.e. an entire solution to a knapsack problem
    - Assume all projects cost the same — without loss of generality. Aggregation method: choose the projects which get the most votes

- Overlap utility: If a voter wants allocation **x** for a budget item and the final chosen budget has an allocation **y** for that item, then the voter's utility from that item is **min{x, y}**
    - Truth-telling is a dominant strategy: A voter can not increase her utility by misreporting her preferred budget

Other Incentive Properties

- Under budget constraints, maximizing overlap utility is equivalent to minimizing the L_1 norm between the chosen budget and the original budget
    - Hence, also truthful if the penalty perceived by a user is the total disagreement between her ideal budget and the final chosen budget
- Under arbitrary additive utilities over (divisible) projects, the best response of a user to everyone else's vote is weakly strategy-proof: If A is winning, and I want A to win, I will put A in my knapsack solution

Knapsack voting for divisible projects

$1 million

50 $200K

$600K 130

$150K 40

35 $70K

60 $250K

$300K 70

Optimum solution to knapsack problem: choose projects in decreasing order of "value for money"

Our comparison based interface as deployed in Chicago

Participatory Budgeting 49th Ward

About Español

Which project gives the best value for the money? [1 / 8]

Shared Bike Lanes on Clark from Howard to Albion

Install permanently marked shared bike lanes in both directions

Estimated cost: $90,000

This Project

New Water Feature at Pottawattomie Park

Replace the old, often non-functioning feature with a new feature with more sprays and a more durable infrastructure.

Estimated cost: $75,000

This Project

I don't know

From Comparisons to Knapsack

- Assume every pair of projects (a,b) gets compared the same number of times. Let w(a,b) be the number of times a beats b.

- **Rank aggregation:** Assume there is a true societal ranking R. If a < b in R, then Prob[a beats b in a comparison] = p > 1/2

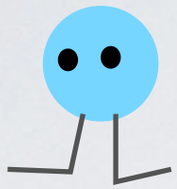
  - The problem of finding the Maximum Likelihood Estimator R given observed w(a,b) is **NP-Hard**

- **Knapsack aggregation:** Assume there is a true knapsack solution S. If a is in S and b is not, then Prob[a beats b in comparison] = p > 1/2; Prob [b beats a in a comparison] = 1-p. Else, the probability is 1/2.

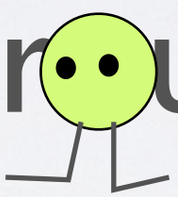
  - **Theorem:** Choosing the B projects that win the most comparisons is the MLE for the observed w(a,b), without any need to know p

Future Directions

- Joint interface for soliciting proposals and voting
- From ordinal approximations to cardinal approximations? Powerful new direction in social choice theory [Anshelevich et al. 2015]
- Empirical evaluation: good aggregation? good long-term decisions?
- More unstructured settings?



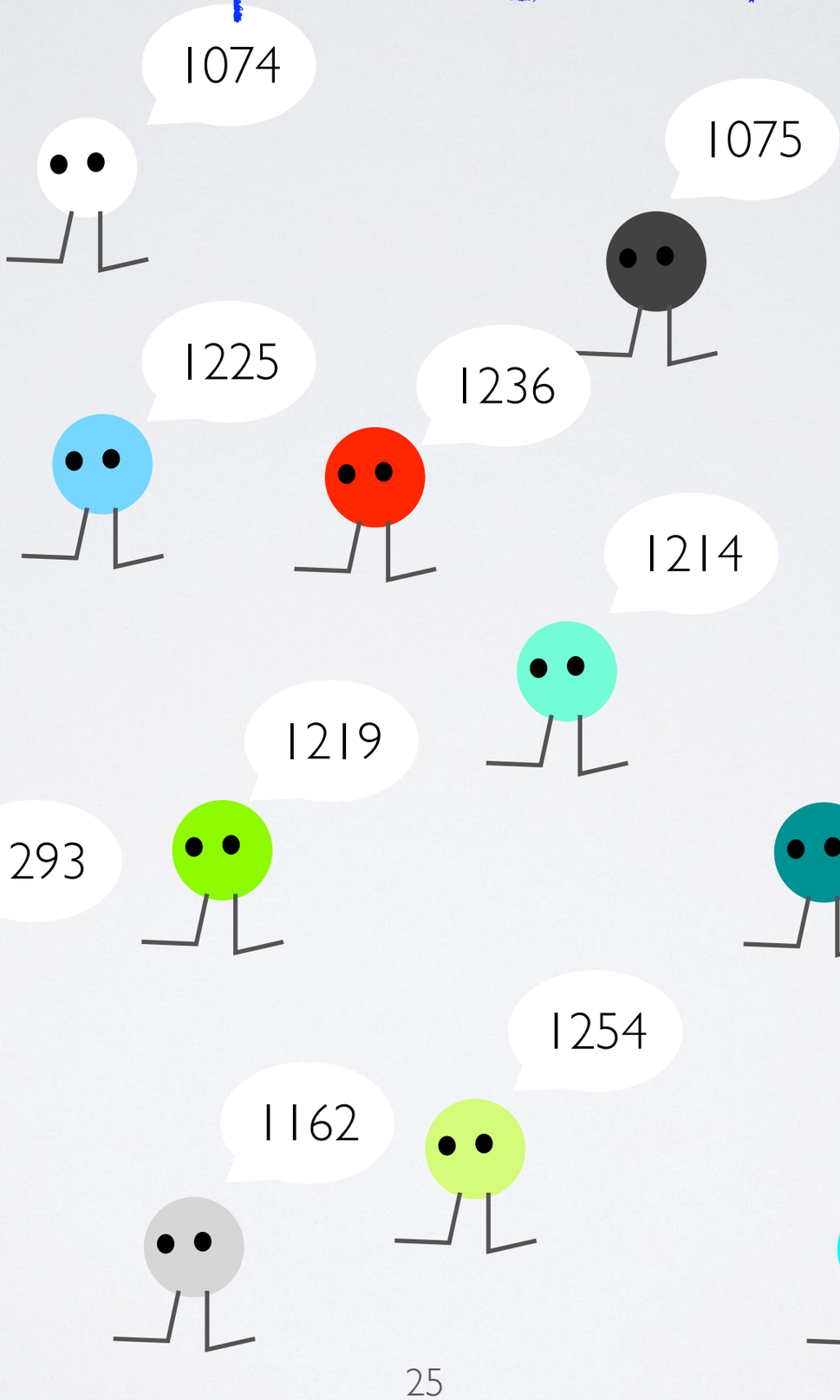
Large-Scale Decision-Making via Small Group Interactions:

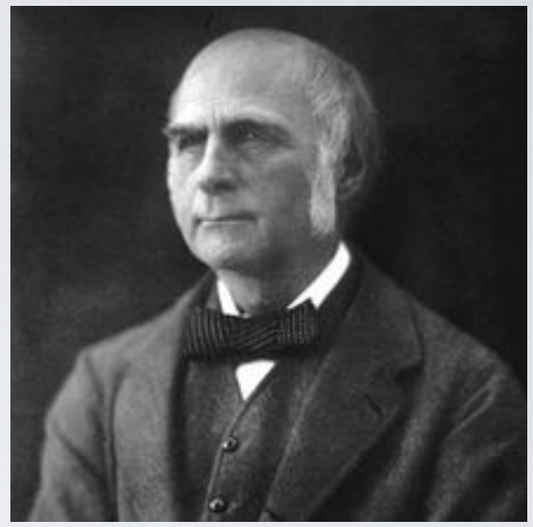


The Importance of Triads

Ashish Goel
David Lee
Stanford University

Median voter theorem: Reporting true values is incentive compatible [Black, 1948]





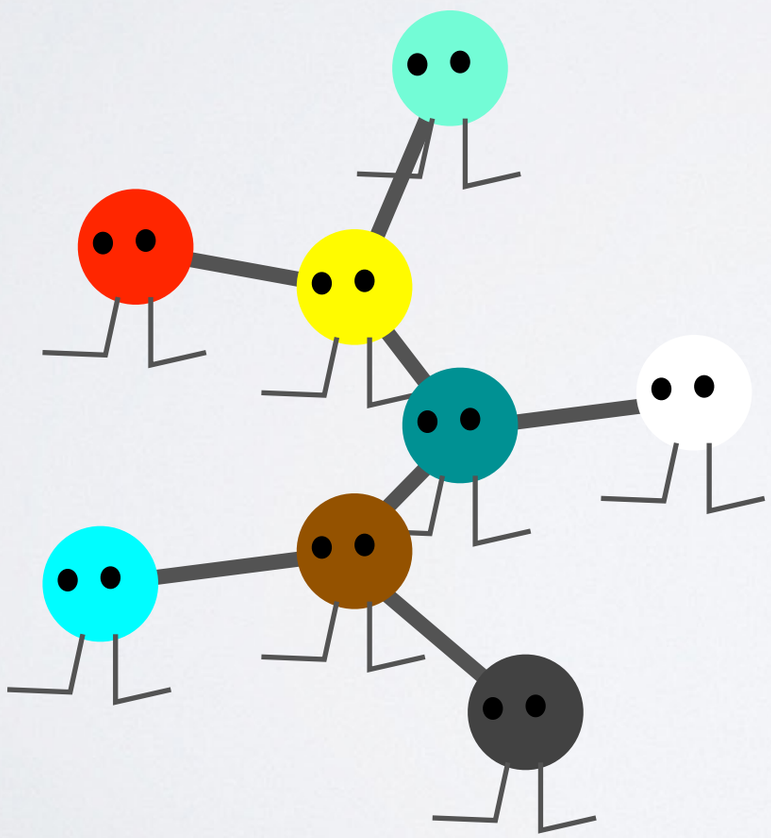
Vox Populi, Nature 1907

How can you aggregate unstructured estimates or where there is no consistent vocabulary for a voter to express her position?

The 1-median

(a.k.a the geometric median or the Fermat-Weber point)

$$m = \arg\min_{x} \sum_{j=1}^{n} d(x, x_j)$$



Related to the Condorcet winner

Wendell and McKelvey 1981

Bandelt and Barthelemy 1984

Saban and Stier-Moses 2012

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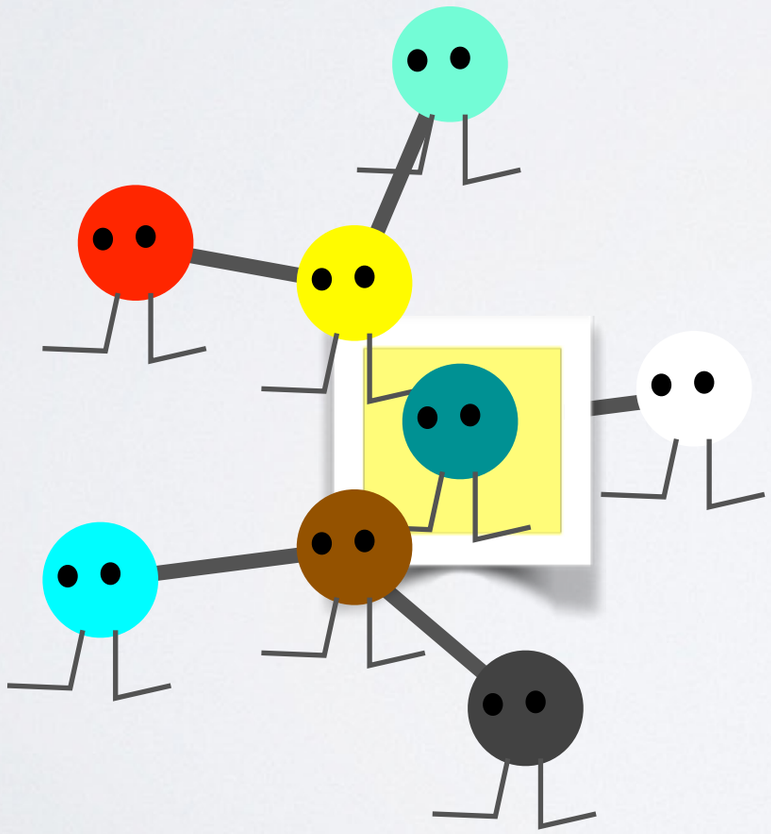

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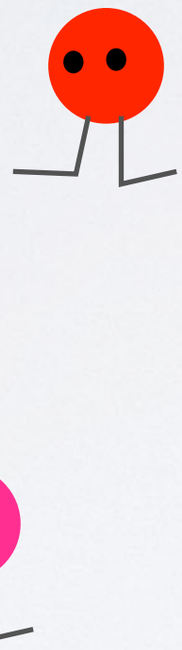
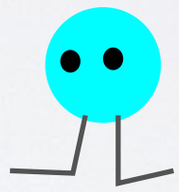
Triadic Interaction

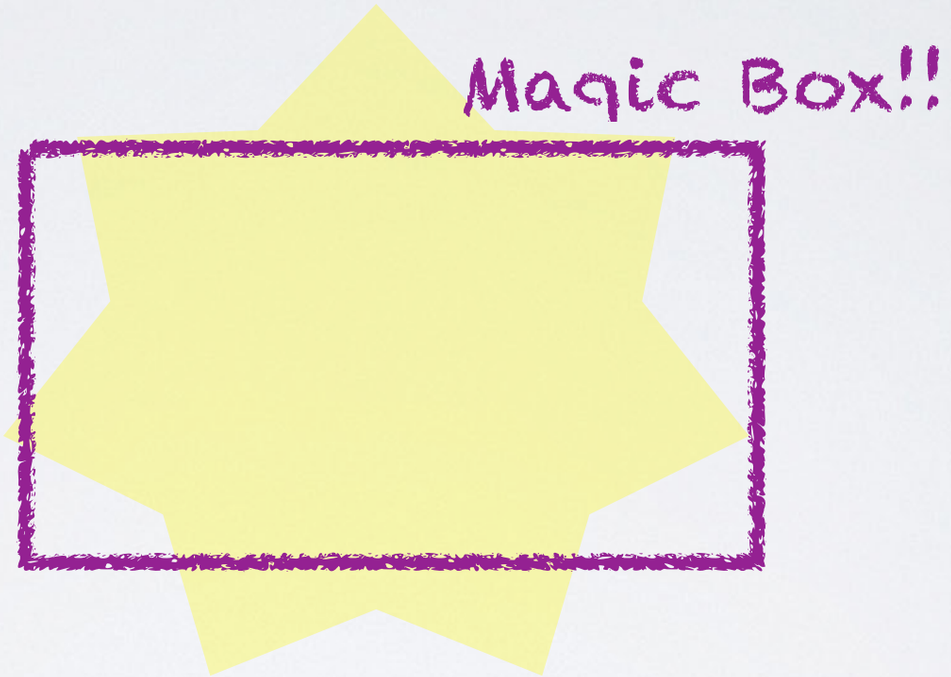




Consensus

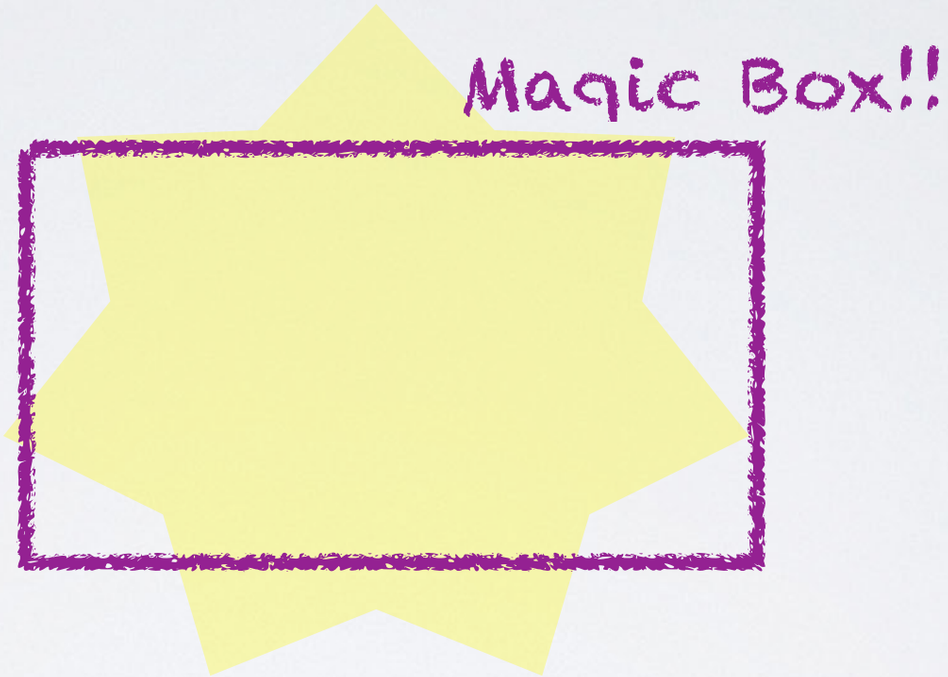
Triadic Interaction

Triadic Interaction



Suppose that triads could find their 1-median.

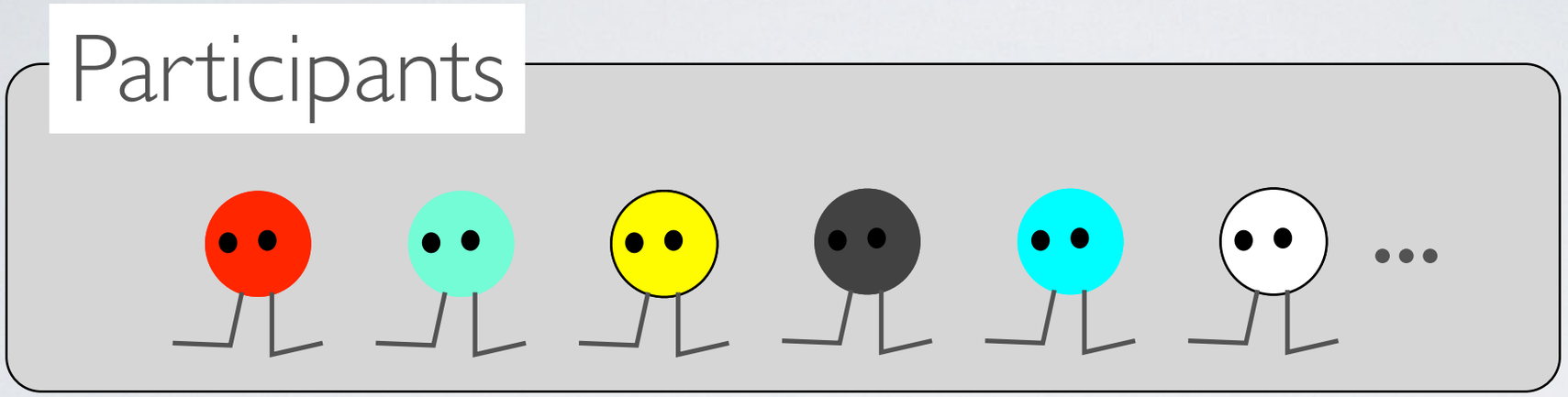
Triadic Interaction

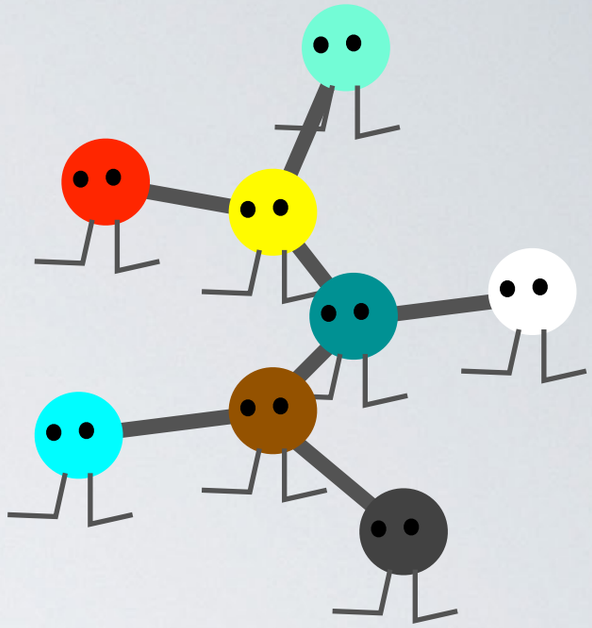


Suppose that triads could
find their 1-median.

(On the line, each member just votes among the other two – back to comparisons!)

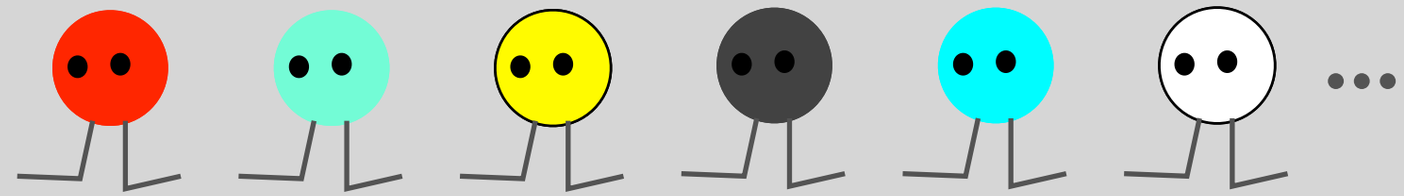
Triadic Consensus

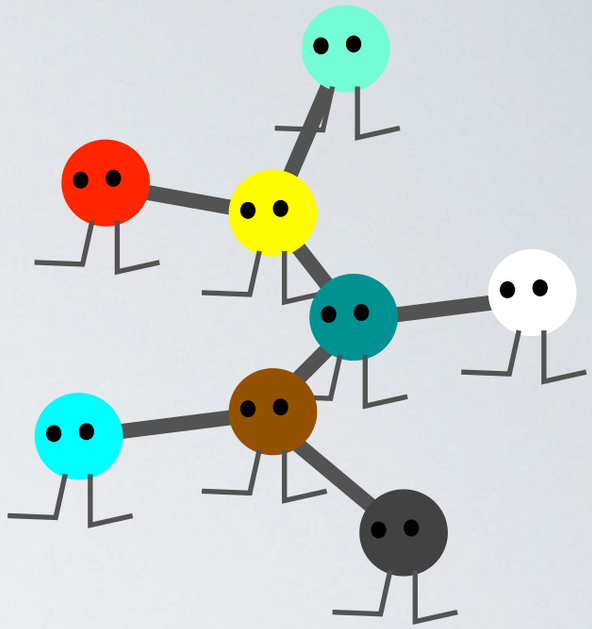




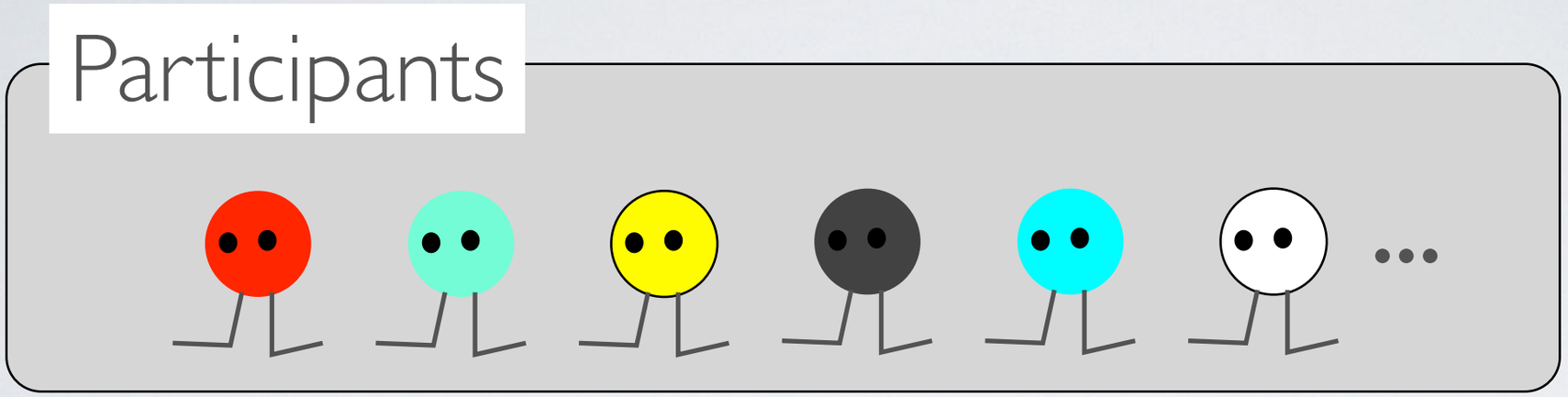
Triadic Consensus

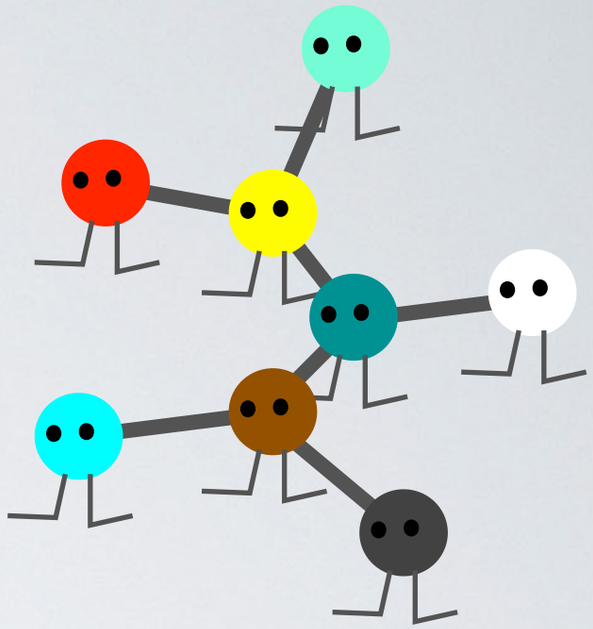
Participants

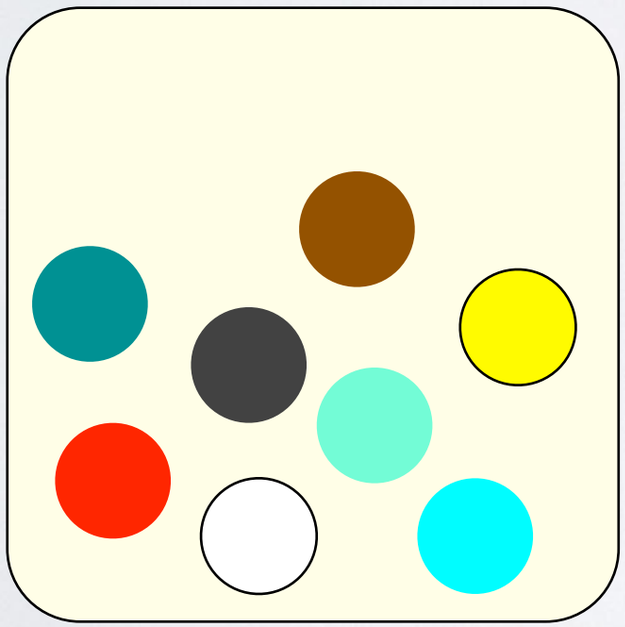




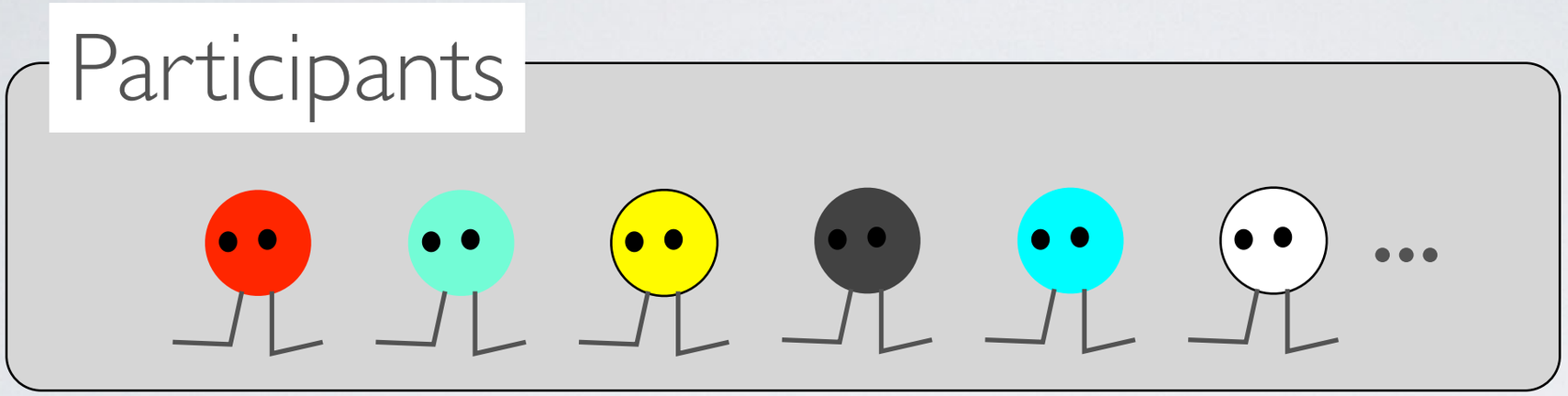
Triadic Consensus

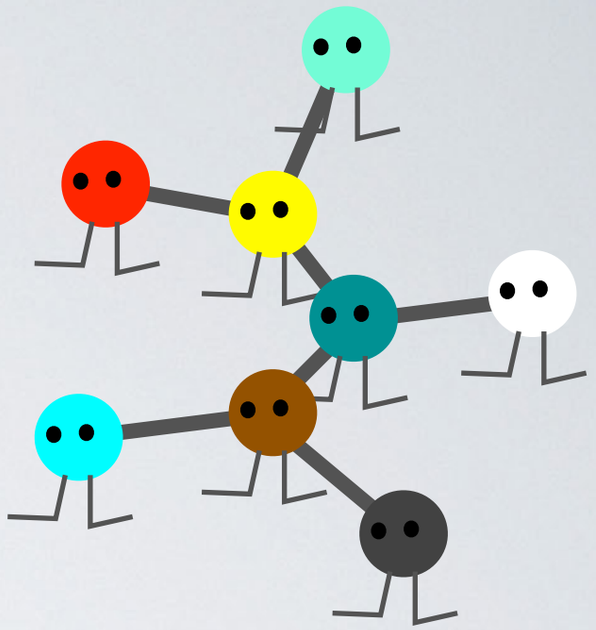


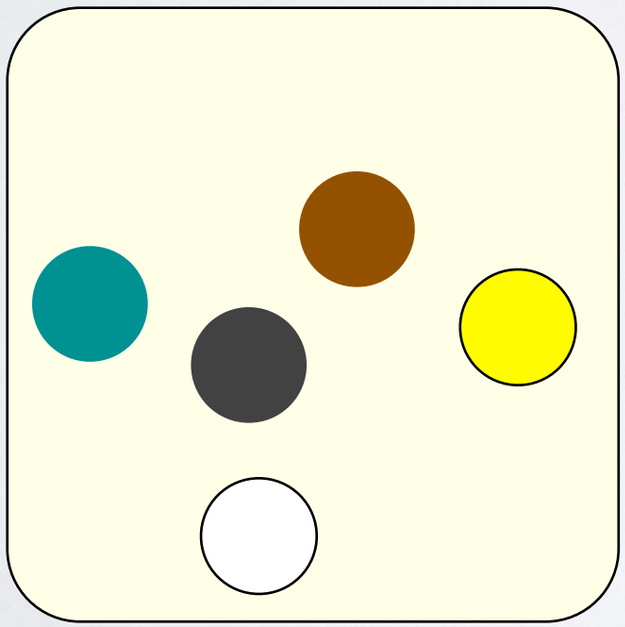


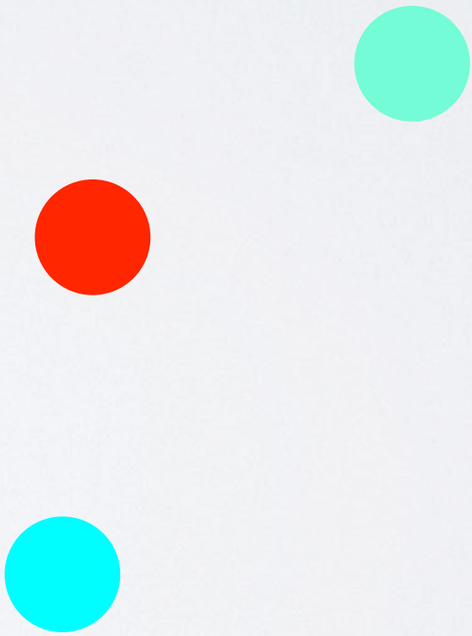


Triadic Consensus









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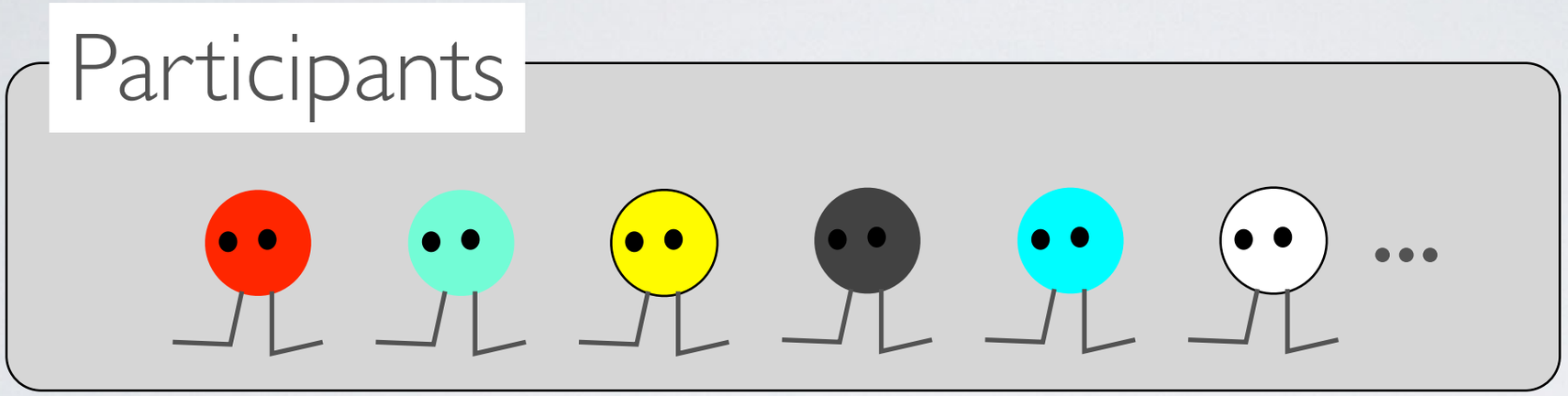

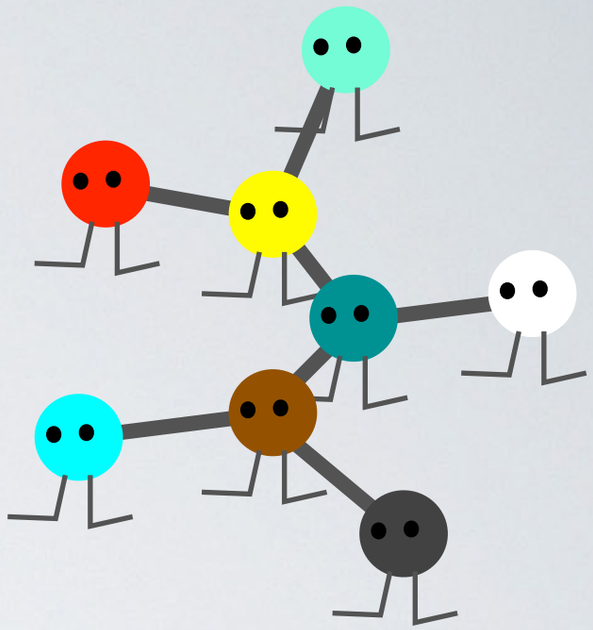

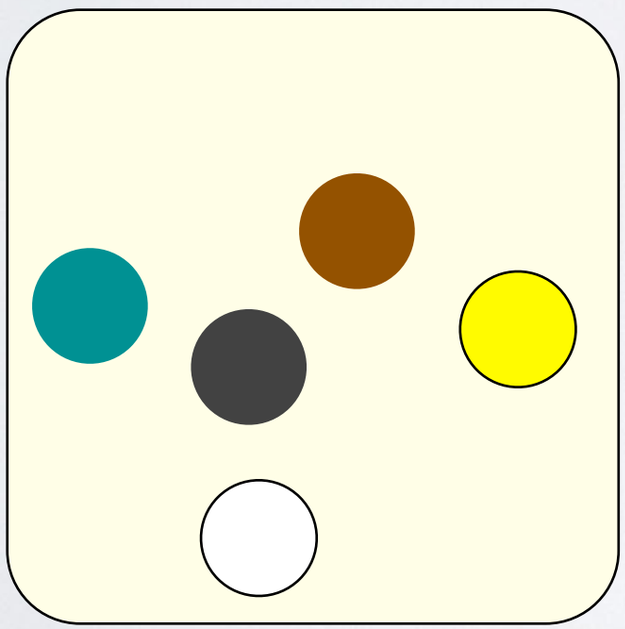

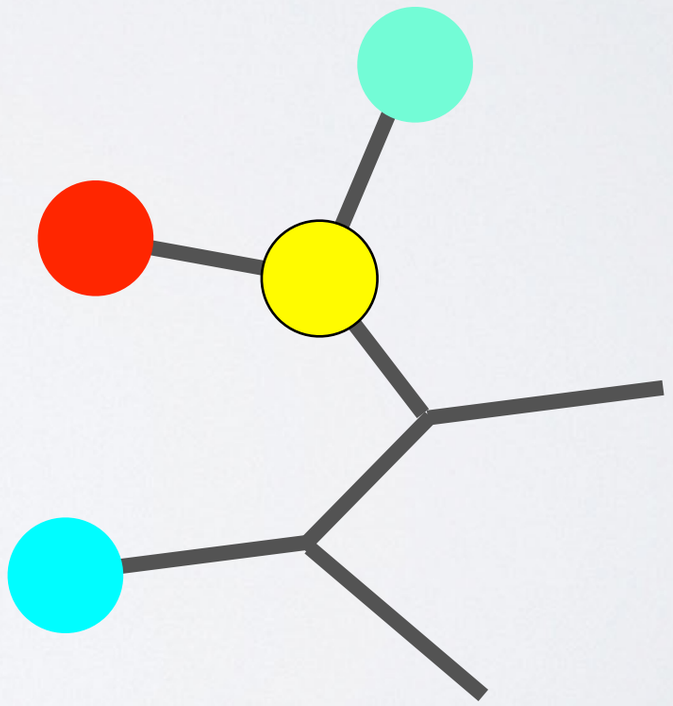

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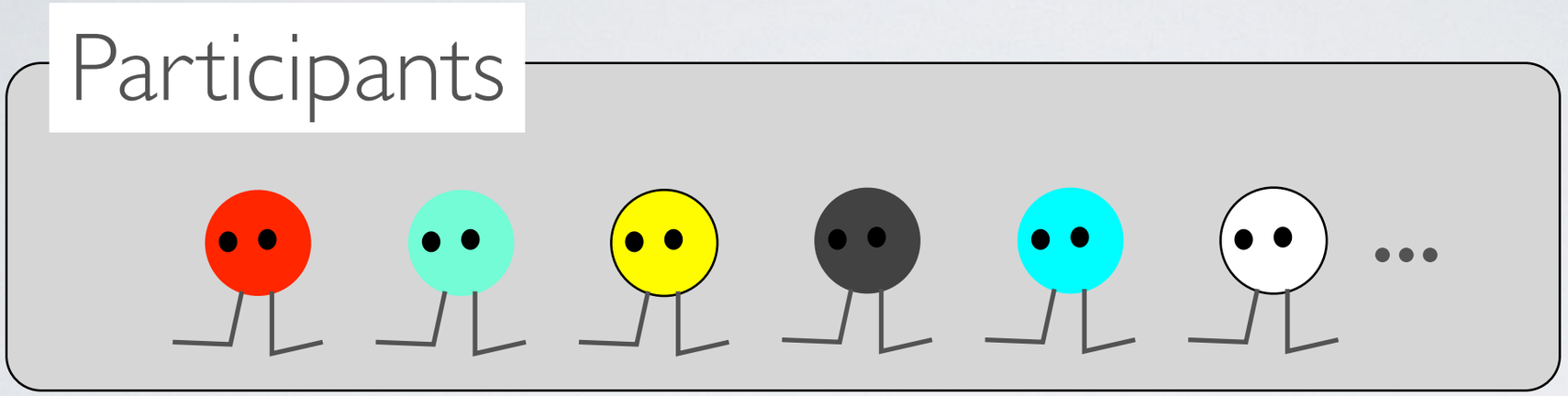

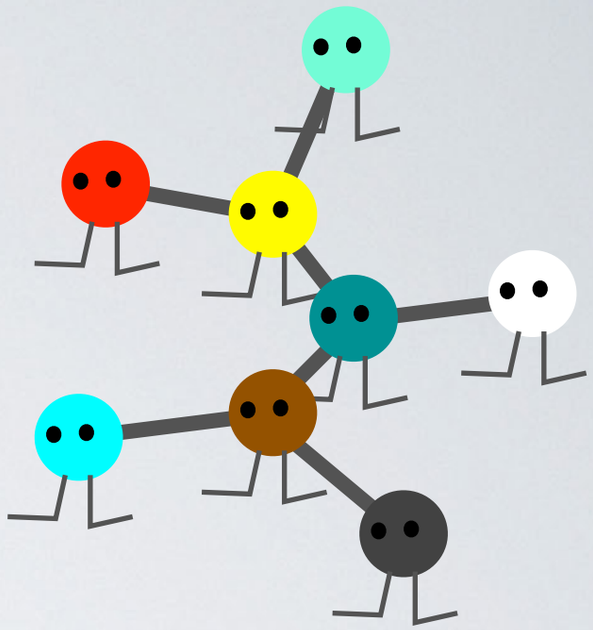

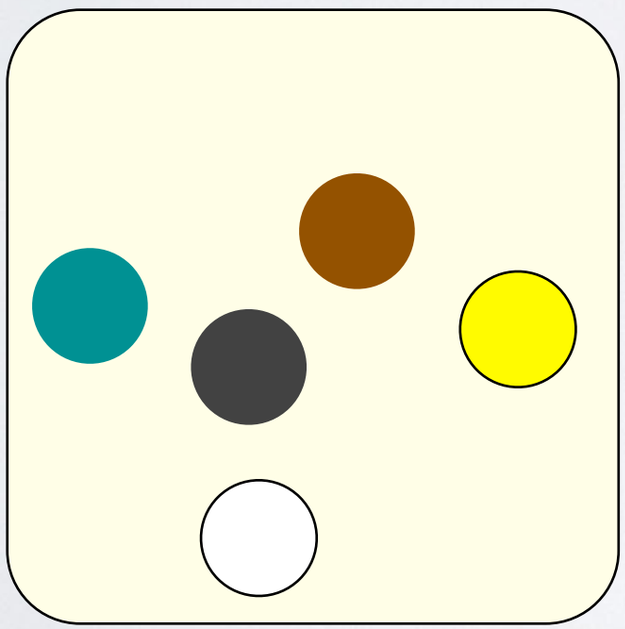

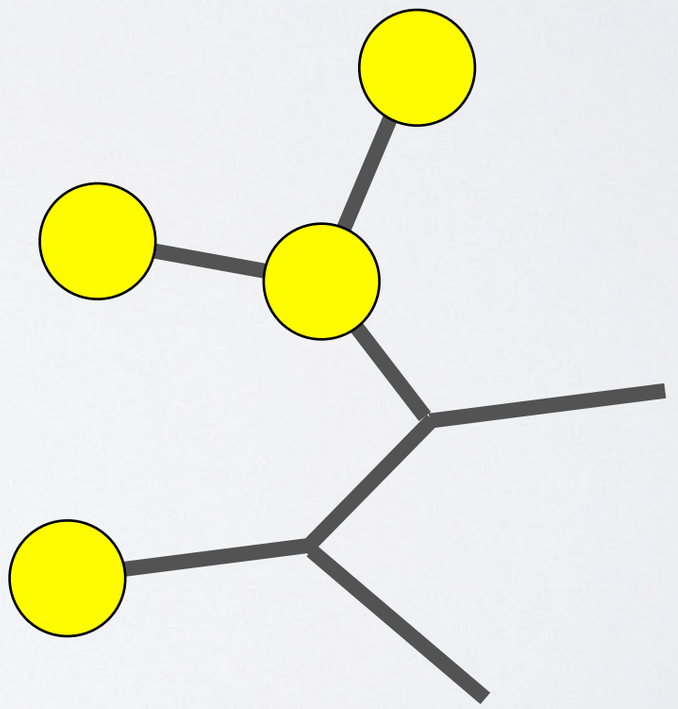

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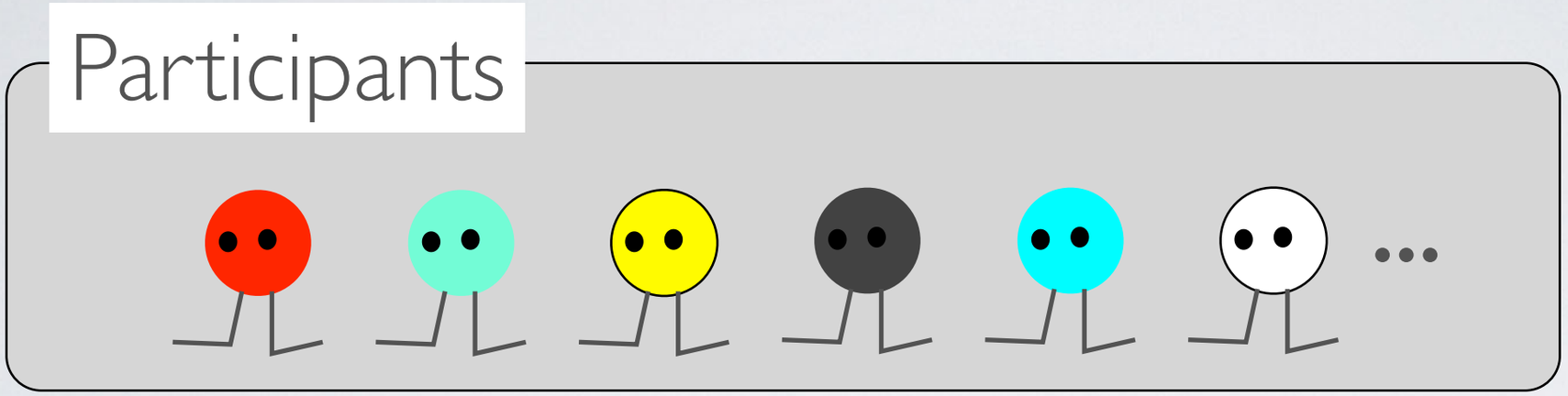

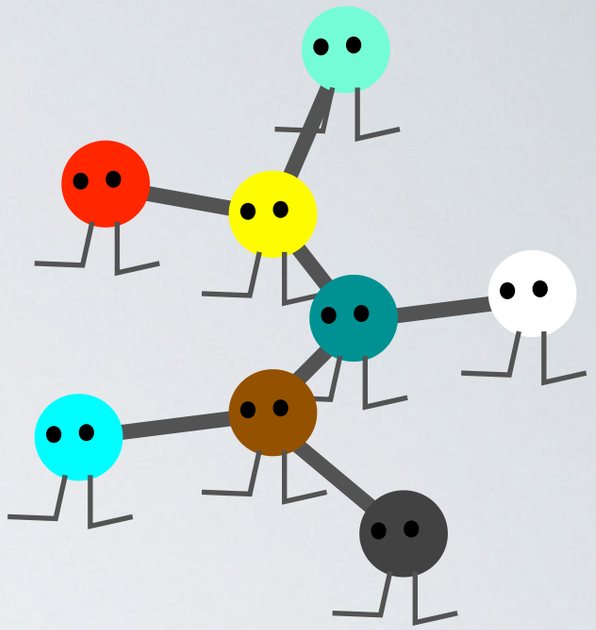

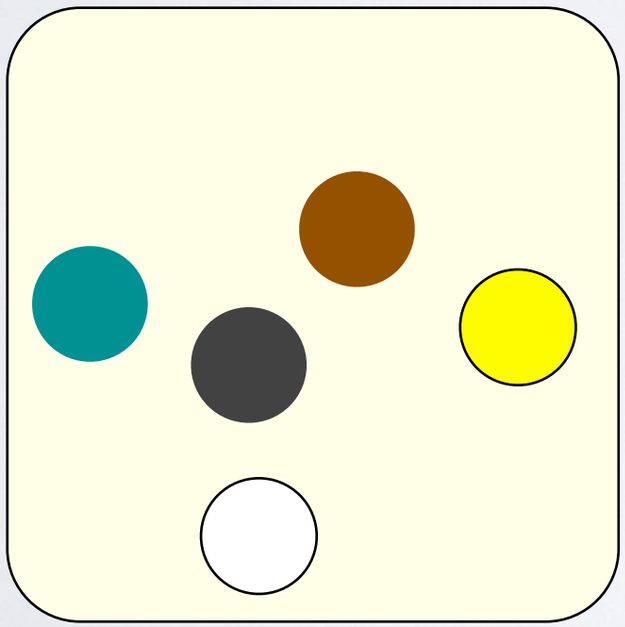

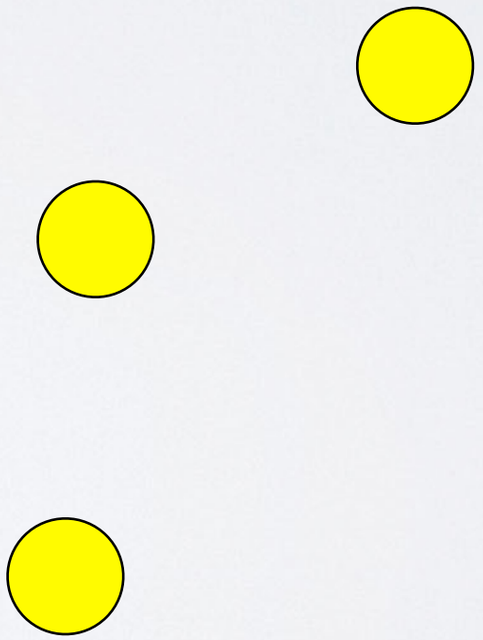

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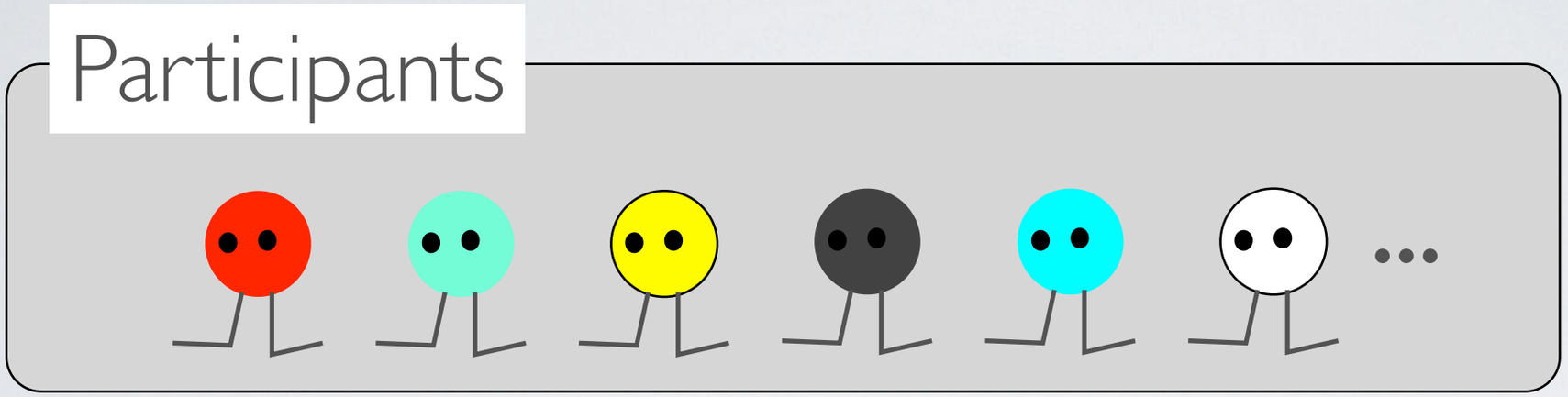
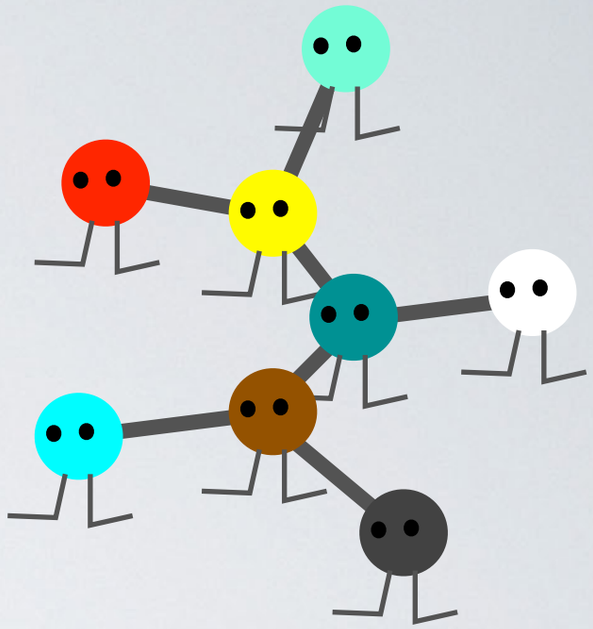
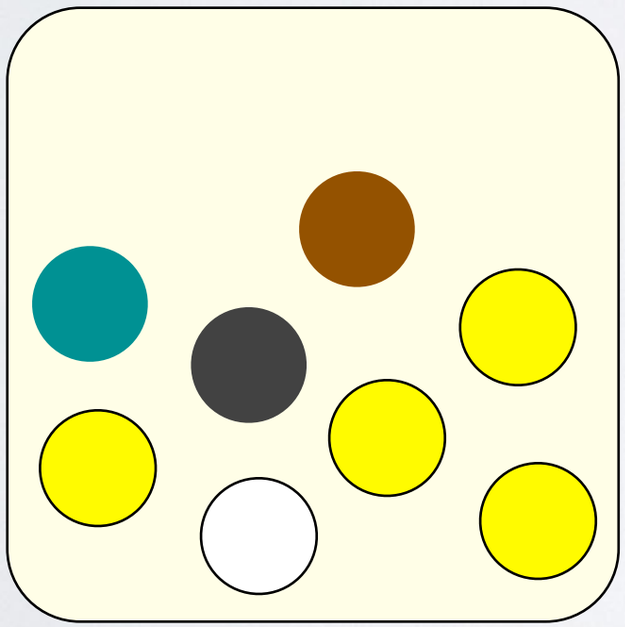
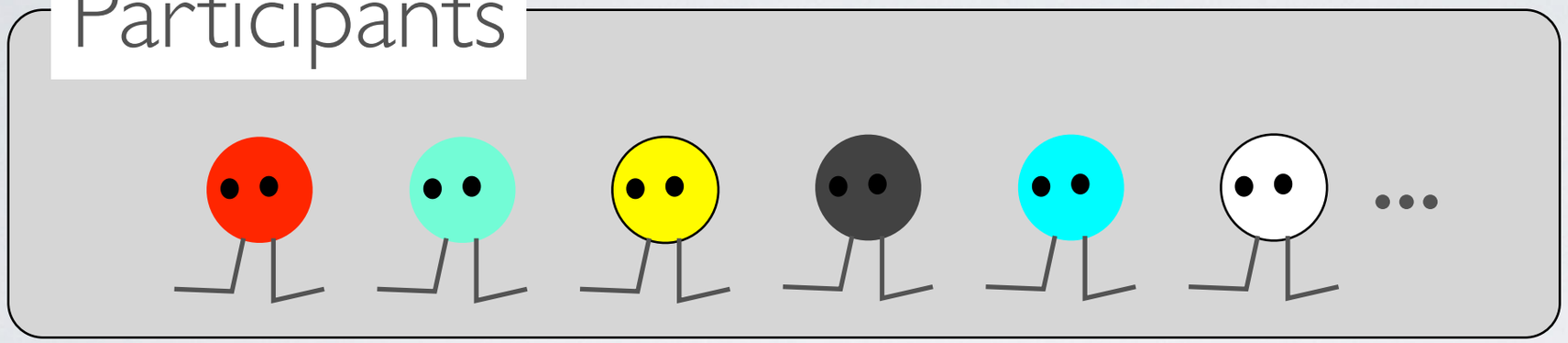


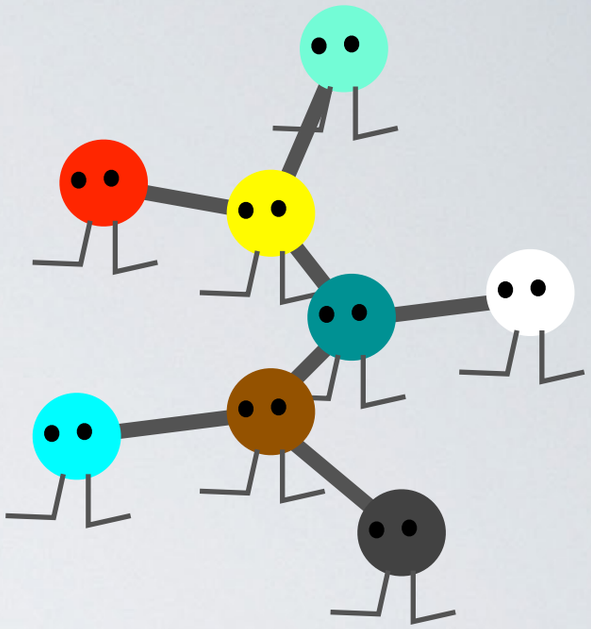


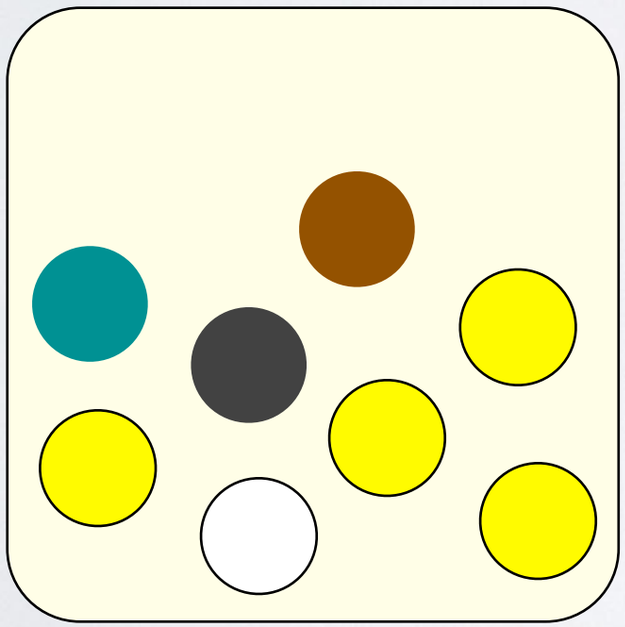


Triadic Consensus

Participants







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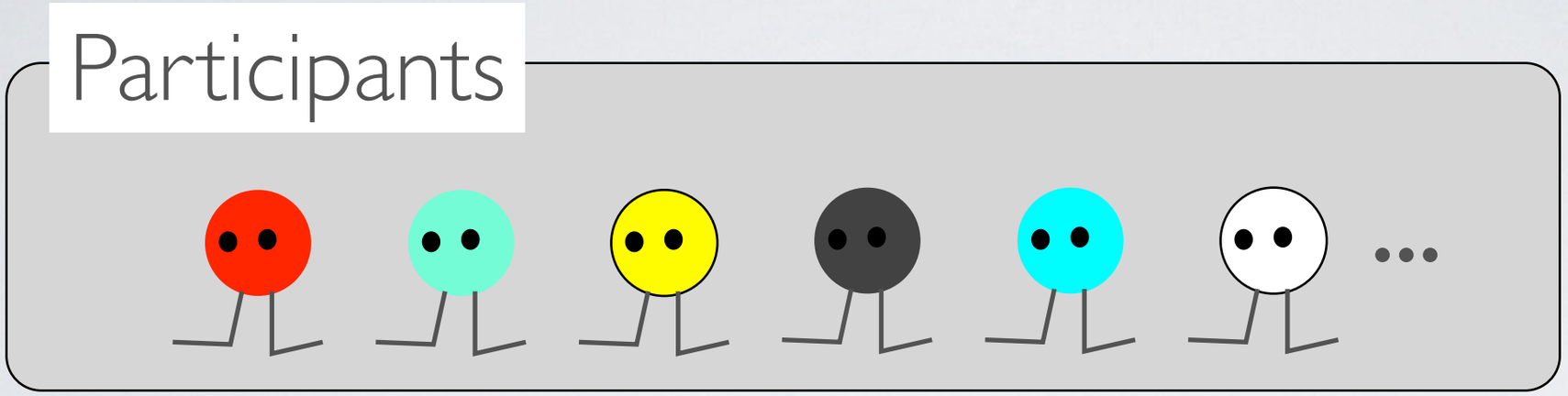

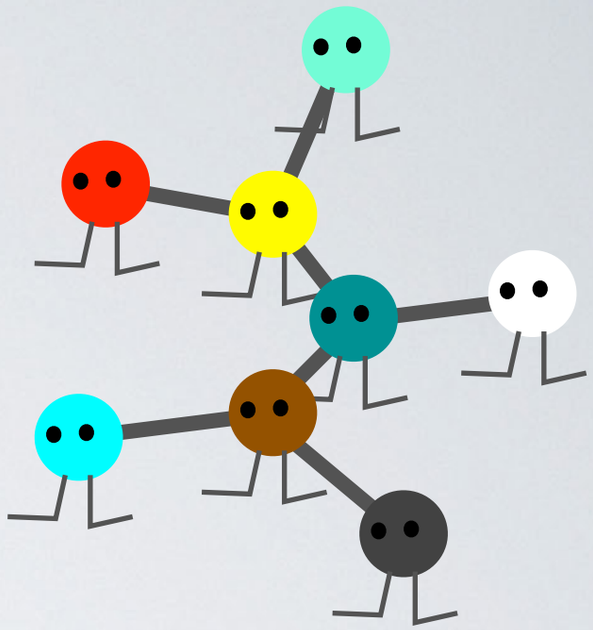

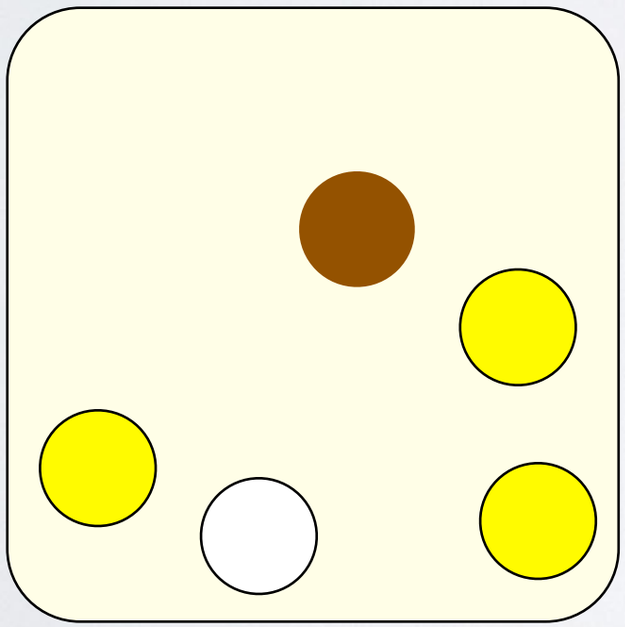

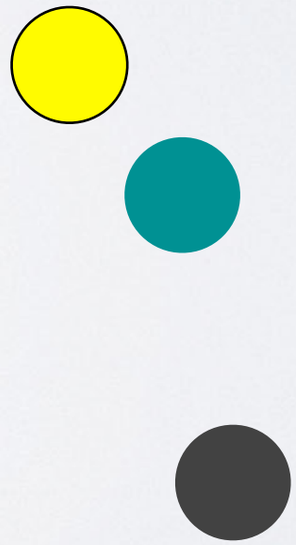

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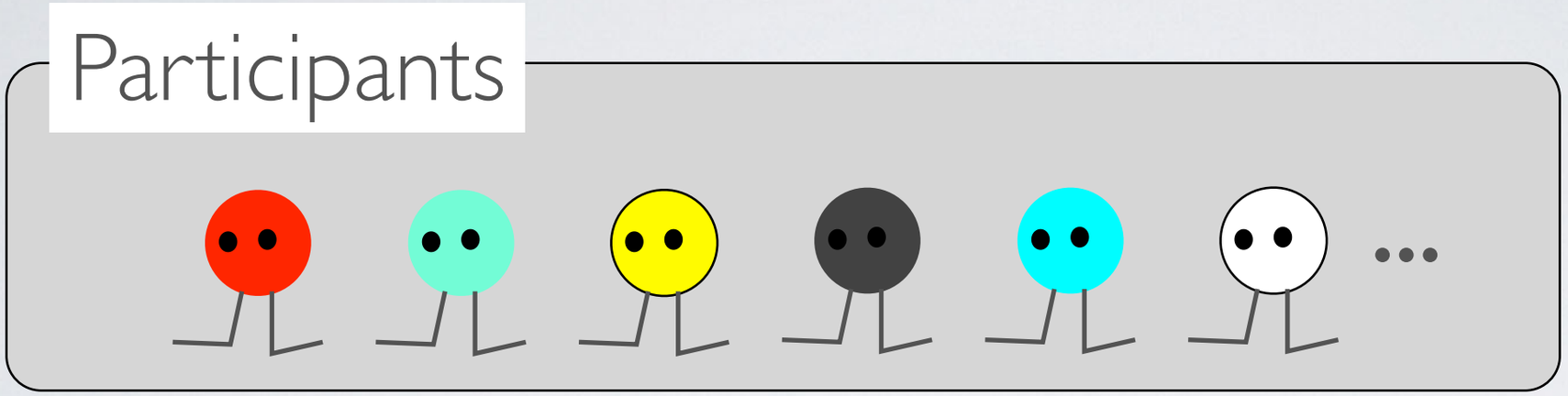

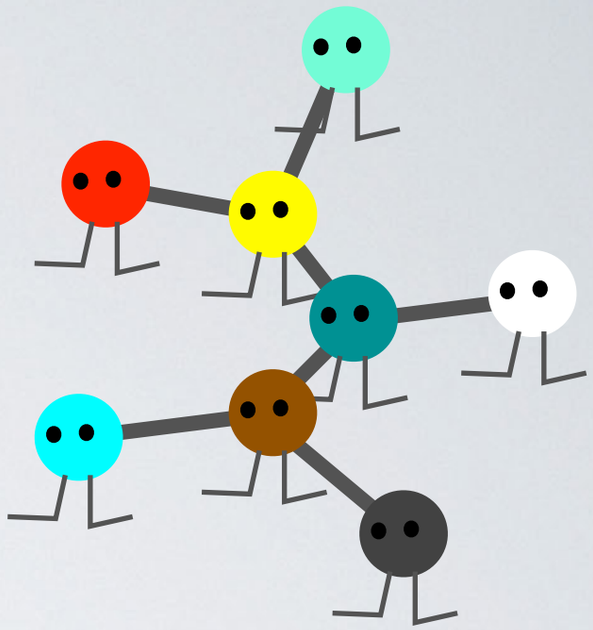

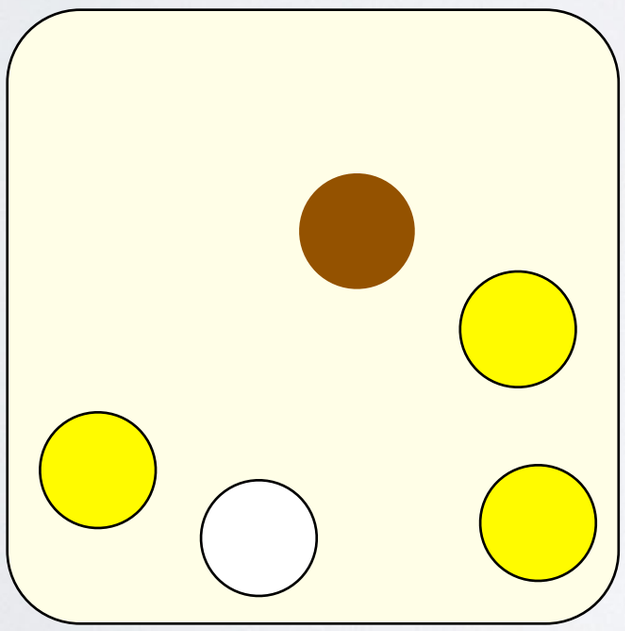

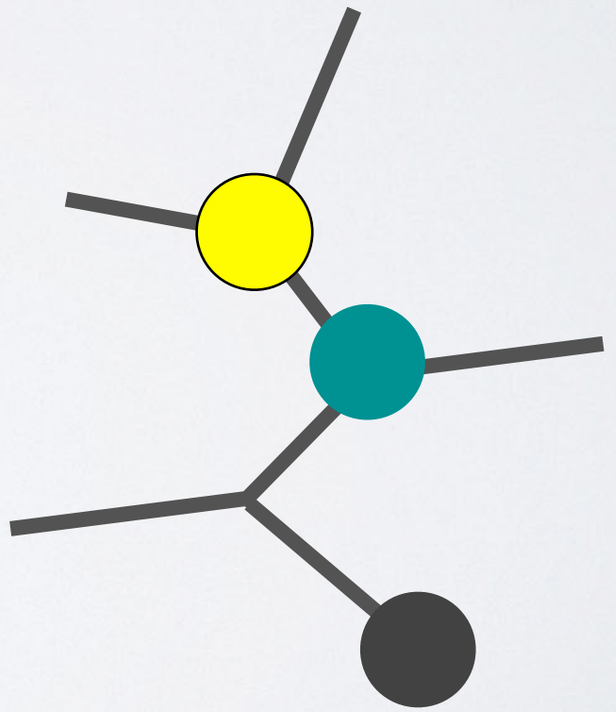

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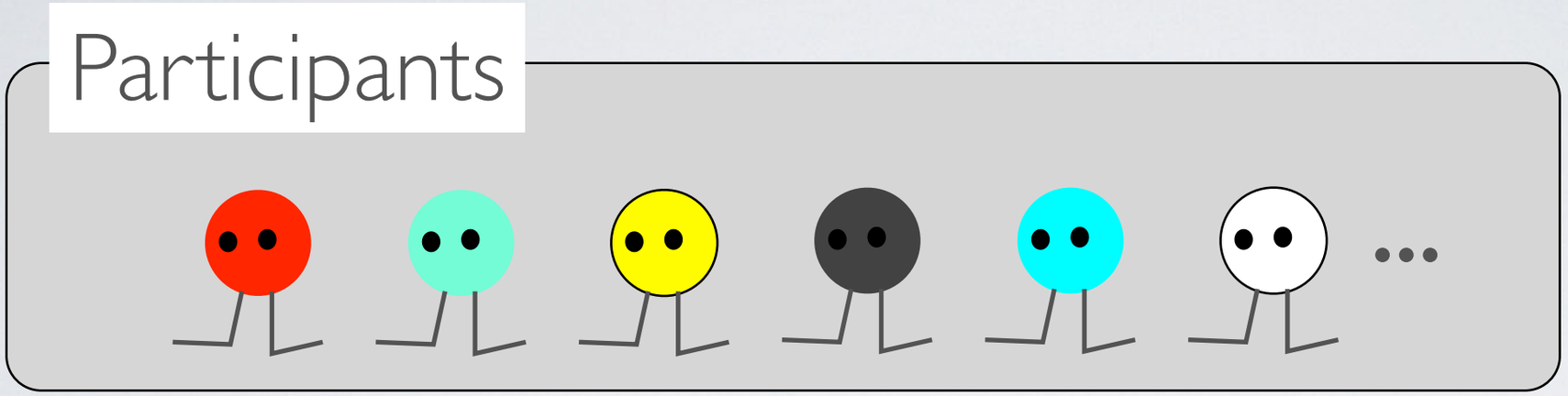

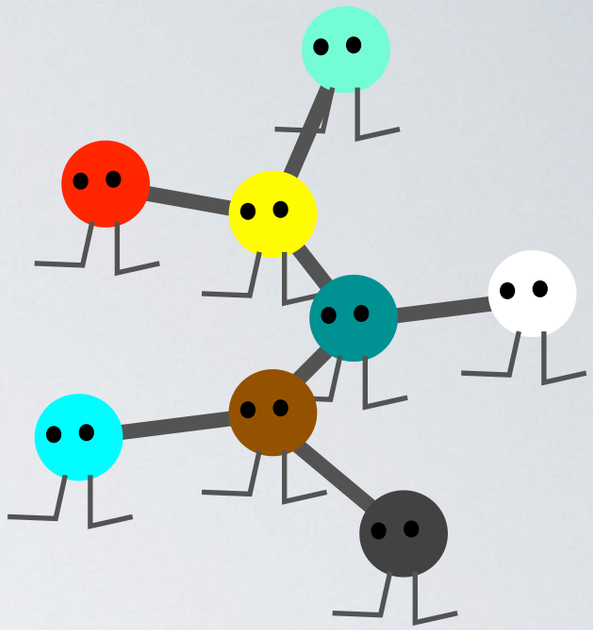

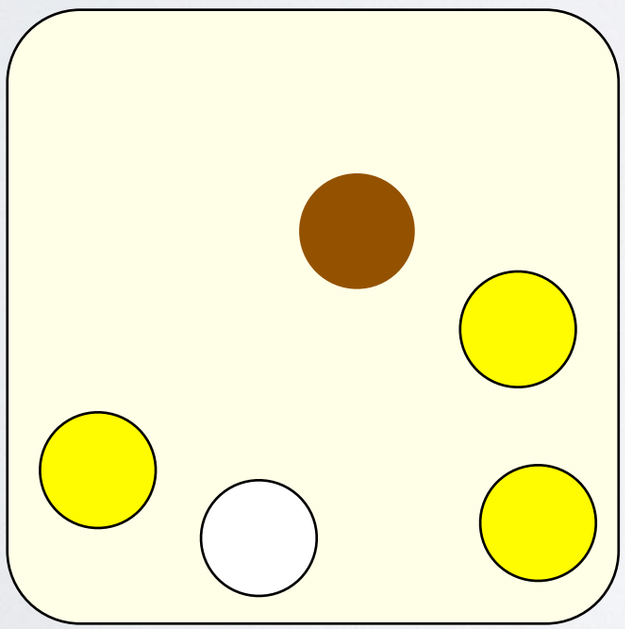

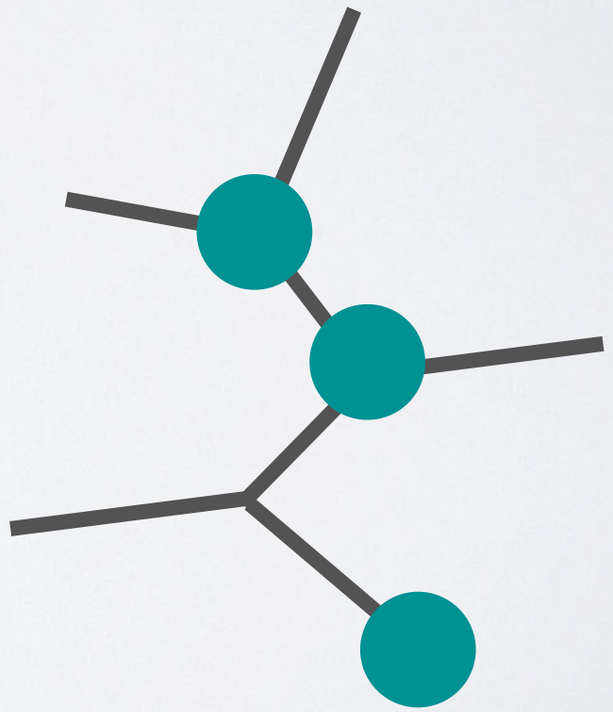

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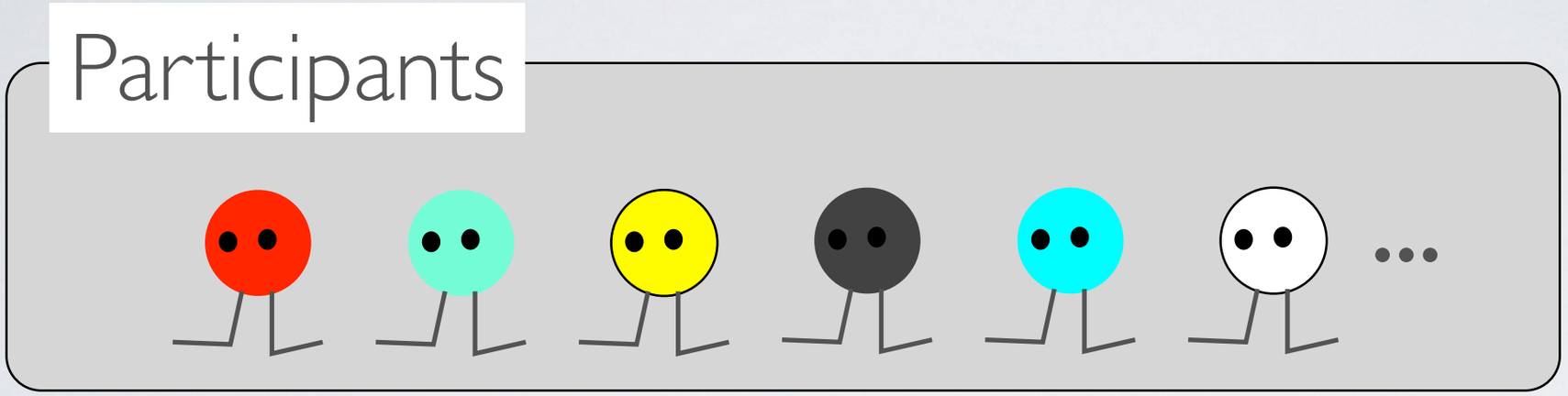

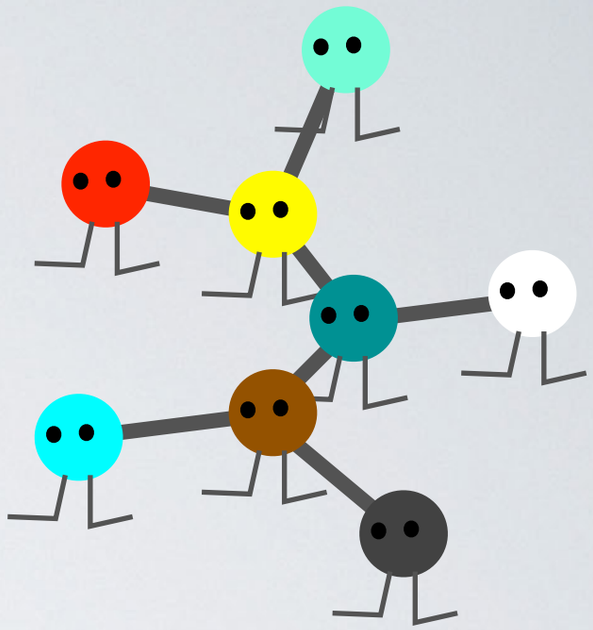

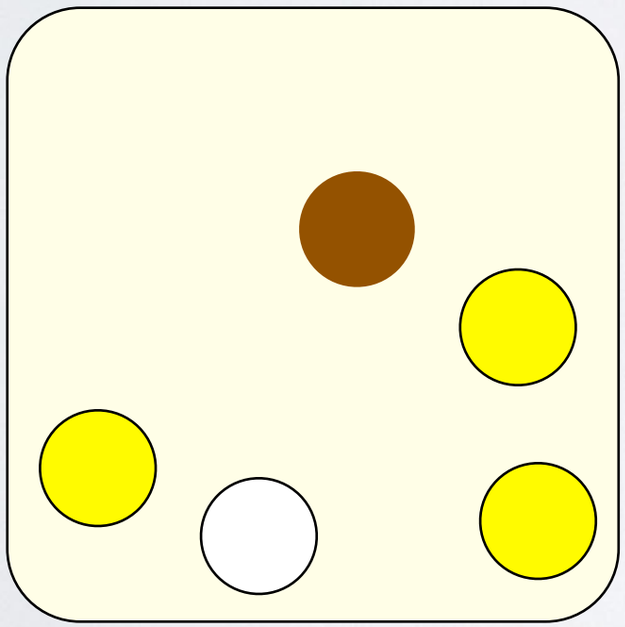

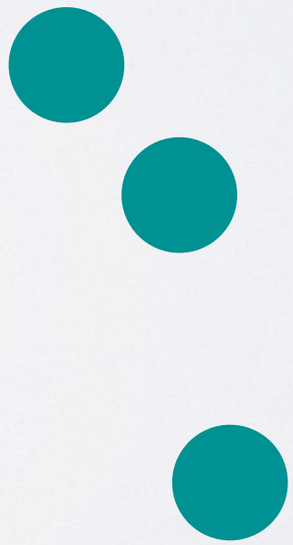

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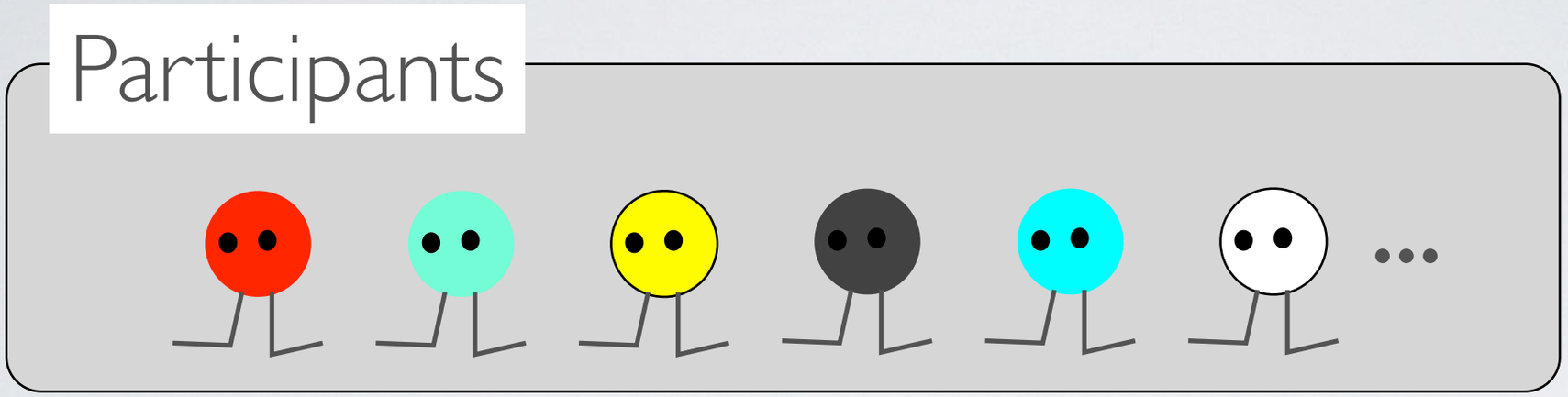

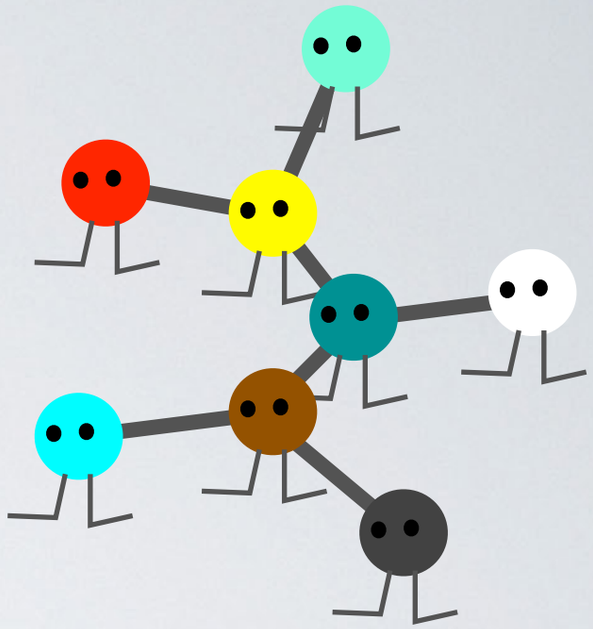

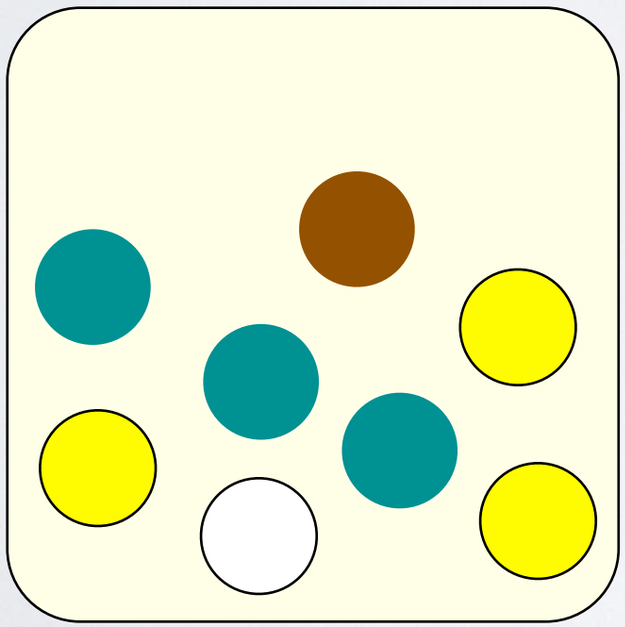

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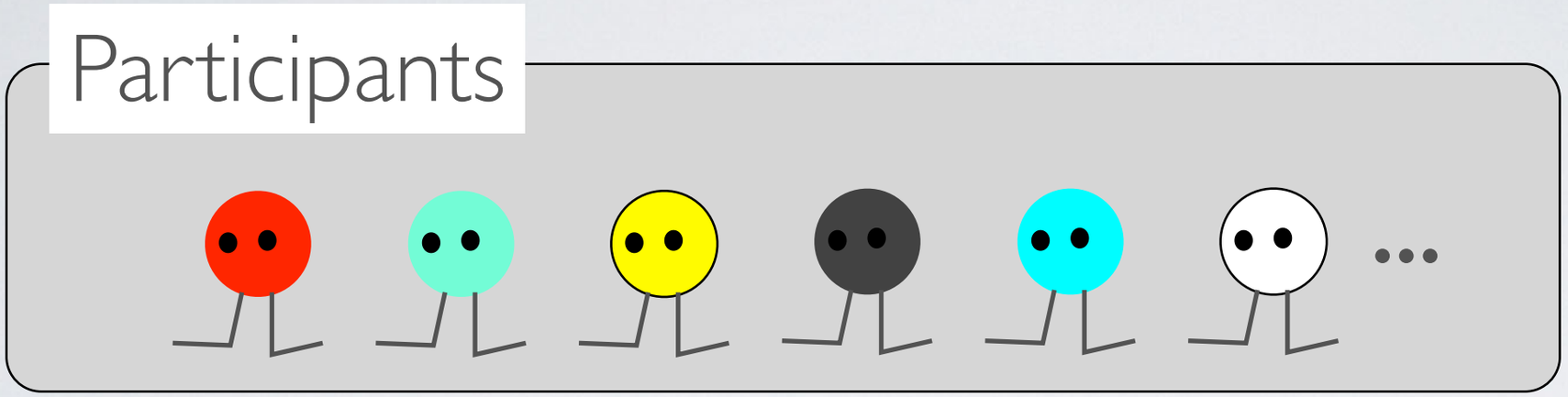

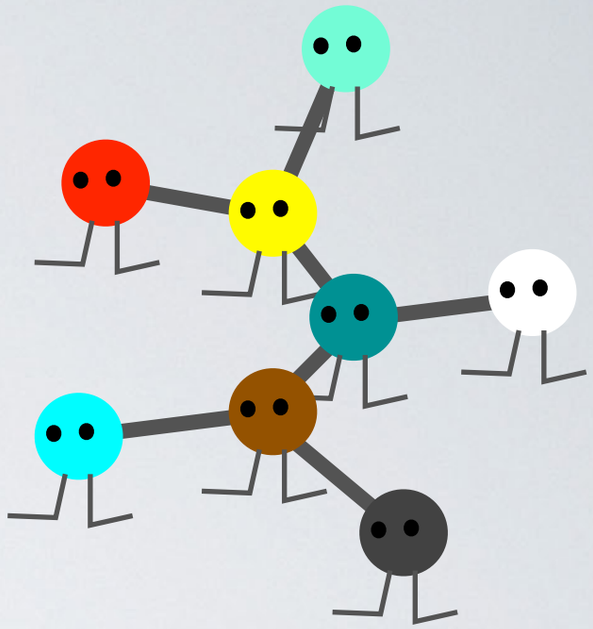

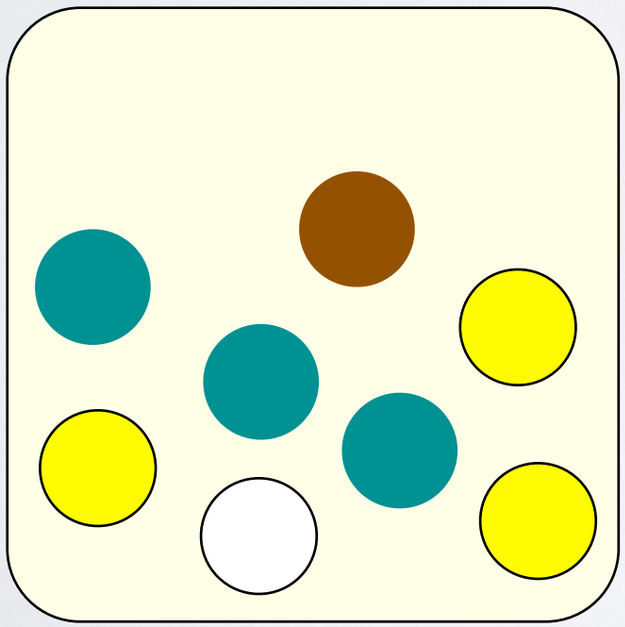

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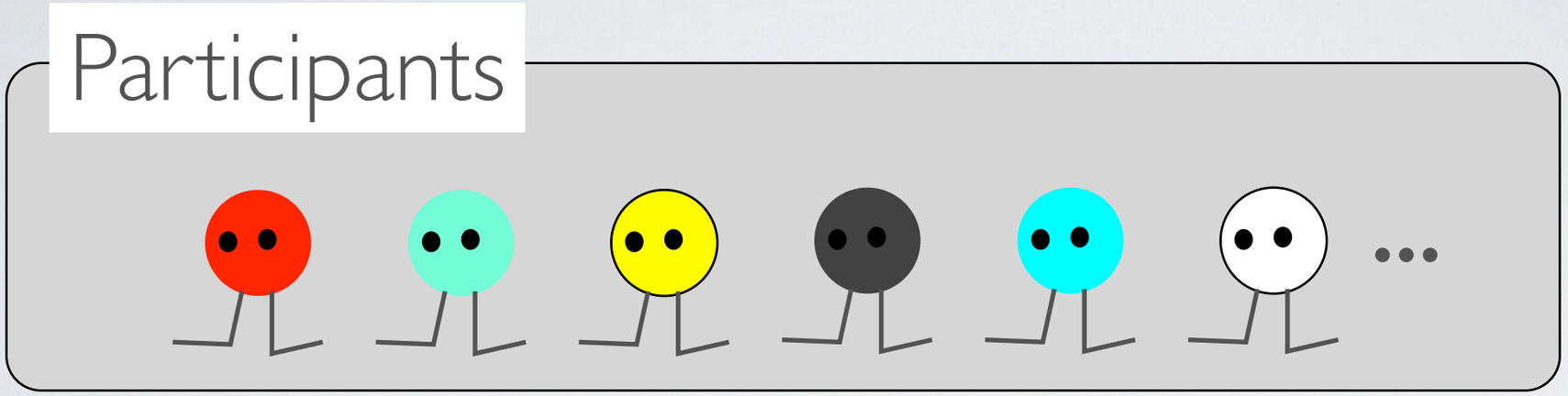

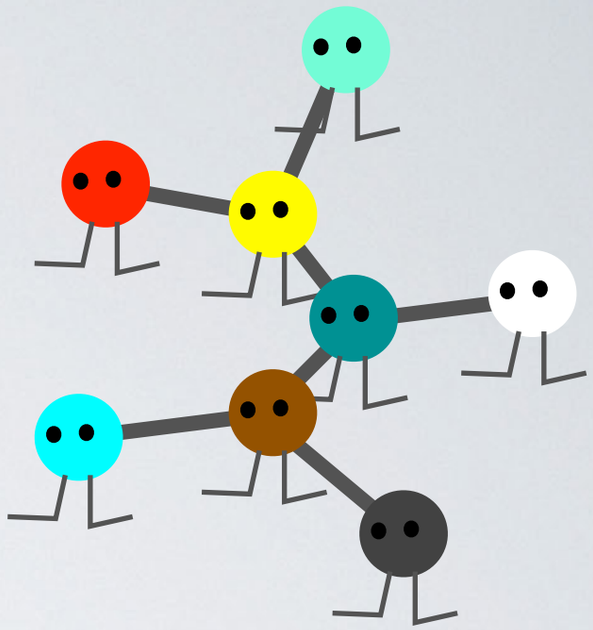

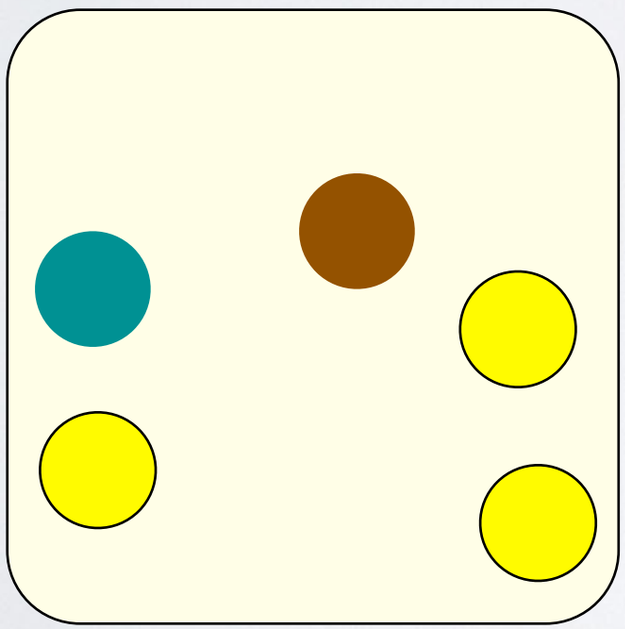

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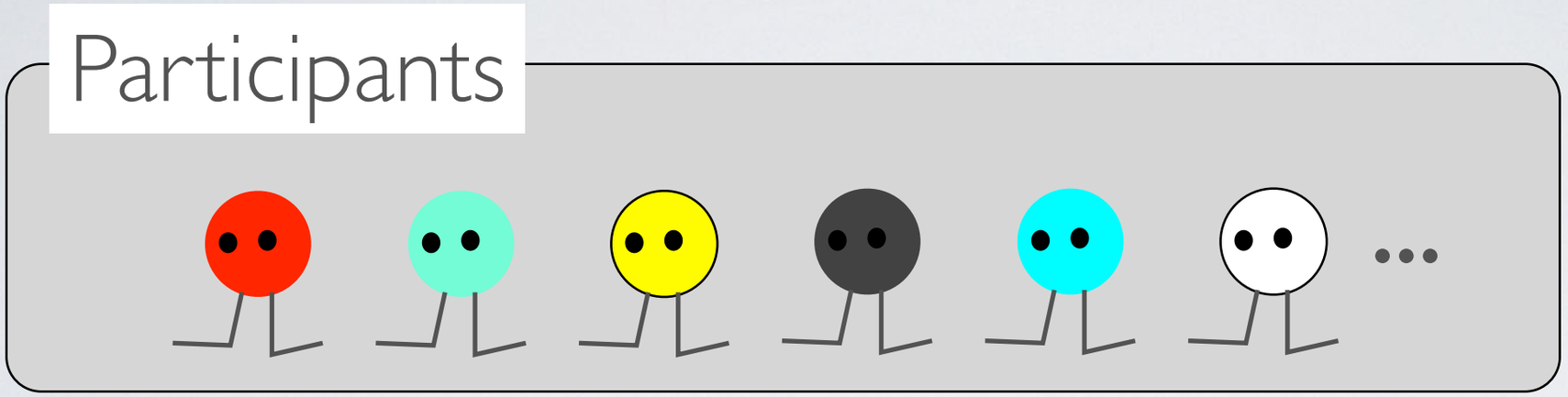

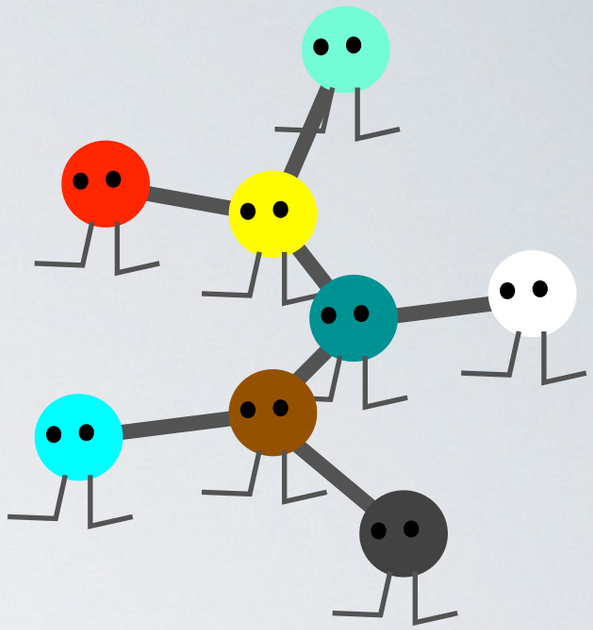

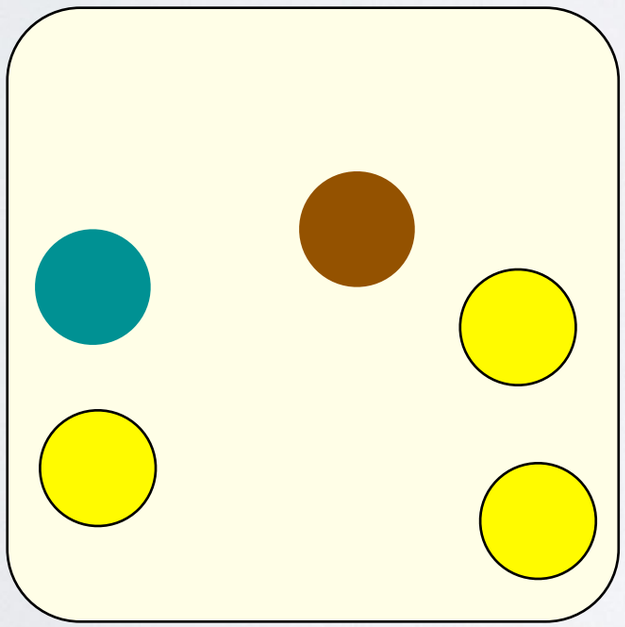

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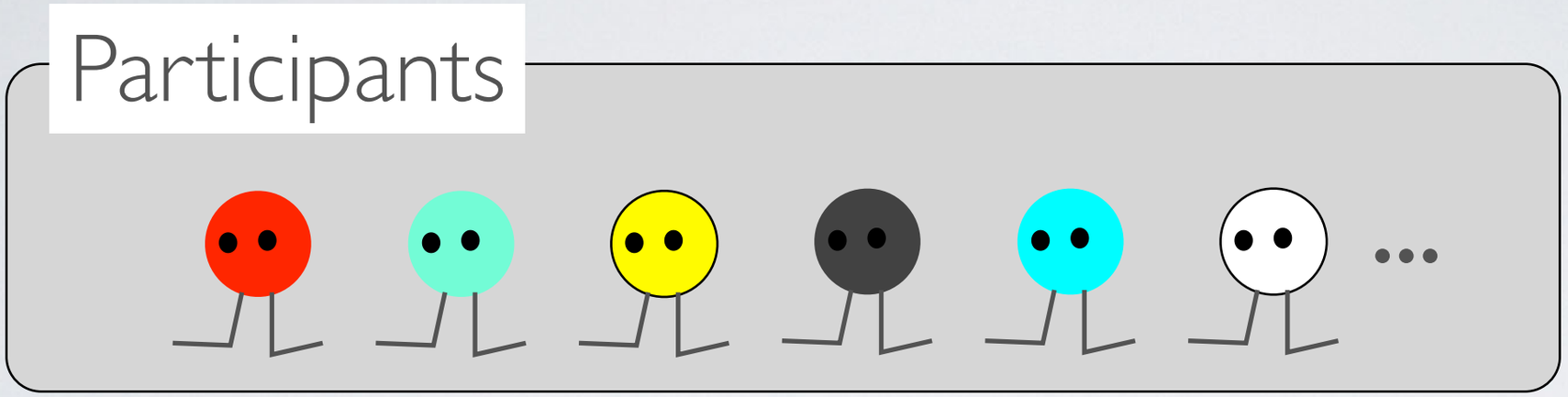

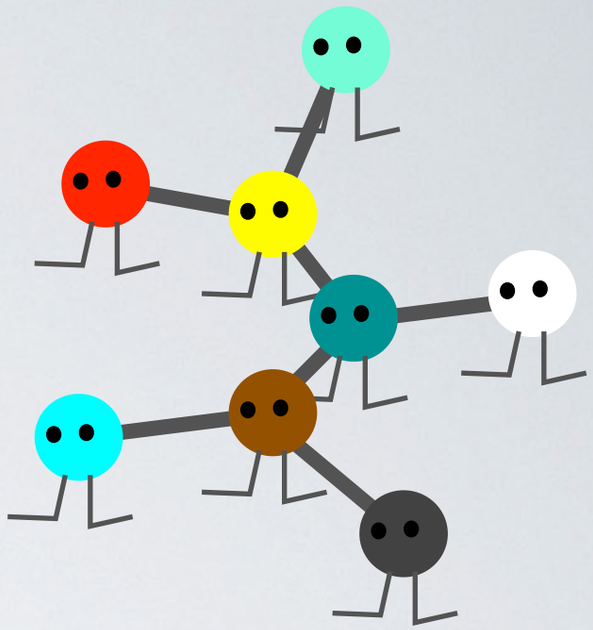

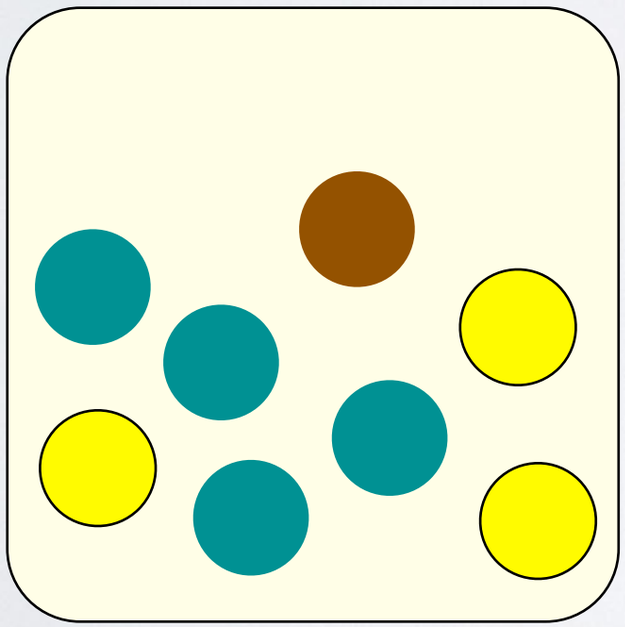

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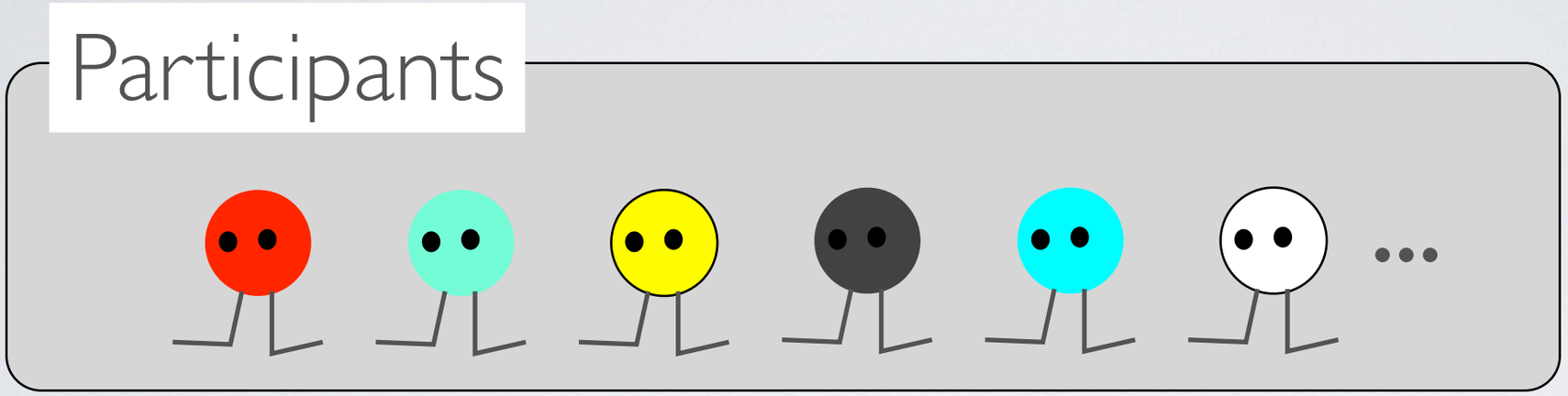
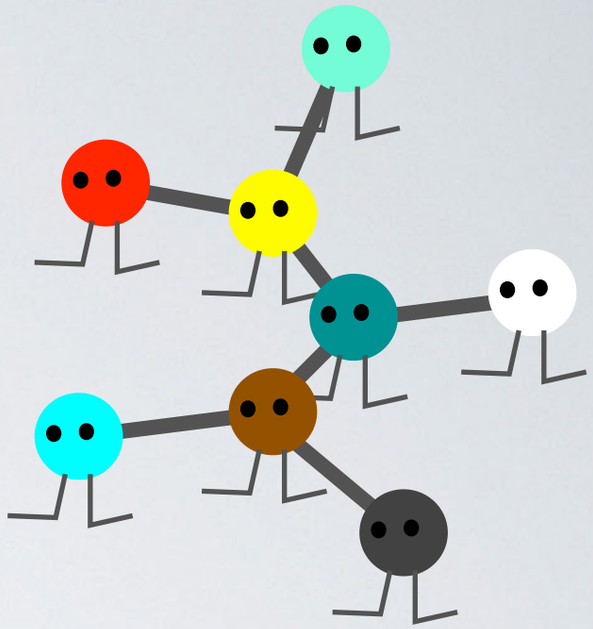
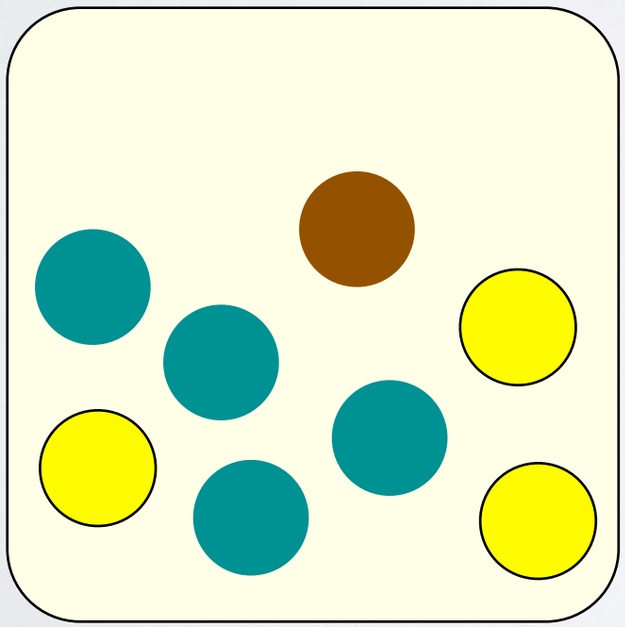


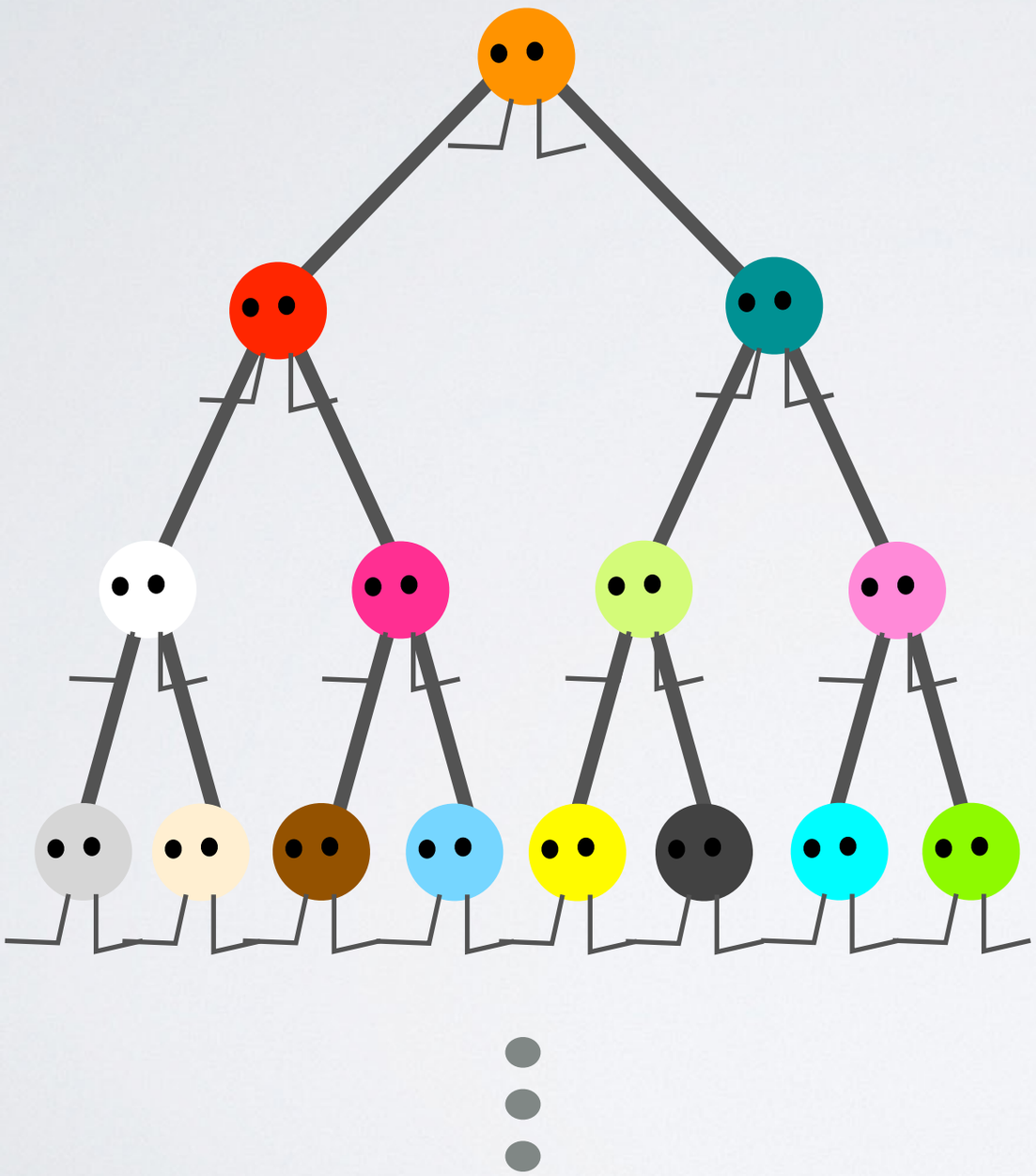




Winner is the remaining color

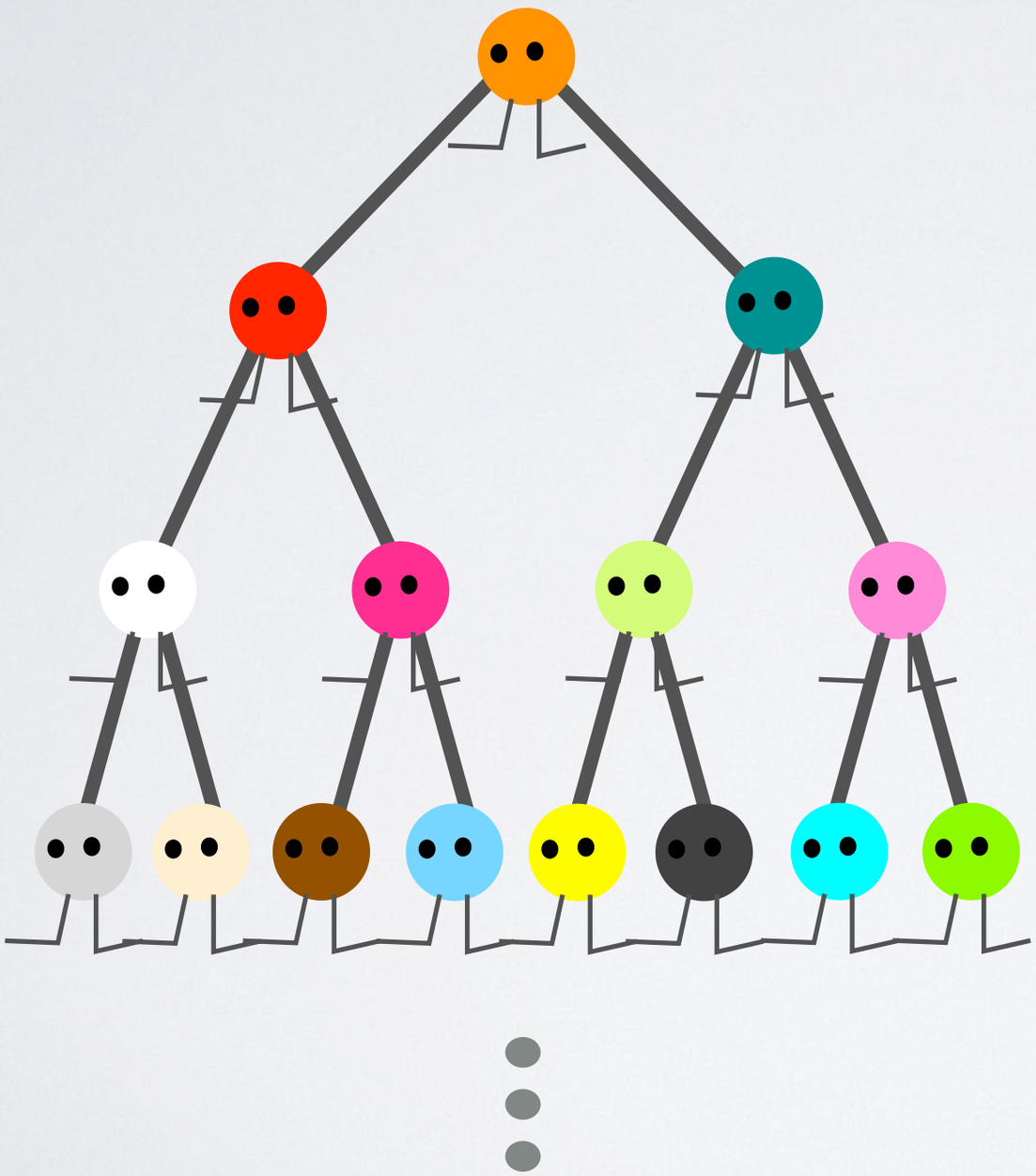
A binary tree

h= 1,000,000



A binary tree

h= 1,000,000

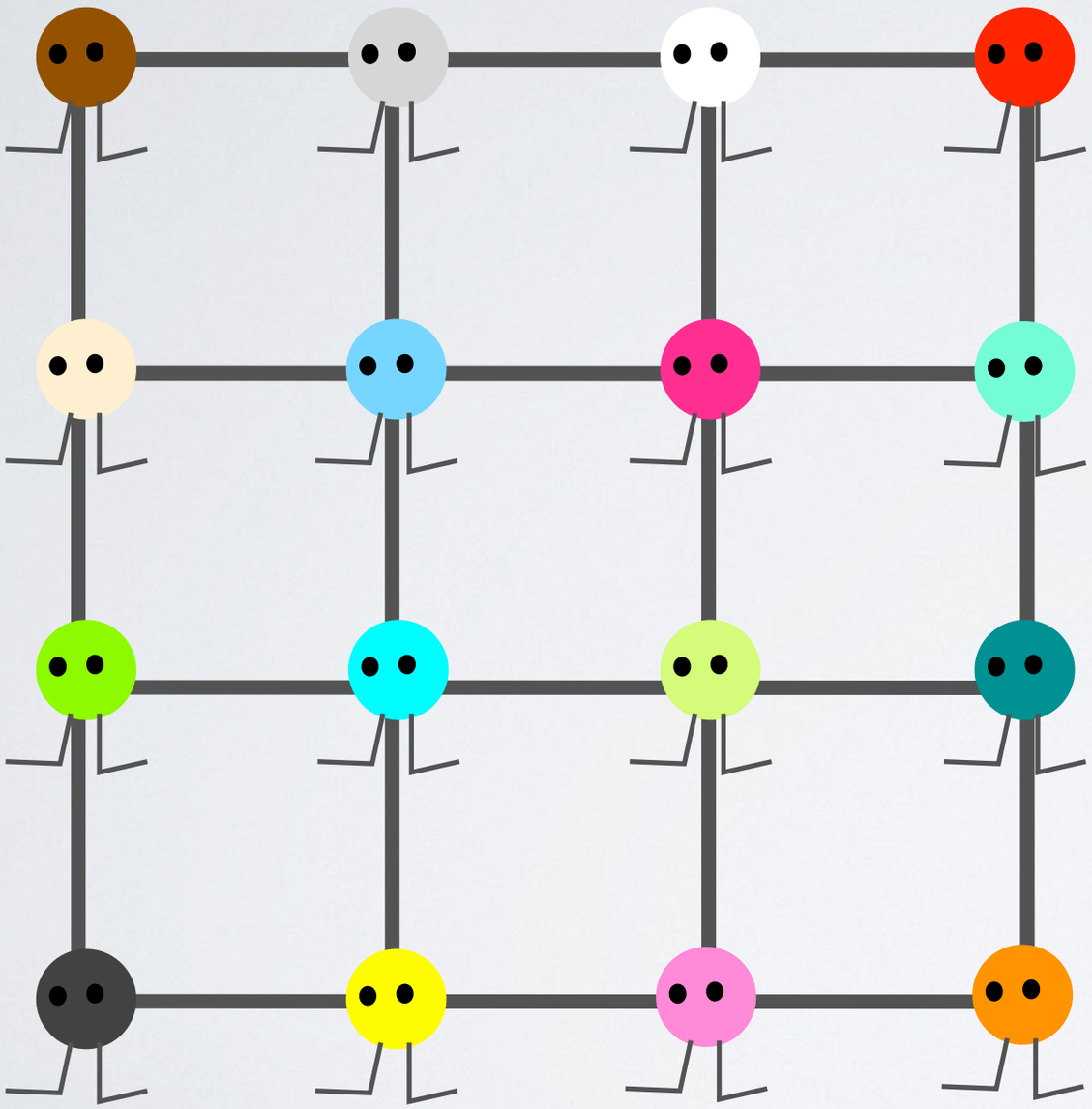


The winner is chosen from the top d levels...

a) d = 500,000
b) d = 1,000
c) d = 20
d) d = 2

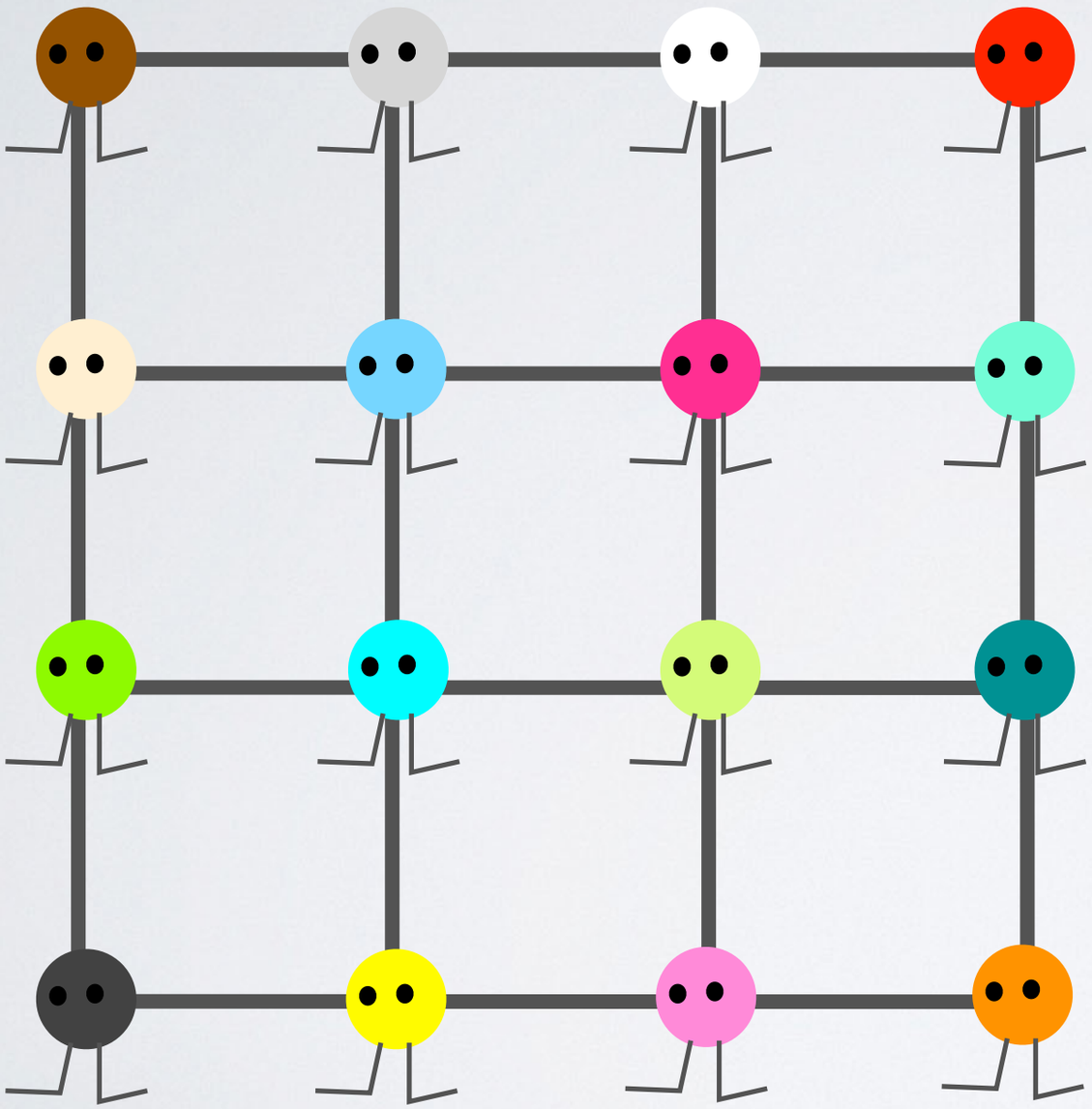
A square grid

n=1,000,001



A square grid

n=1,000,001



With 99%, the winner is in the center dxd square...

a) d = 500,000
b) d = 1,000
c) d = 20
d) d = 3

For any median graph, Triadic Consensus finds a $\left(1 + O\left(\sqrt{\frac{\log n}{n}}\right)\right)$-approximate 1-median in $O(n \log n)$ triads.

For any three nodes A, B, C, there is a unique node X that lies on the pairwise shortest paths between A, B, and C

For any median graph, Triadic Consensus finds a $\left(1 + O\left(\sqrt{\frac{\log n}{n}}\right)\right)$-approximate 1-median in $O(n \log n)$ triads.

NASH EQUILIBRIUM?

- Assumption: Participants are not strategic in declaring their positions

- Strategy space: the votes cast in any triadic interaction

- Truthful voting is a Nash equilibrium in extensive form game

Assumption too strong? Think of designing the right mechanism as an open Problem, and interpret this result only as an analysis of the dynamics with truthful voters.

REPLACE MEDIAN FINDING WITH COMPARISONS?

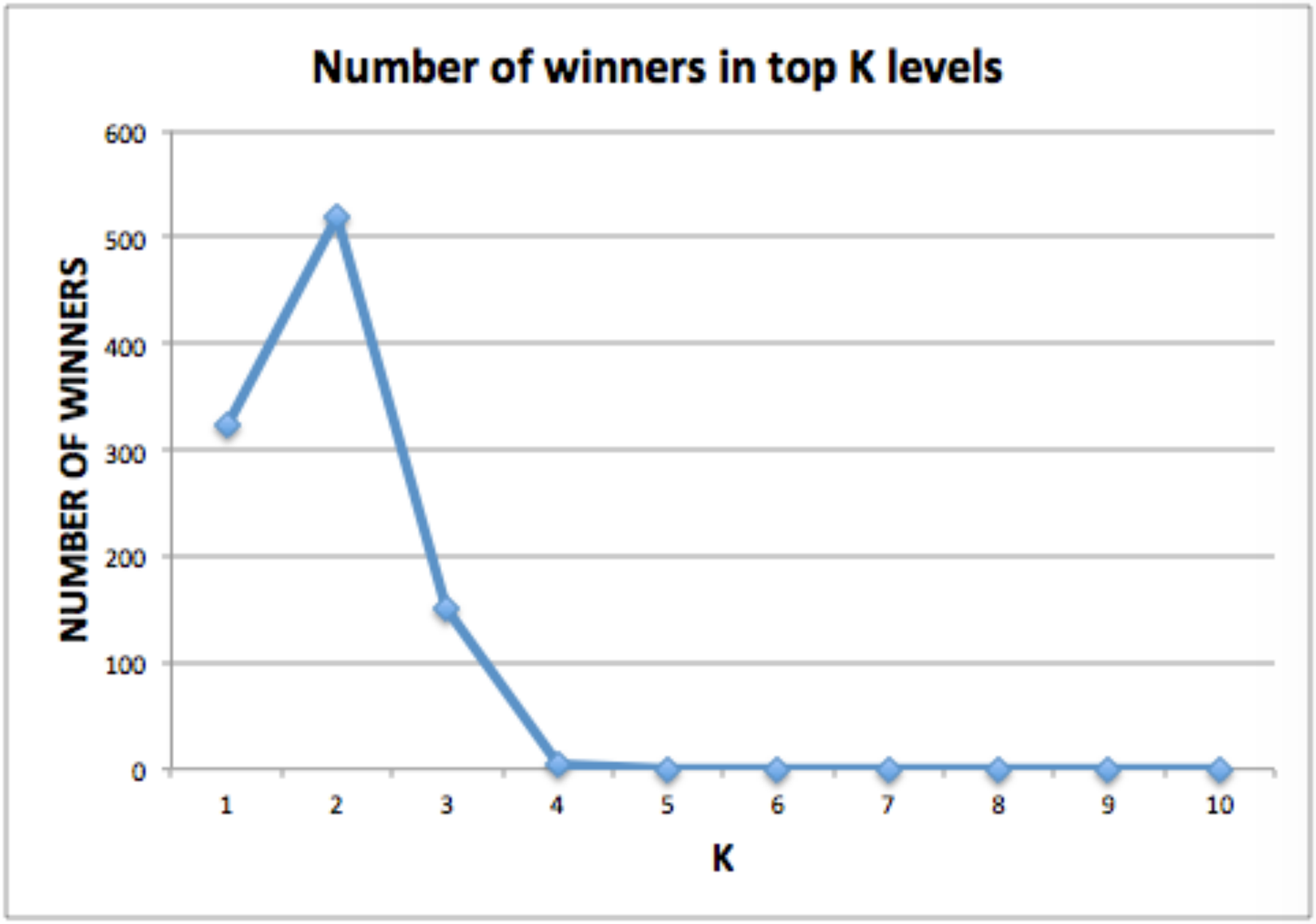
- Simulation Evidence for trees and grids — take a 21x21 grid or a 10 level full binary tree, and run the modified triadic process to completion
  - 1000 trials
  - Optional: replicate every node many times before running the trial
- Interesting future direction: Provable bounds

CHOOSING A WINNER FROM THE TRIAD

TEN LEVEL BINARY TREE, WITH NO REPLICATION

Mass in the top 3 nodes = 84.3%

Mass in the top 7 nodes = 99.5%

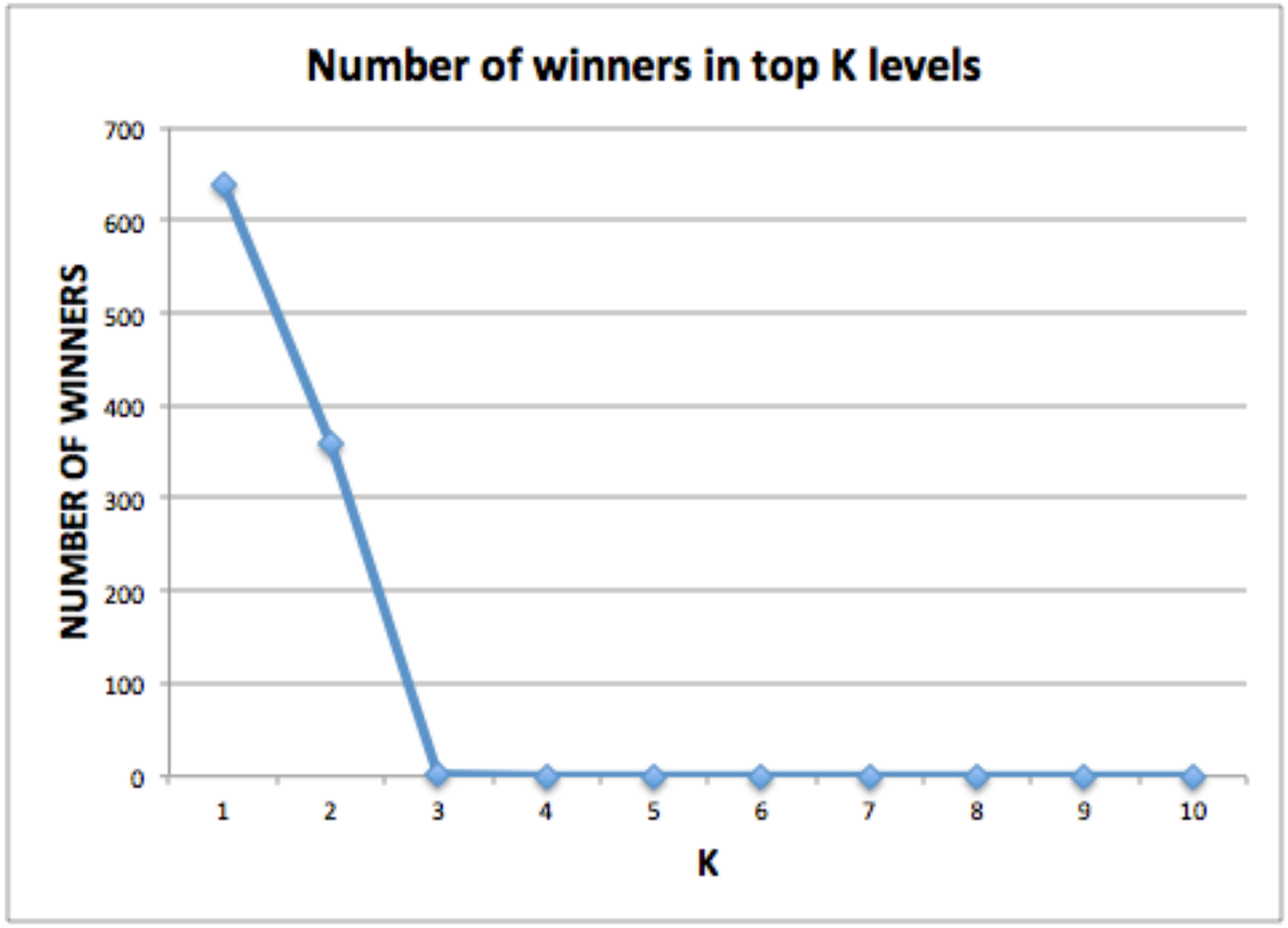


CHOOSING A WINNER FROM THE TRIAD

TEN LEVEL BINARY TREE, WITH EVERY NODE REPLICATED FIVE TIMES

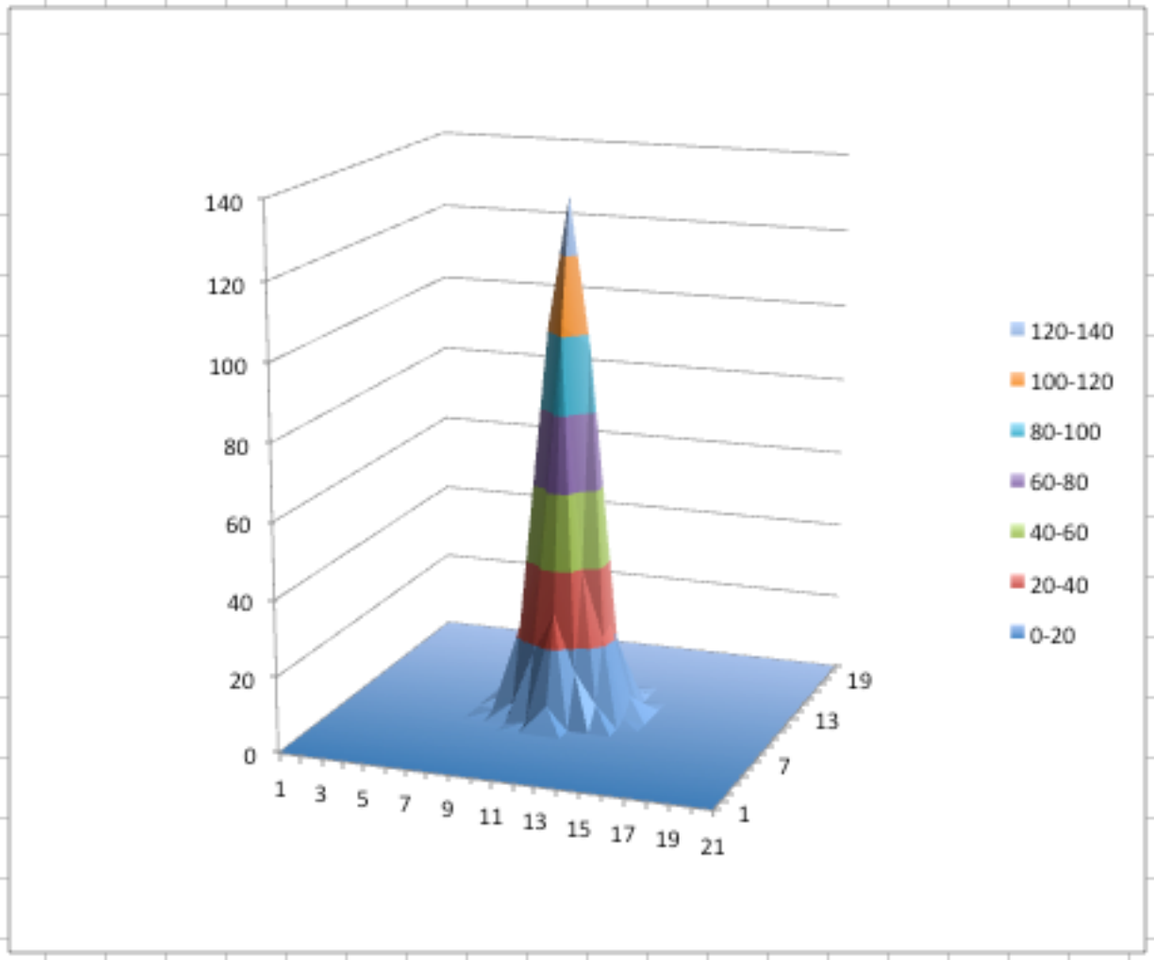
Mass in the top 3 nodes = 99.9%

Mass in the top 7 nodes = 100.0%



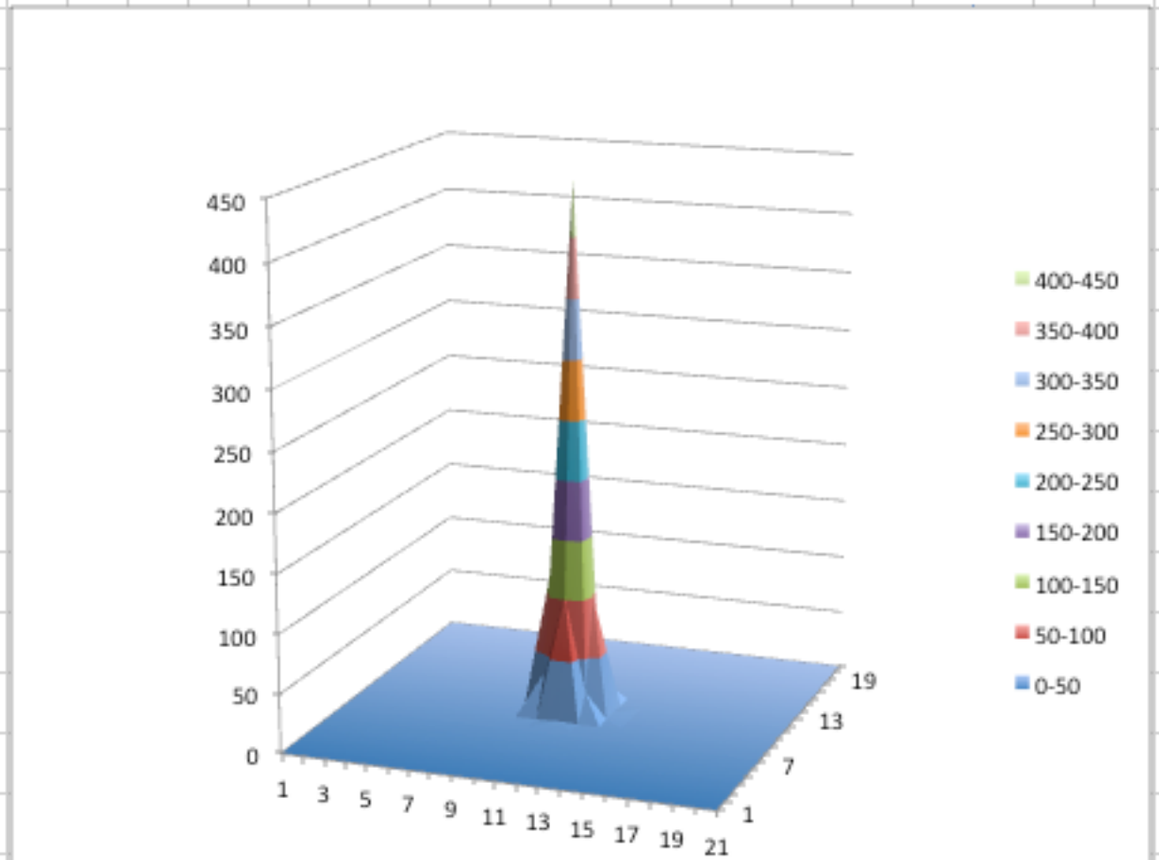
21x21 grid, no replication, winner from triad

x➔ / y➘	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	1	6	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	1	4	18	29	14	6	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	23	46	103	40	22	1	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	2	18	101	135	97	24	5	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	1	12	58	87	52	18	1	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	2	2	20	21	19	3	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	3	2	2	1	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

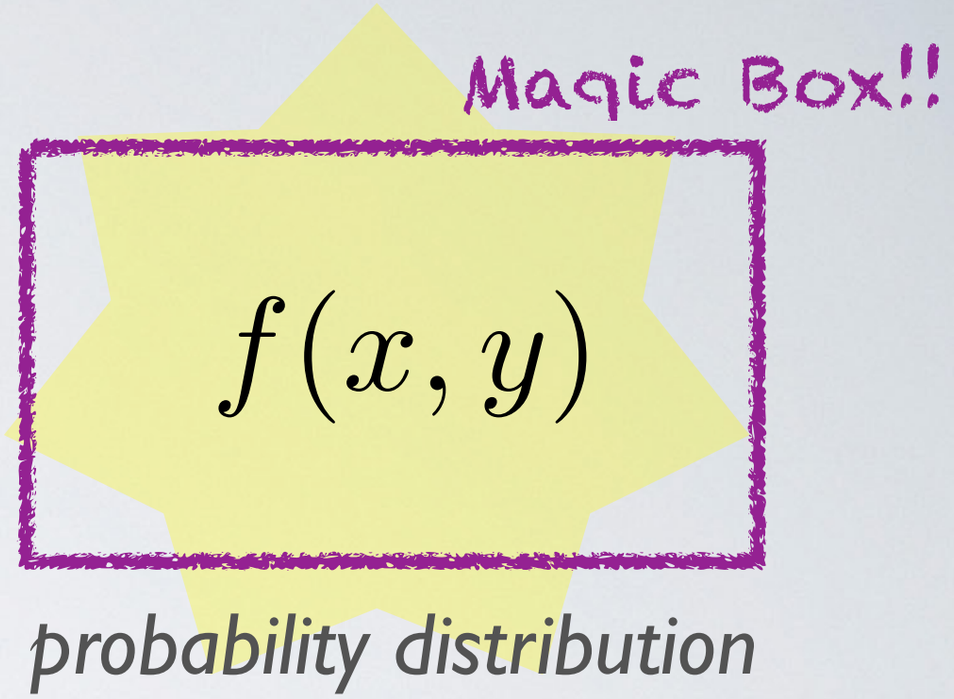


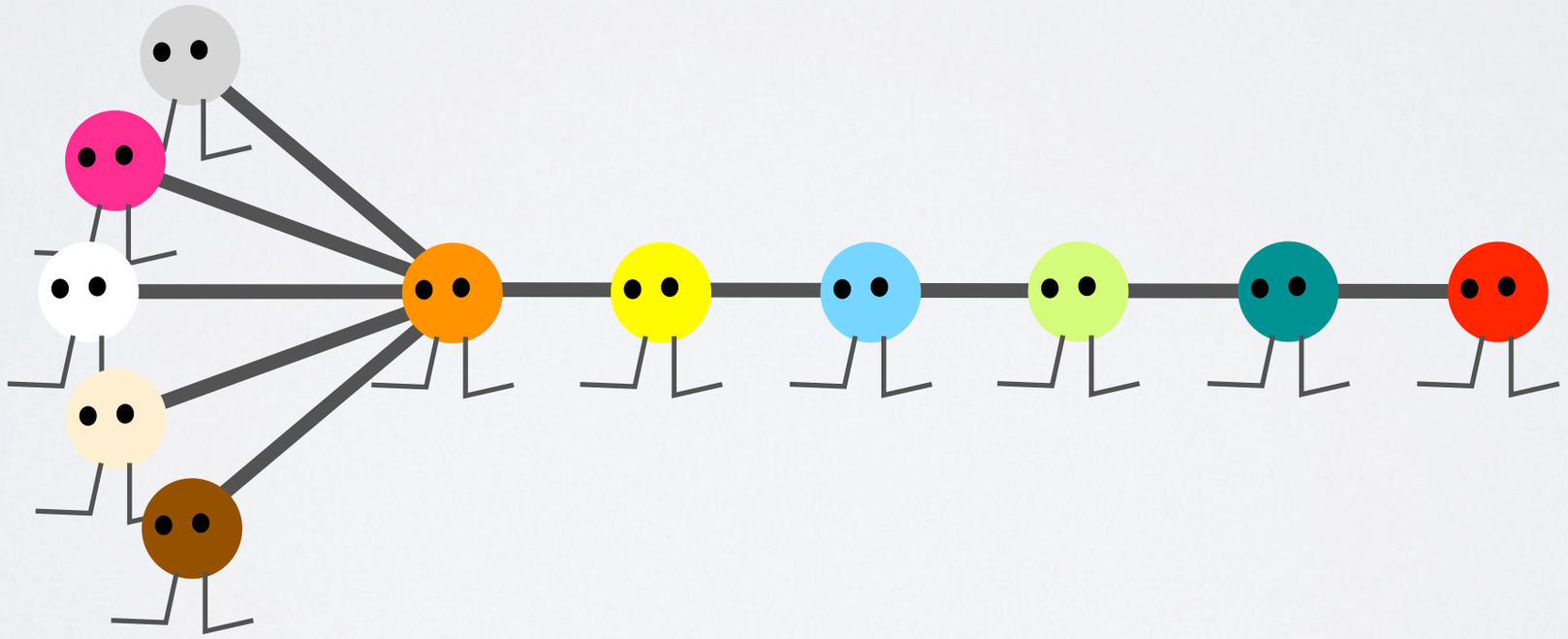
21x21 grid, 5x replication, winner from triad

x→ / y↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	35	103	22	1	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	122	447	100	1	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	34	109	24	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

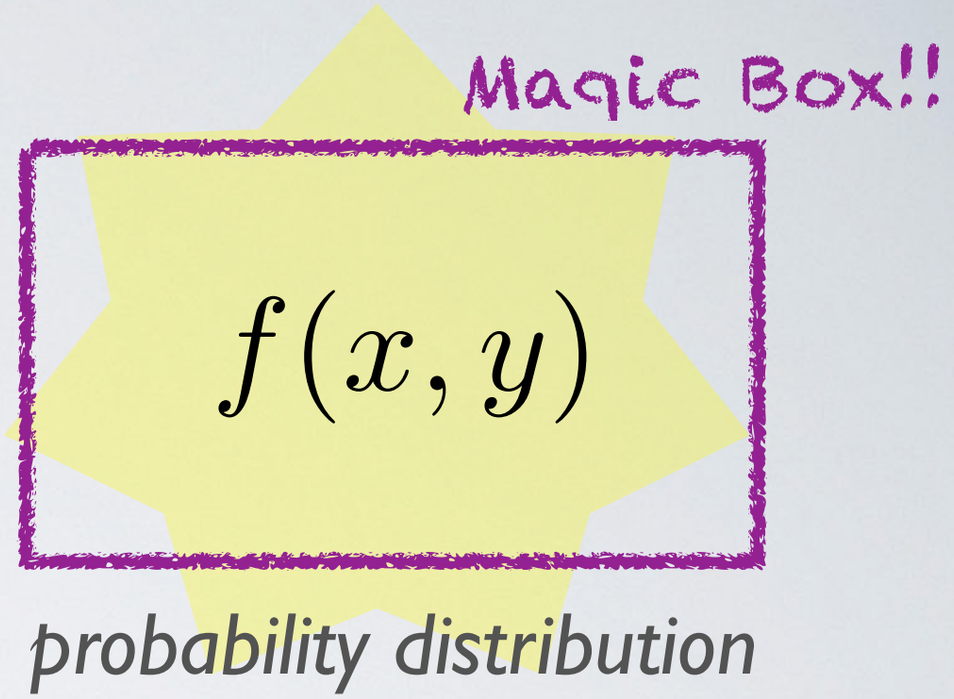


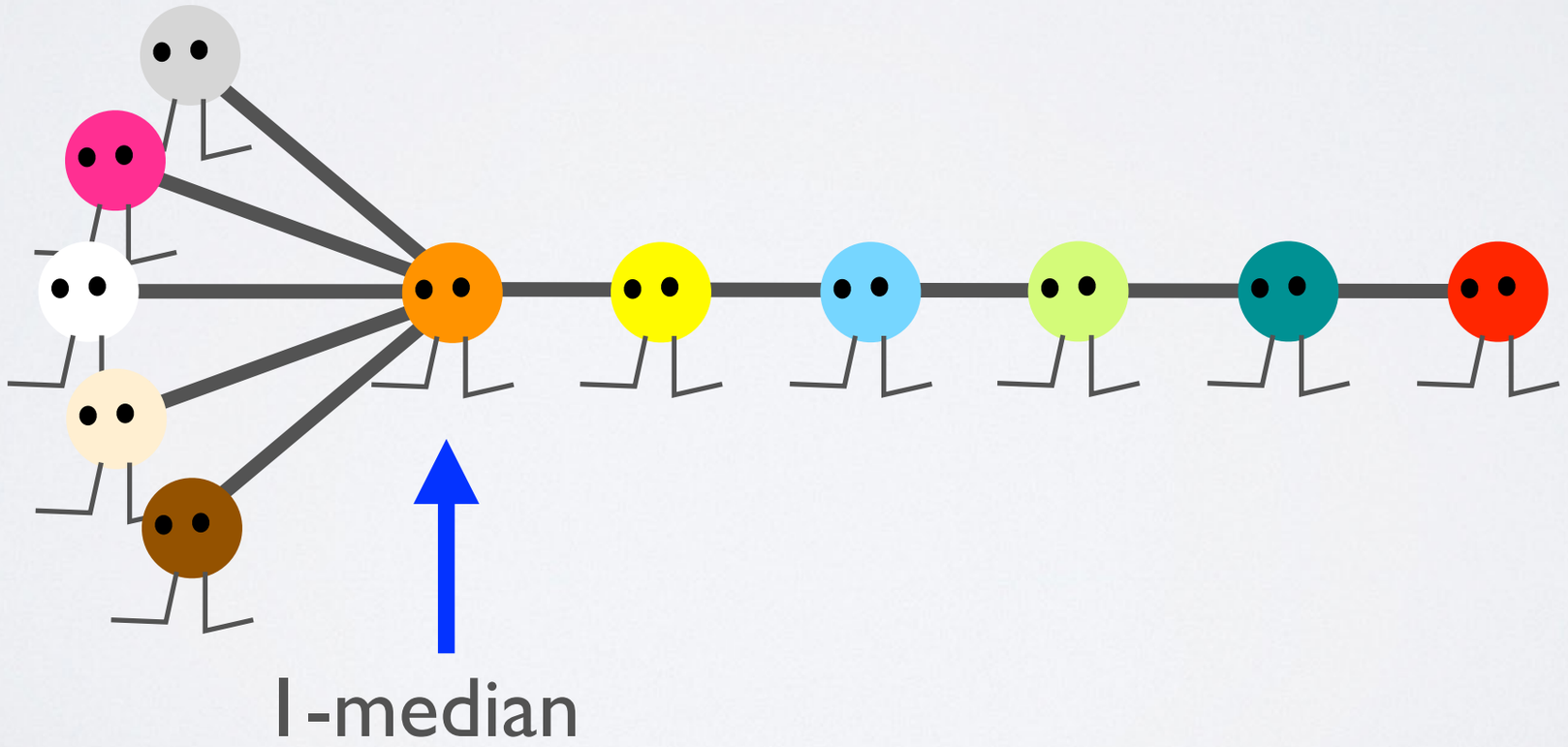
A two person reduction?



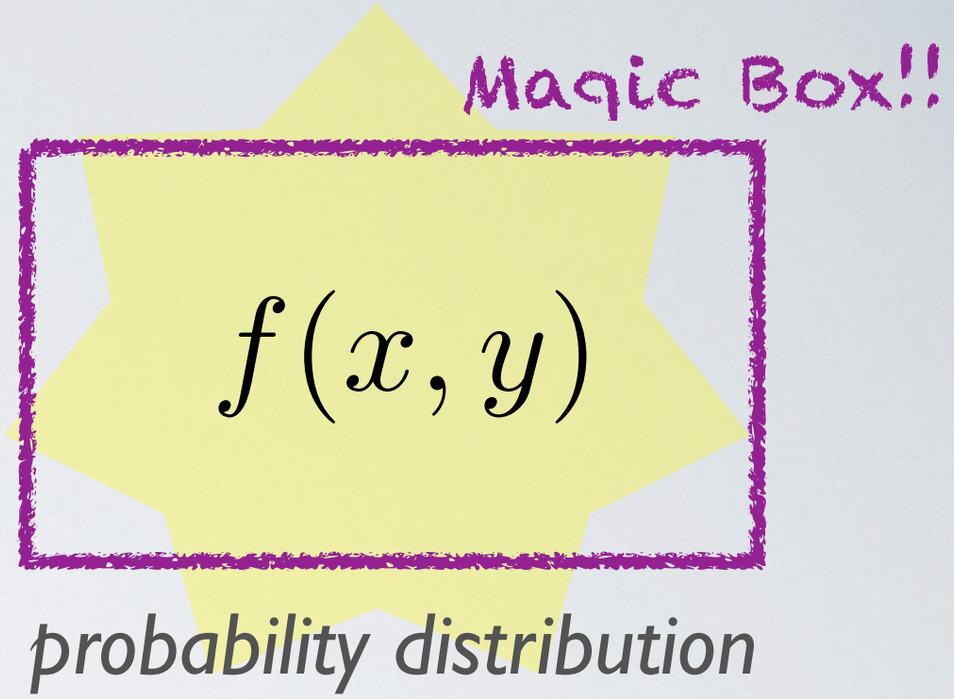


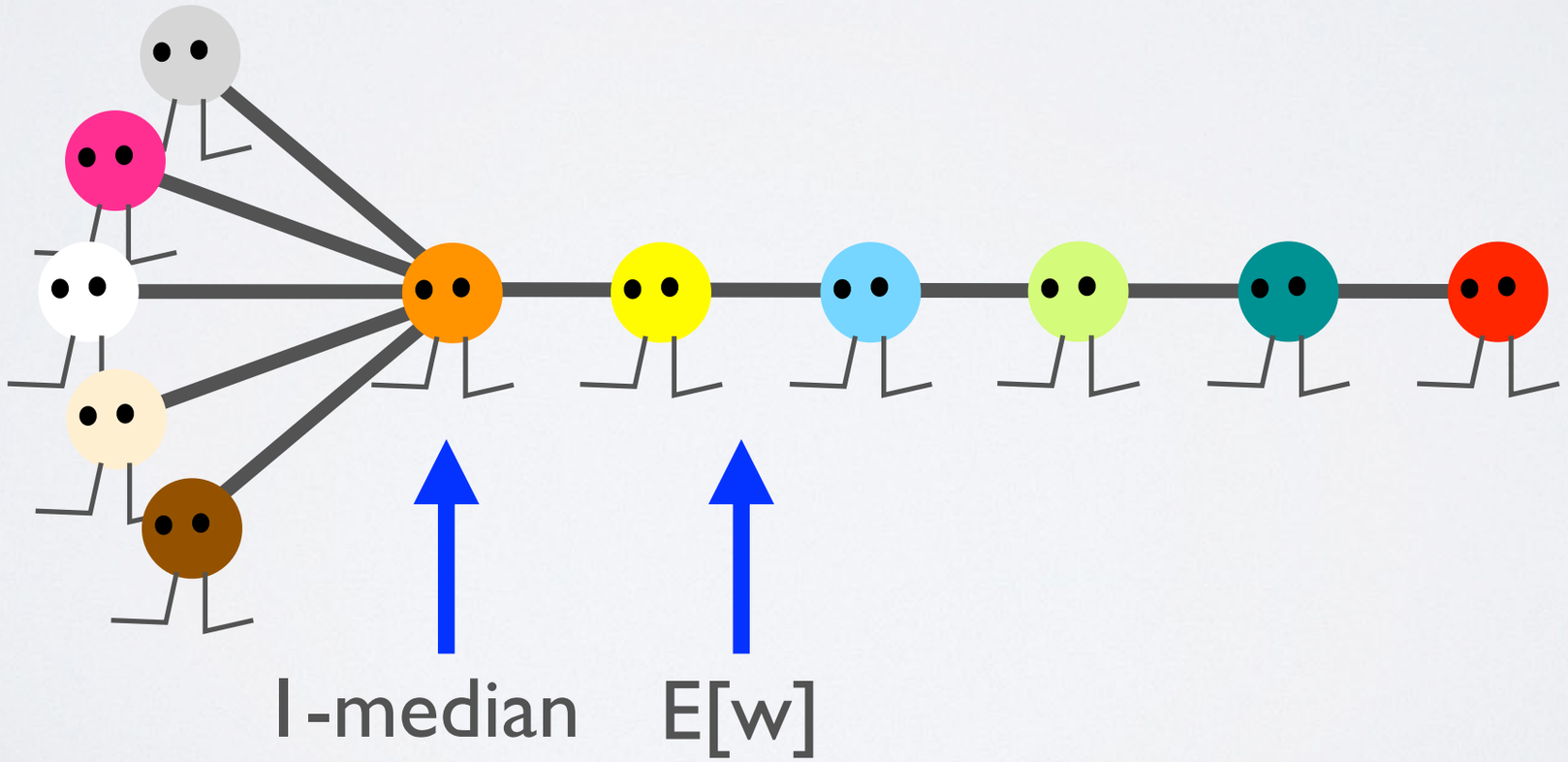
A two person reduction?





A two person reduction?





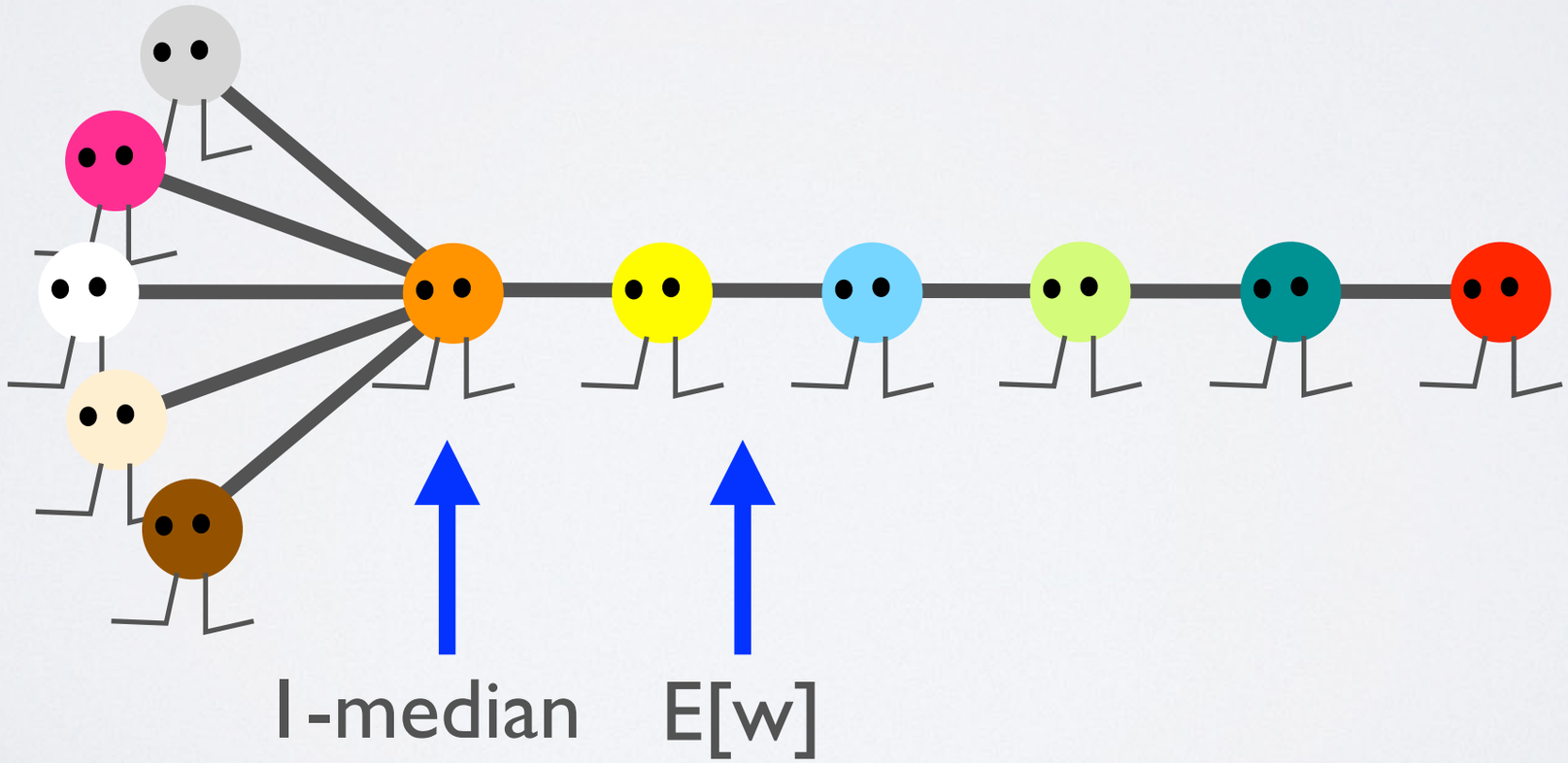
A two person reduction?

No 2-person locally-consistent reduction can approximate the 1-median up to (1+o(1))

Magic Box!!

$f(x, y)$

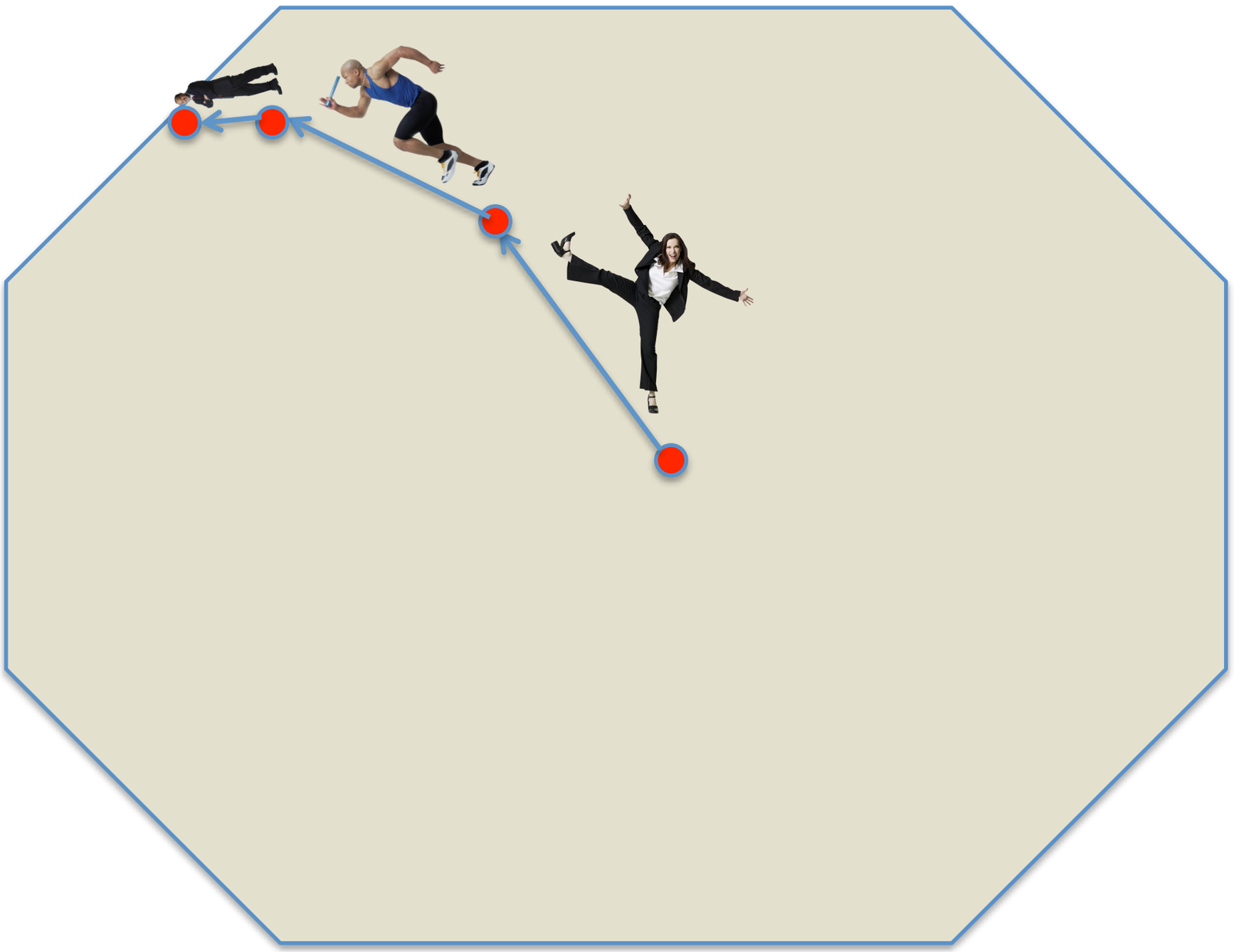
probability distribution over outcomes given x,y



FUTURE

- Generalize past median spaces and to other "magic boxes"
- Experimental studies of small group consensus
- Other small group mechanisms Fiorina and Plott 1978
- An axiomatic treatment of the problem, or more detailed models
- Advertising for diversity?

$f(x) = x^2$



CONVEX OPTIMIZATION COULD BE COLLABORATIVE

Food for thought: Probabilistic Voting

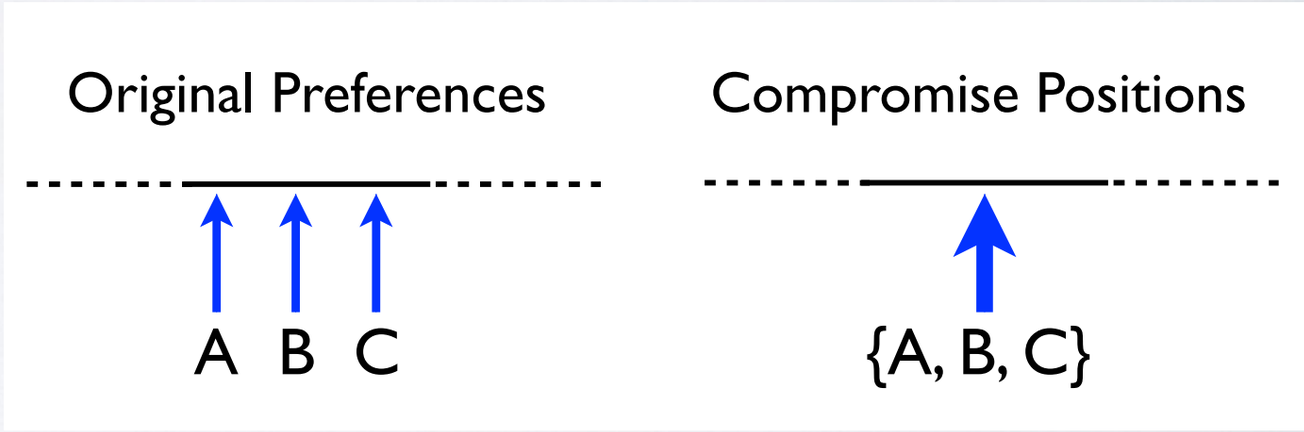
A user can make as many proposals to solve a complex problem as she likes

Each user has only one vote, but she can change her vote as many times as she likes

A proposal that ends up with K votes gets K*K tickets to a lottery

Goal: Reward consensus-building

Problem: The Nash equilibria of this game are not well behaved, even on the line



By moving to a compromise position, the total weight for A, B, and C in the lottery goes from 3 to 9.

Finland off-road traffic law

Tanja Aitamurto, Helen Landermore, David Lee

Comparing

Complete the following actions:

16 / 101 completed

Compare Instructions

Question

Location of routes. What would be the best way to situate snowmobile routes?

Ideas

Undeveloped land beyond residential areas should be made available as free-riding zones. [Background Information]

Better Idea

There should be more routes in the Eastern and Northern parts of Finland.

Better Idea

Scoring

Complete the following actions:

18 / 101 completed

Rate Instructions

Question

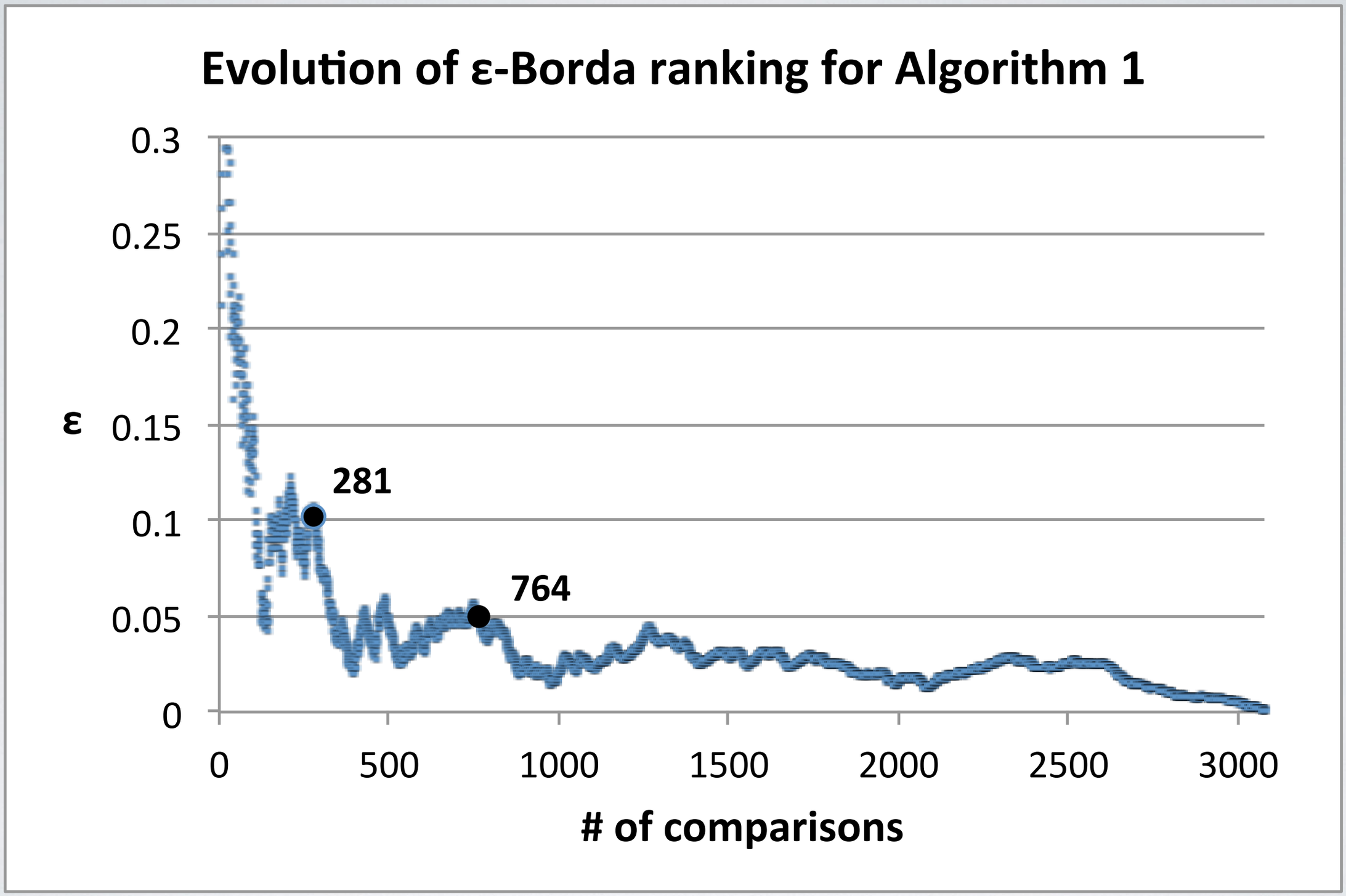
Establishing new routes. *What would be the best way to determine the time frame of route permits?*

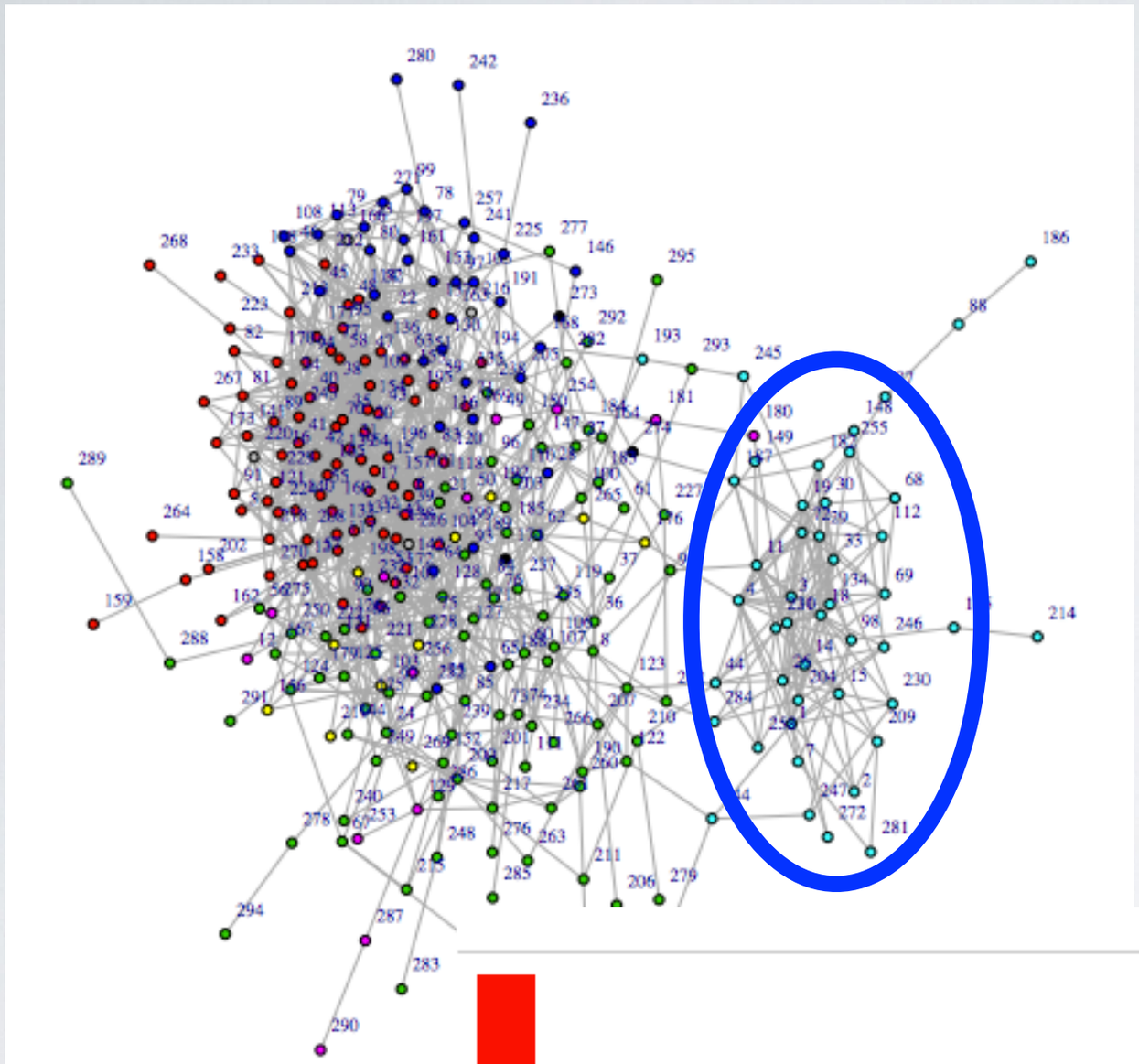
Ideas

If a route permit has been granted but the feasibility of the route plan is not inspected on a yearly basis, the route permit should be automatically annulled. [Background Information]

Arvioi idea valitsemalla sopiva määrä tähtiä.

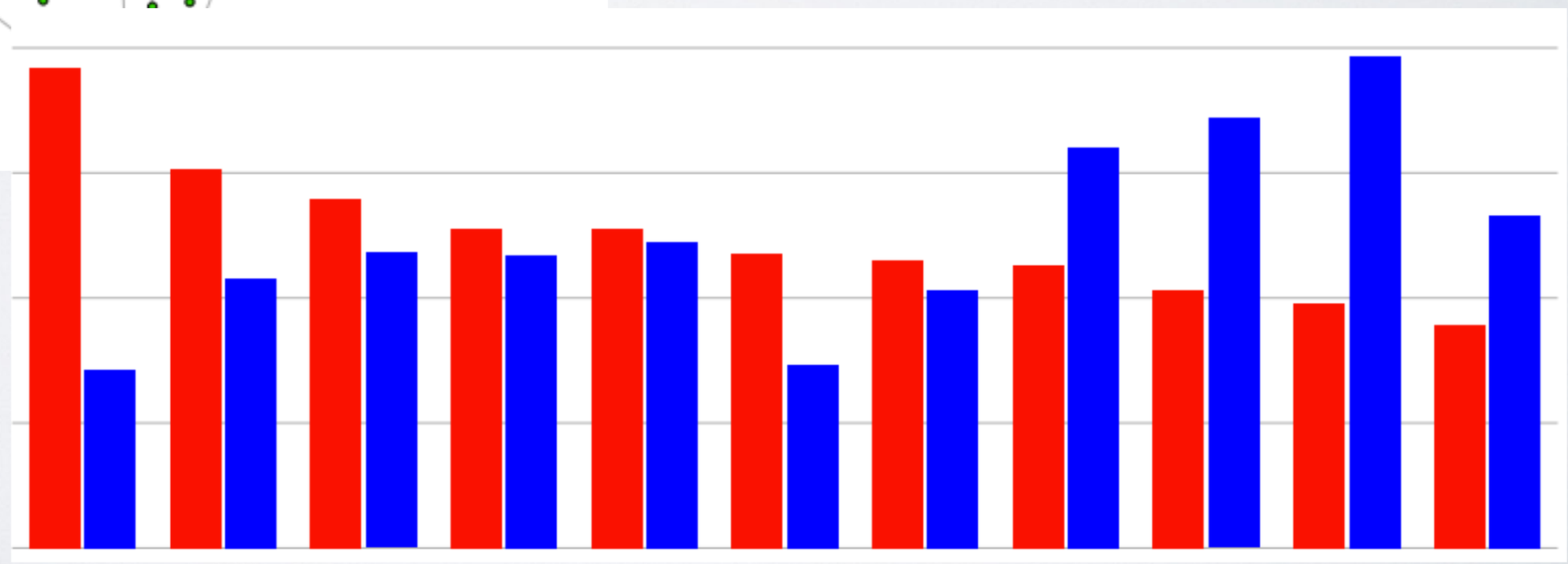
Submit


Evolution of ε-Borda ranking for Algorithm 1
0.3
0.25
0.2
0.15
0.1
0.05
0
ε
281
764
0
500
1000
1500
2000
2500
3000
# of comparisons



Detection of a minority cluster

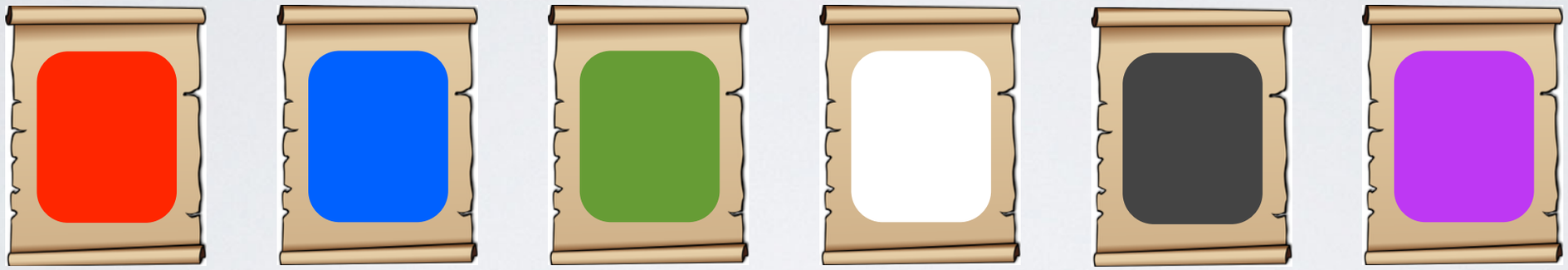
(ratings are key)



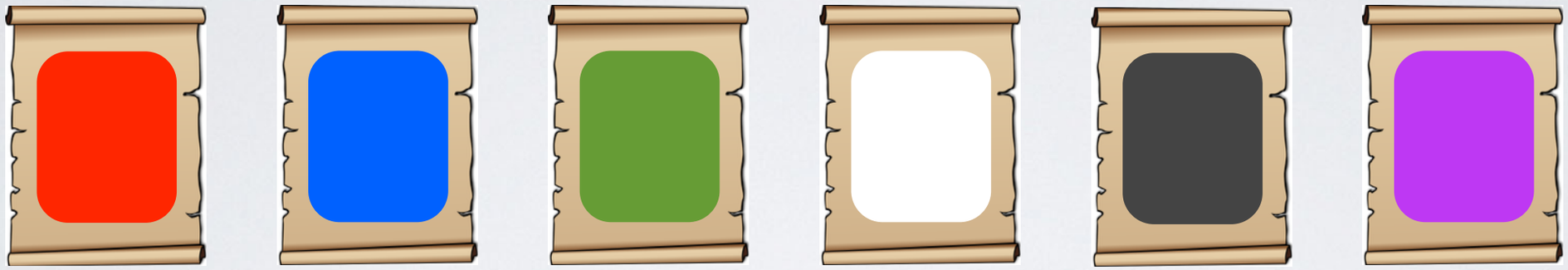
THANKS!

QUESTIONS?

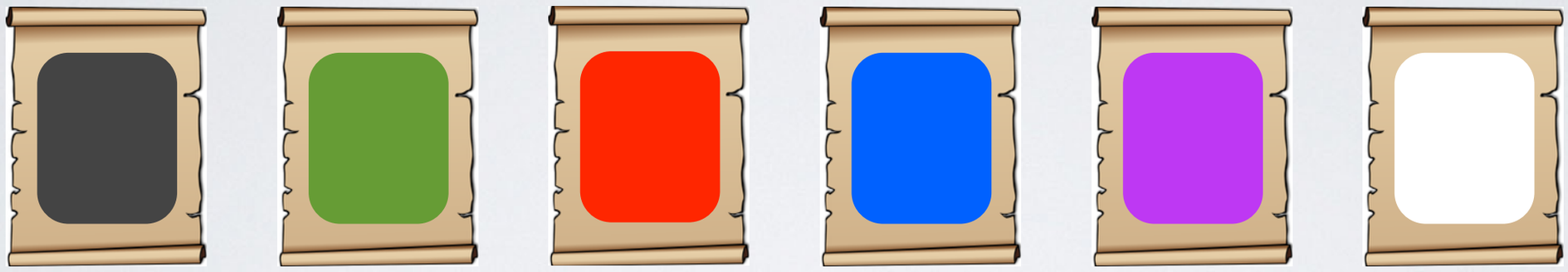
Social choice functions



Social choice functions

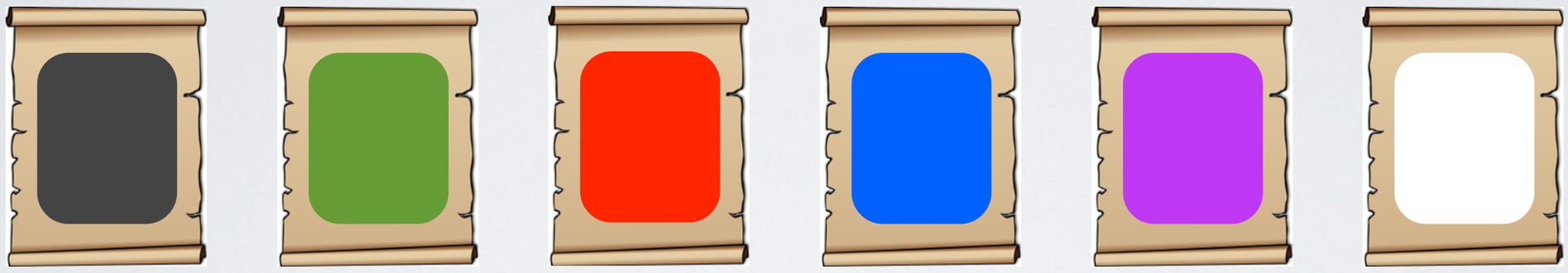


Social choice functions



1 2 3 4 5 6

Social choice functions



1 2 3 4 5 6

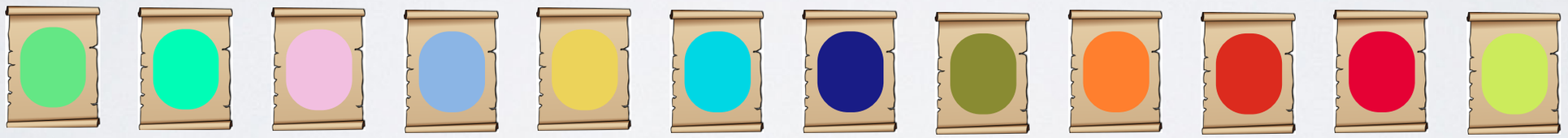
1 2 3 4 5 6

4 2 1 3 5 6

$f(\text{rankings})$

output ranking

Problem



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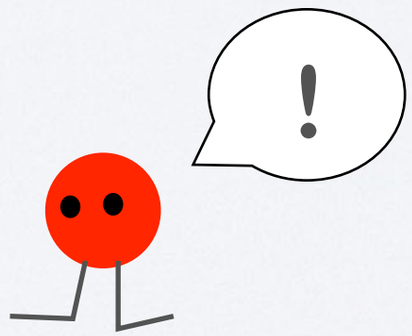
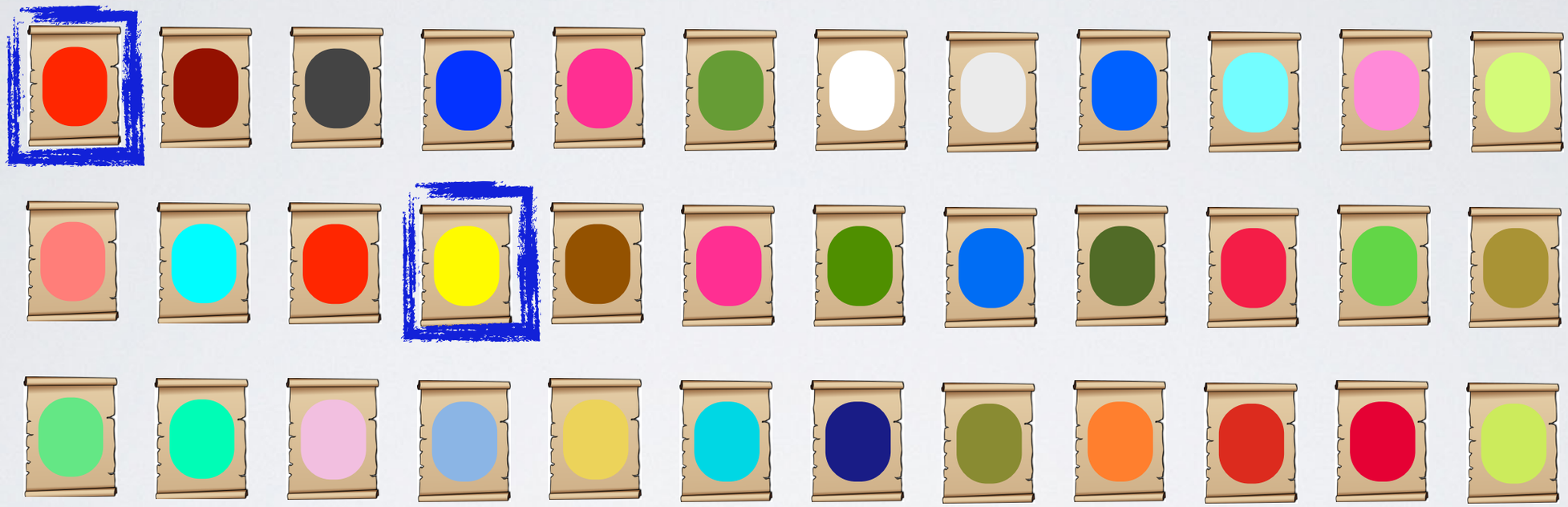
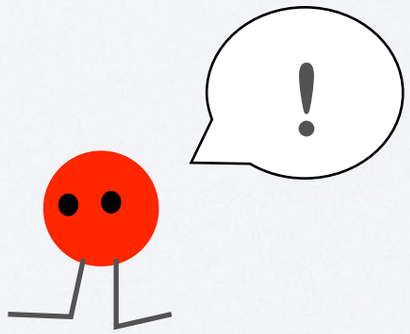


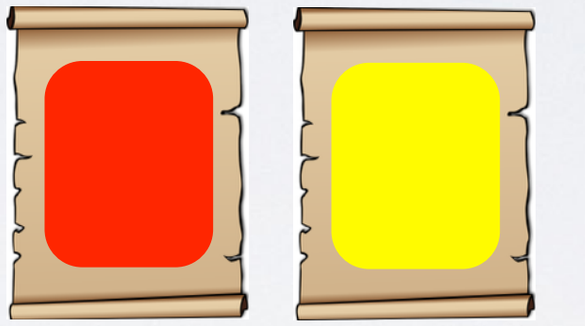
Enter preference elicitation

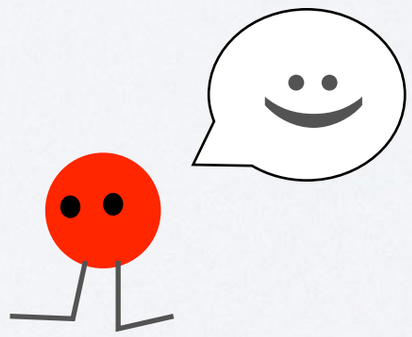


...

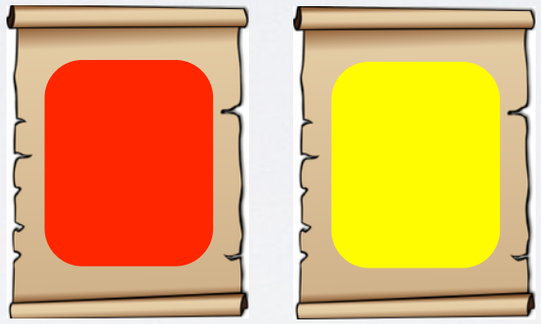


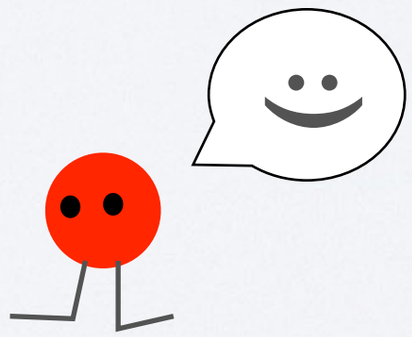
Enter preference elicitation





Can we find the output ranking with a small number of comparisons?





Can we find the output ranking with a small number of comparisons?

Many negative theoretical results.

Conitzer, Sandholm 2002, 2005

Service, Adams 2012

m proposals, n voters

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	Borda	Condorcet
CS '05	$\Omega(nm \log m)$	$\Omega(nm)$

m proposals, n voters

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Many negative theoretical results.

Conitzer, Sandholm 2002, 2005 | Service, Adams 2012

	Borda	Condorcet
CS '05	$\Omega(nm \log m)$	$\Omega(nm)$
SA '12	$\Omega(nm \log \frac{1}{\epsilon})$	N/A

m proposals, n voters

Can we "approximate" the output ranking
with a
small number of comparisons?

	Borda	Condorcet
CS '05	$\Omega(nm \log m)$	$\Omega(nm)$
SA '12	$\Omega(nm \log \frac{1}{\epsilon})$	N/A
Our Results	$O(\frac{m}{\epsilon^2} \log \frac{m}{\delta})$	$O(\frac{m}{\epsilon^4} \log^2 \frac{m}{\delta\epsilon^2})$

Ask a voter to compare random pairs
of proposals

Eliciting the Borda winner

Eliciting the Borda winner

Definition. *Let* $s(x) = \#\,comparisons\ won.$

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Definition. *Let $s(x) = \#$ comparisons won.*
Then w is a Borda winner if

$$w = \arg\max_x s(x),$$

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Definition. *Let* $s(x) = \#\,comparisons\ won.$
Then w *is a Borda winner if*

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and $\hat{w}$ *is an* ϵ*-Borda winner if*

$$s(\hat{w}) \geq (1-\epsilon)s(w).$$

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Definition. *Let* $s(x) = \#\,comparisons\ won.$
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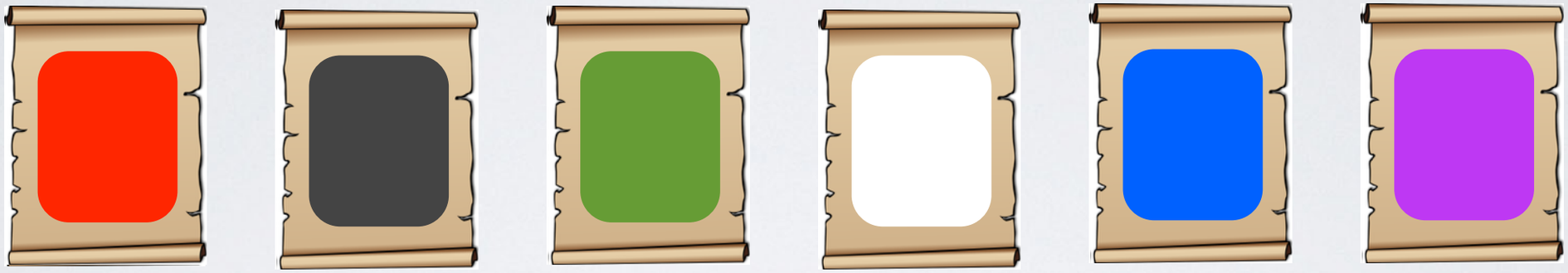
and $\hat{w}$ *is an* ϵ*-Borda winner if*

$$s(\hat{w}) \geq (1-\epsilon)s(w).$$

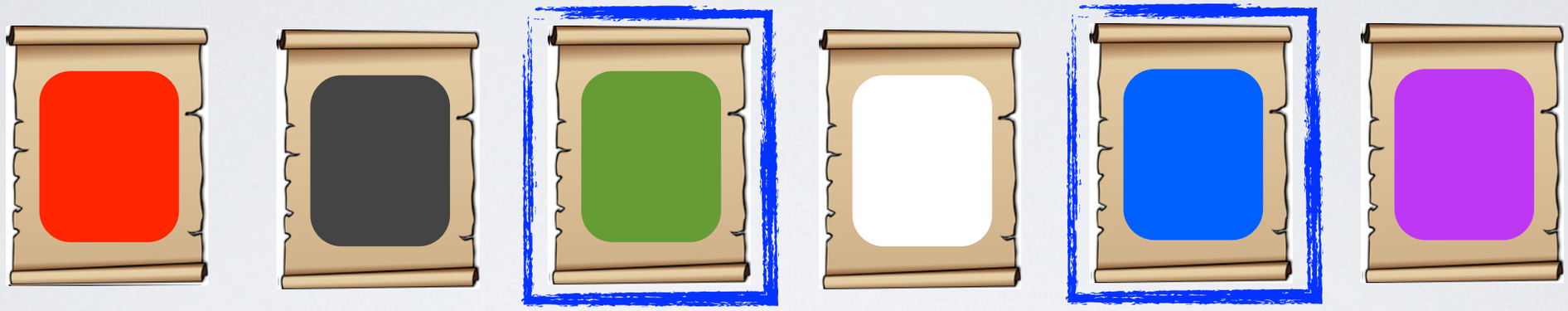
Randomly sample comparisons!

Algorithm for Eliciting Borda



0 0 0 0 0 0

Algorithm for Eliciting Borda



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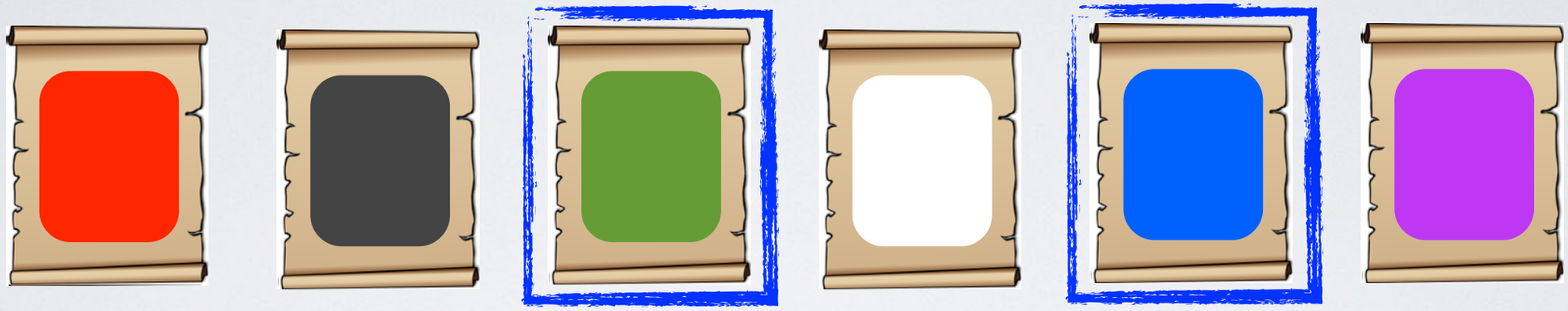

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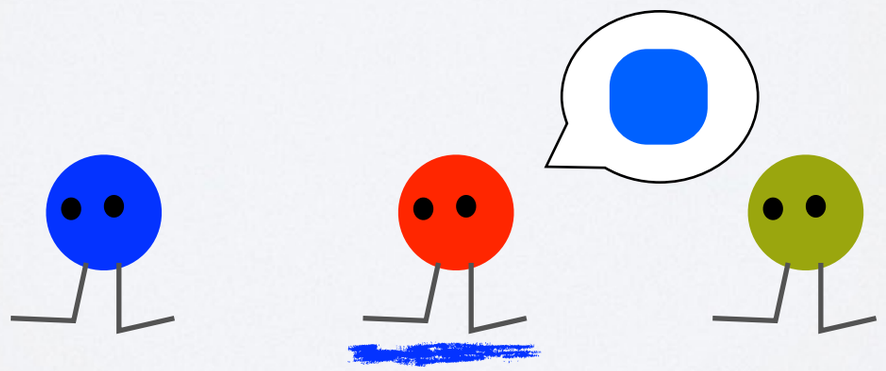

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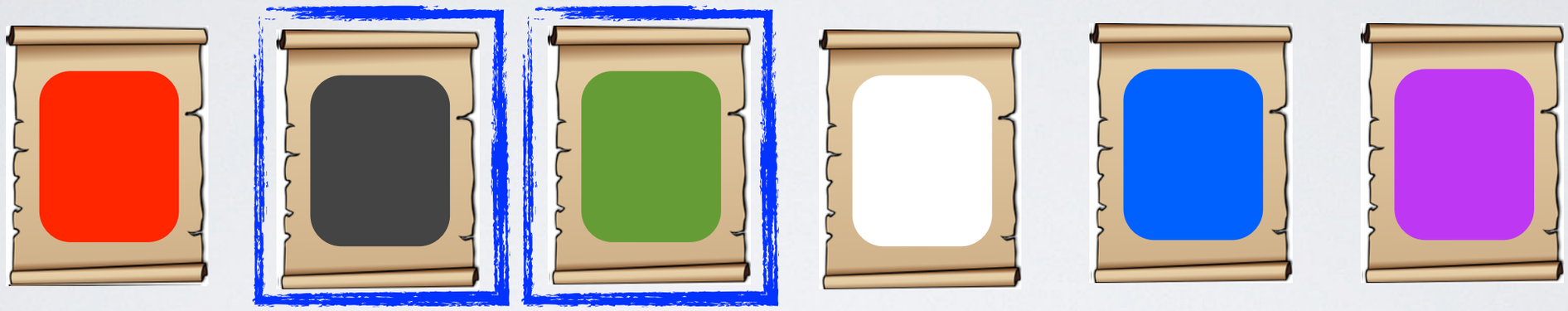
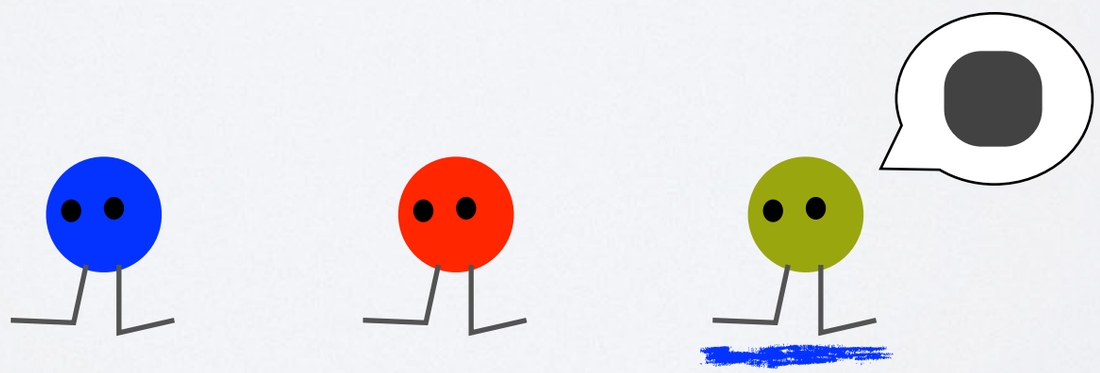


0 1 0 0 1 0



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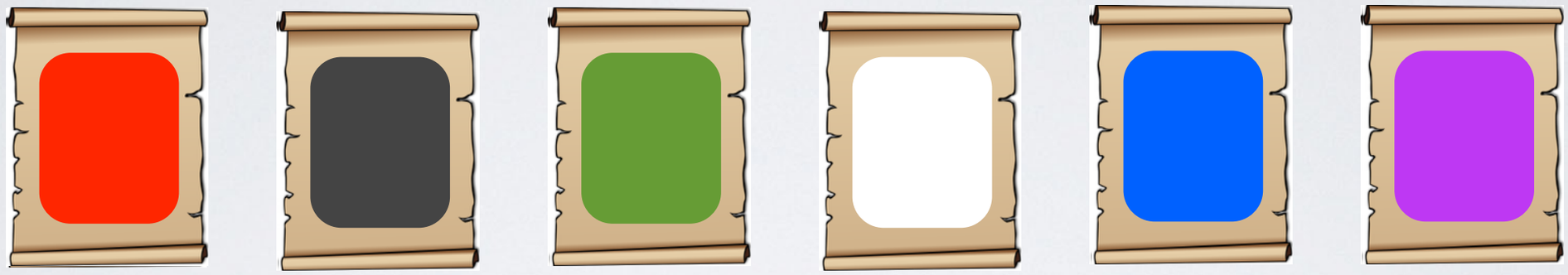

6 5 7 3 1 0

Theorem. *$O(\frac{m}{\epsilon^2} \ln \frac{m}{\delta})$ comparisons are sufficient to find an ϵ-Borda winner w.p. at least $1 - \delta$.*

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Notes:

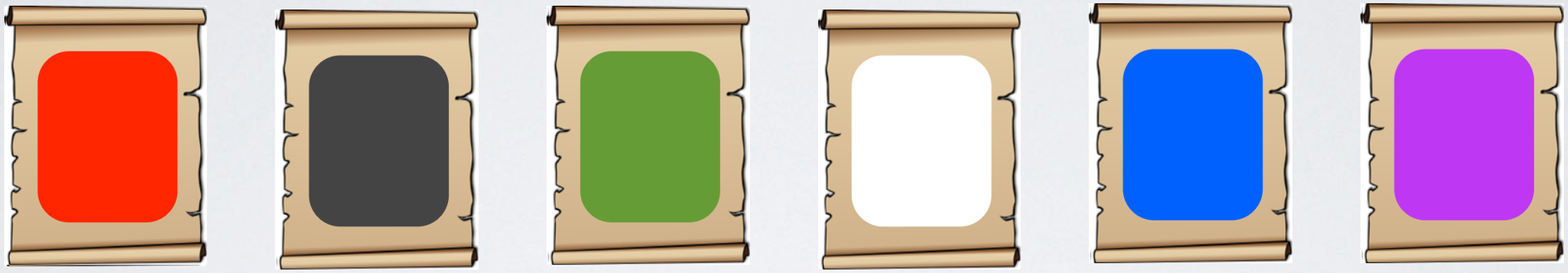
- Independent of n
- O(log m) per person if m < n

Eliciting the Condorcet winner

Eliciting the Condorcet winner

Definition. *$\hat{w}$ is an ϵ-Condorcet winner if it receives $(1-\epsilon)\frac{n}{2}$ votes against $(1-\epsilon)(m-1)$ candidates.*

Algorithm for Eliciting Condorcet



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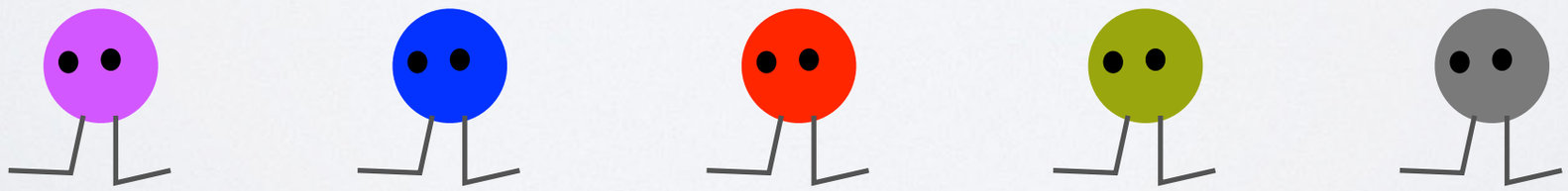
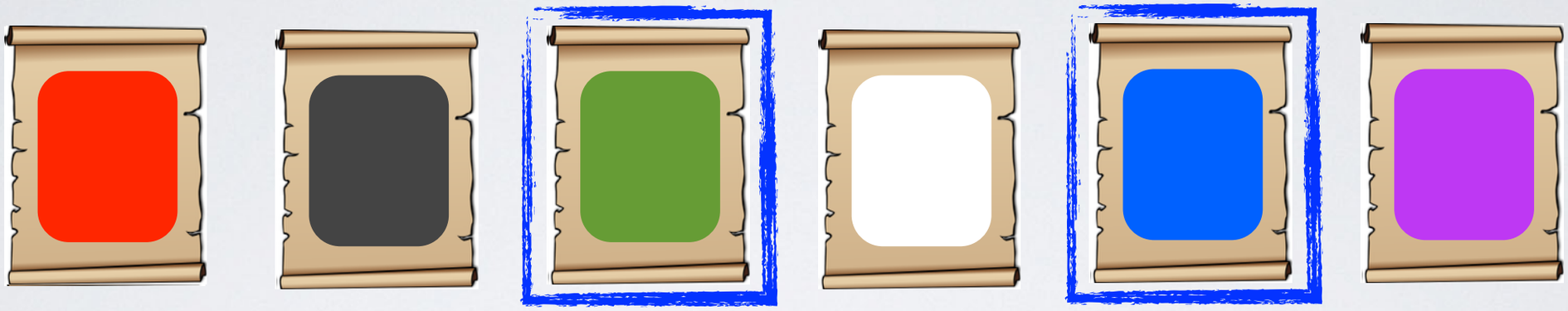
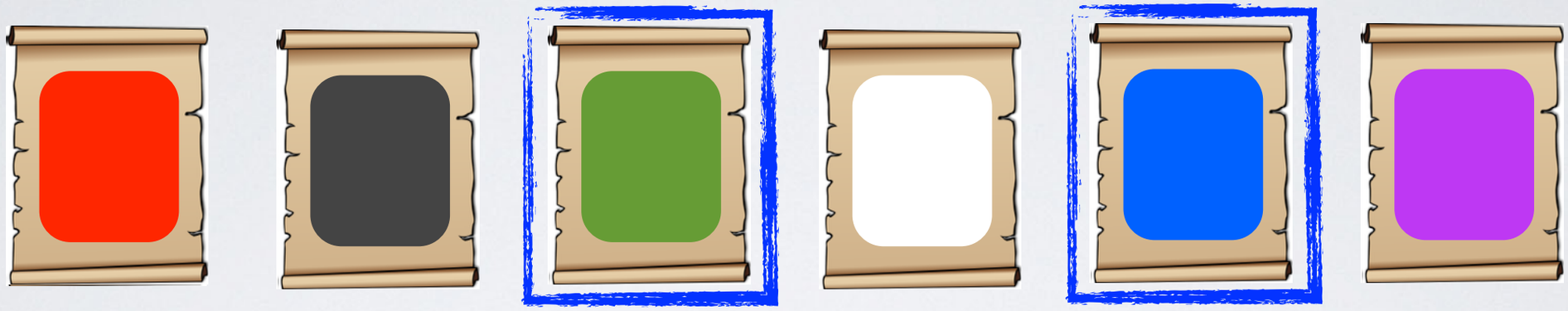


Algorithm for Eliciting Condorcet

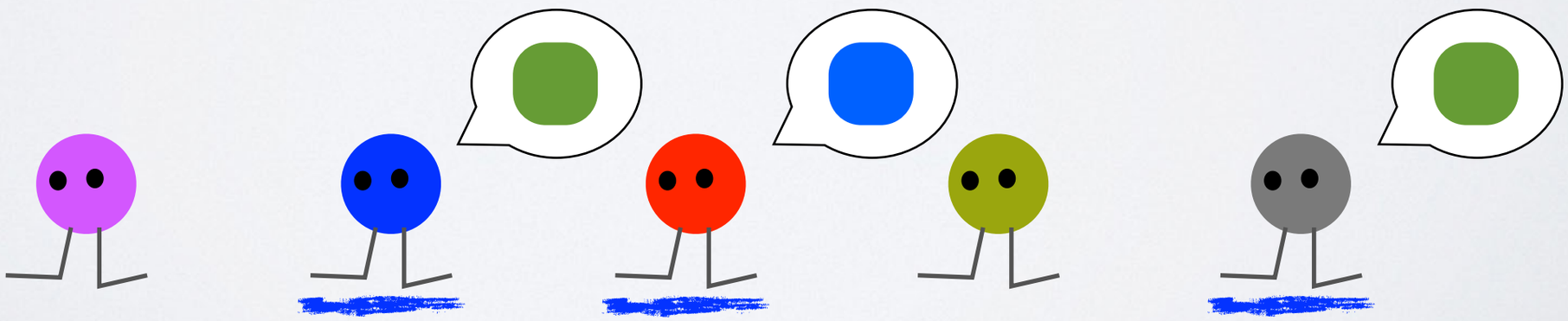


0 0 0 0 0 0

Algorithm for Eliciting Condorcet



0 0 1 0 0 0



Theorem. *If an ϵ-Condorcet winner exists, then $O(\frac{m}{\epsilon^4} \ln^2 \frac{m}{\delta\epsilon^2})$ comparisons are sufficient to find a 2ϵ-Condorcet winner w.p. at least $1 - \delta$.*

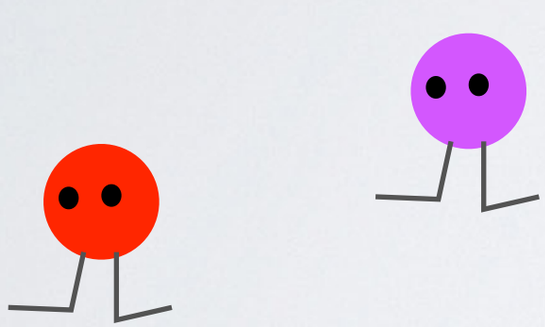
Again, note:

- Independent of n
- $O(\log^2 m)$ per person if $m < n$

There is hope for efficient scaling!

Honesty, trust, and efficient elicitation

We want verification



This means $\Omega(mn)$...

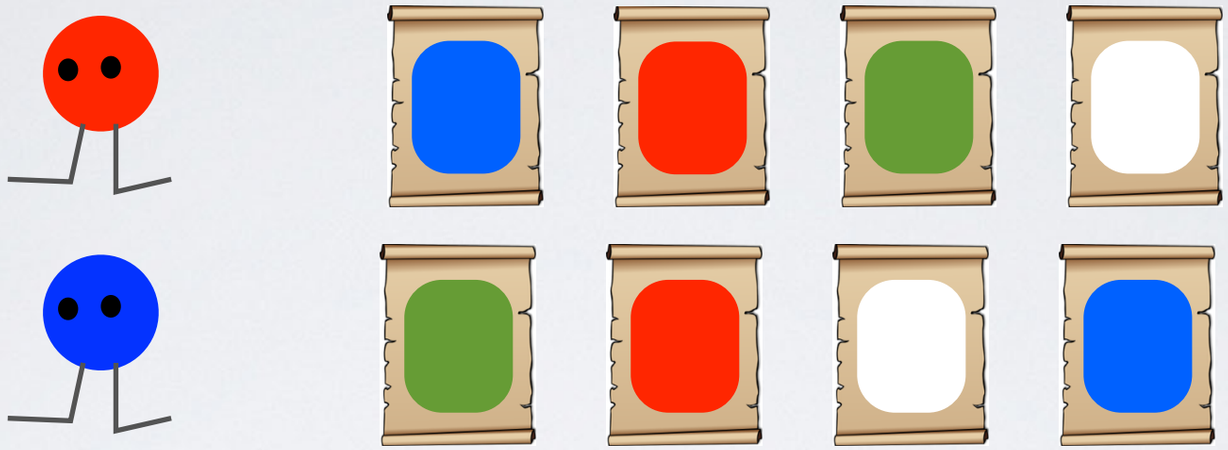
What about other social choice functions?

What about other social choice functions?

We need a more general notion of approximation

An approximate winner is one that could have *resulted from a small perturbation* in voter preferences

Ex 1: Kendall-tau distance



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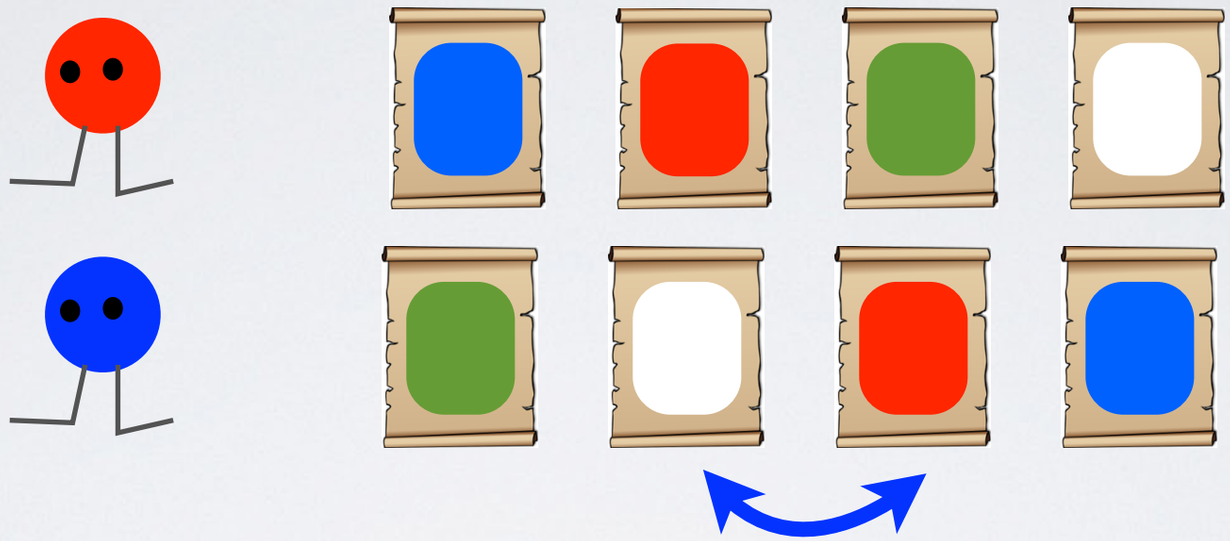

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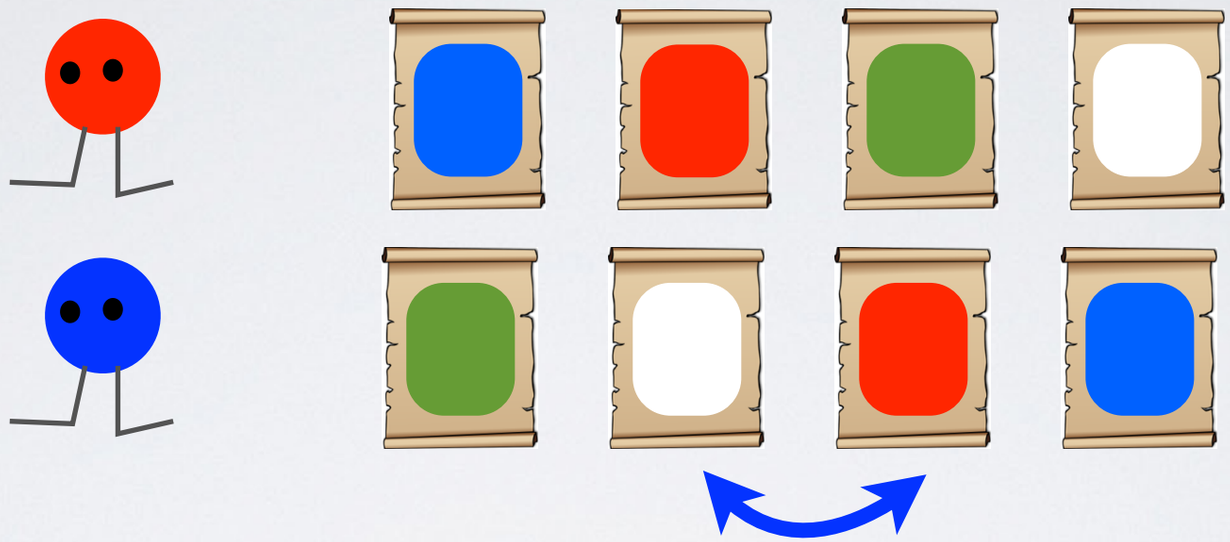

Definition. *$\hat{w}$ is an ϵ winner for $f, \mathcal{P}$ if*

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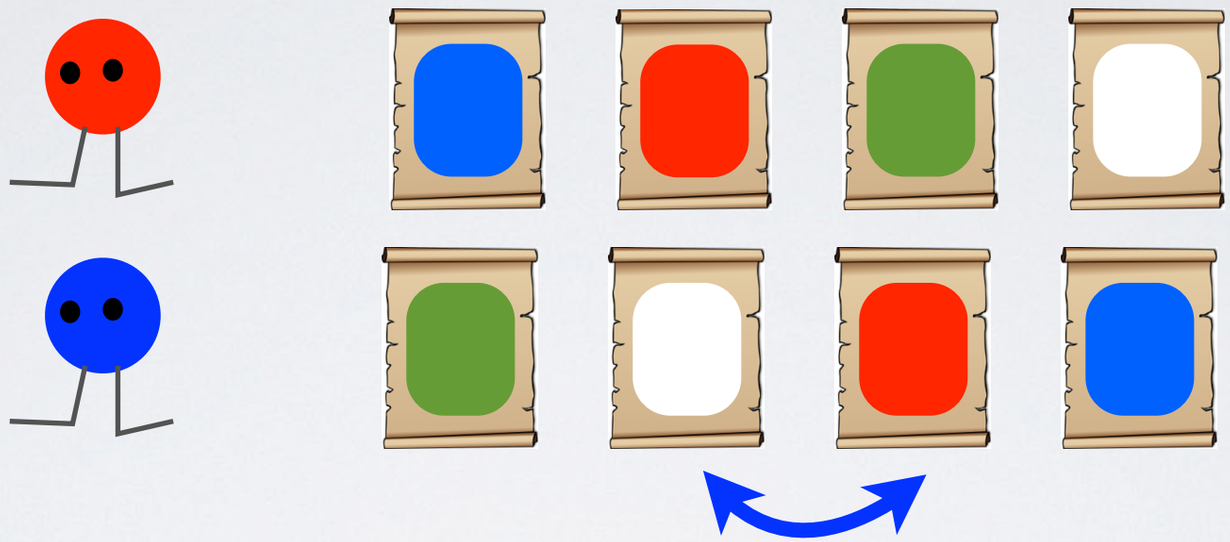

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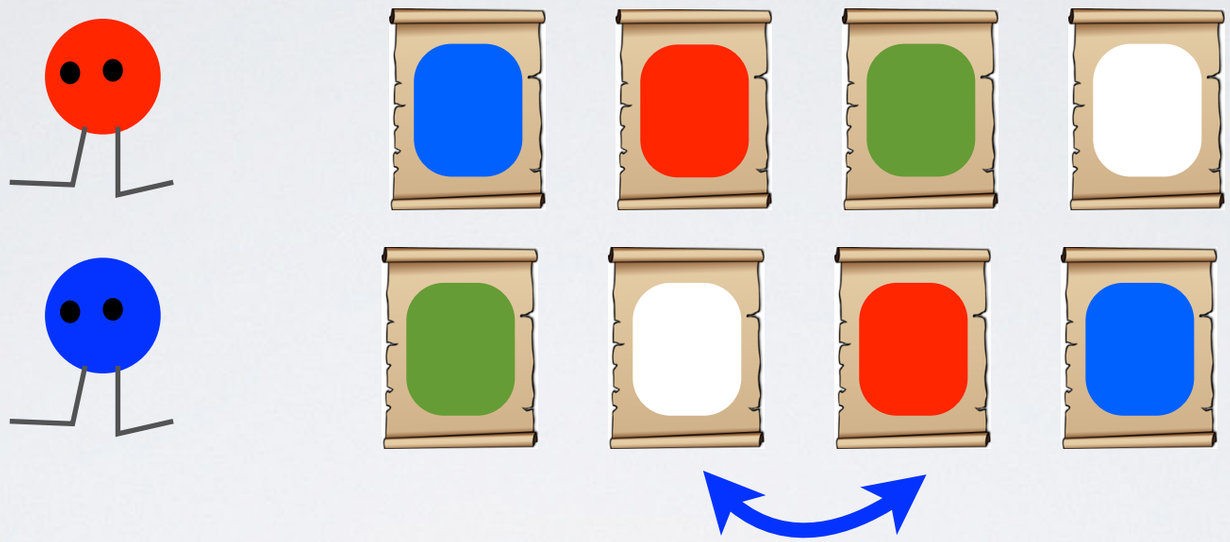

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1. *$\exists\, \mathcal{P}'$ such that $\hat{w} = f(\mathcal{P}')$*
2. *$\mathcal{P}$ and $\mathcal{P}'$ differ in at most $\epsilon n \binom{m}{2}$ comparisons.*

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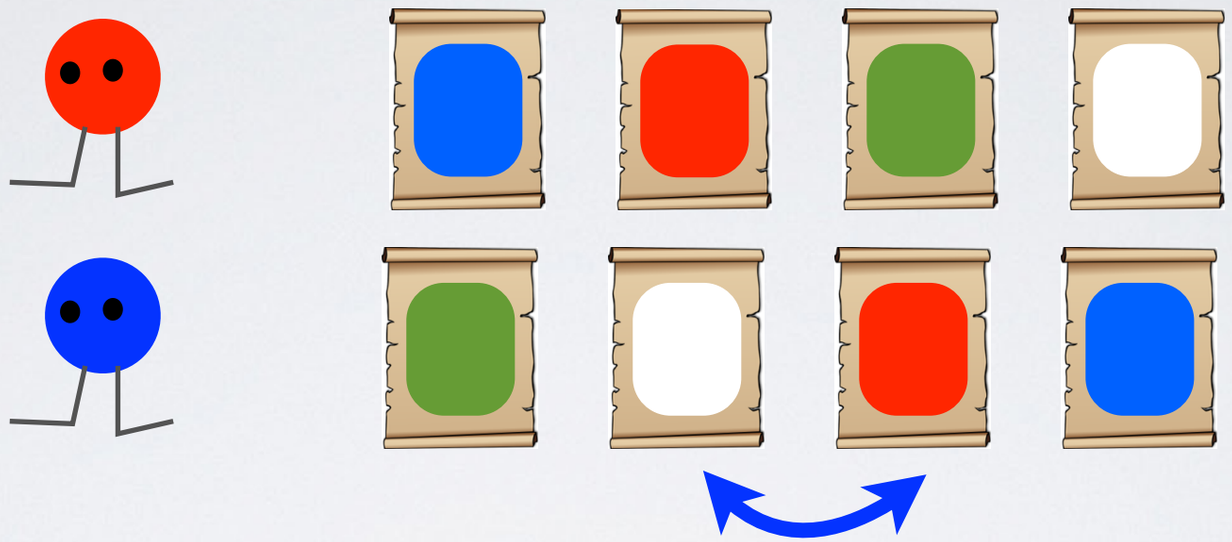

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✗

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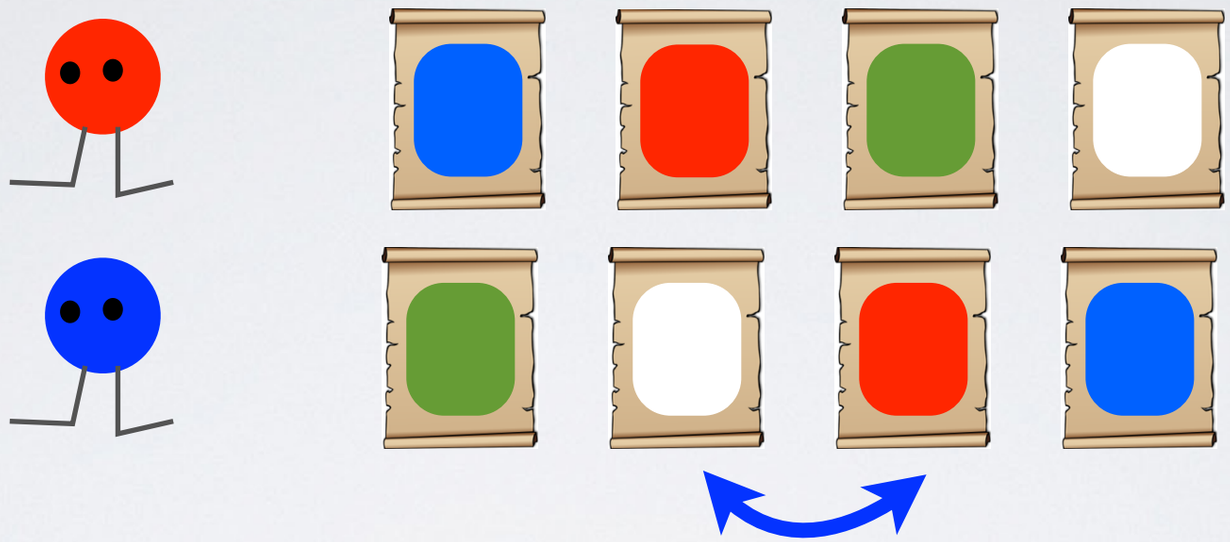

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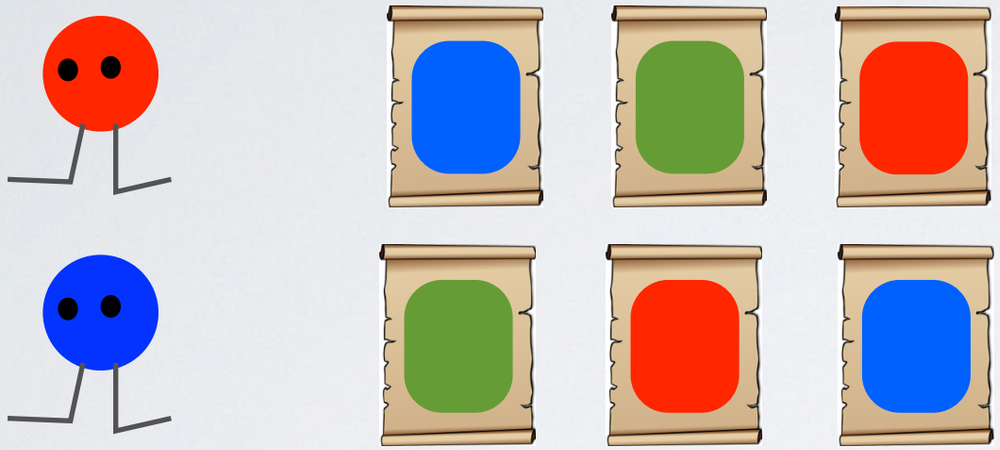
- Prior algorithm still works for Borda
- But not easy to think about others

Ex 2: Tournament distance



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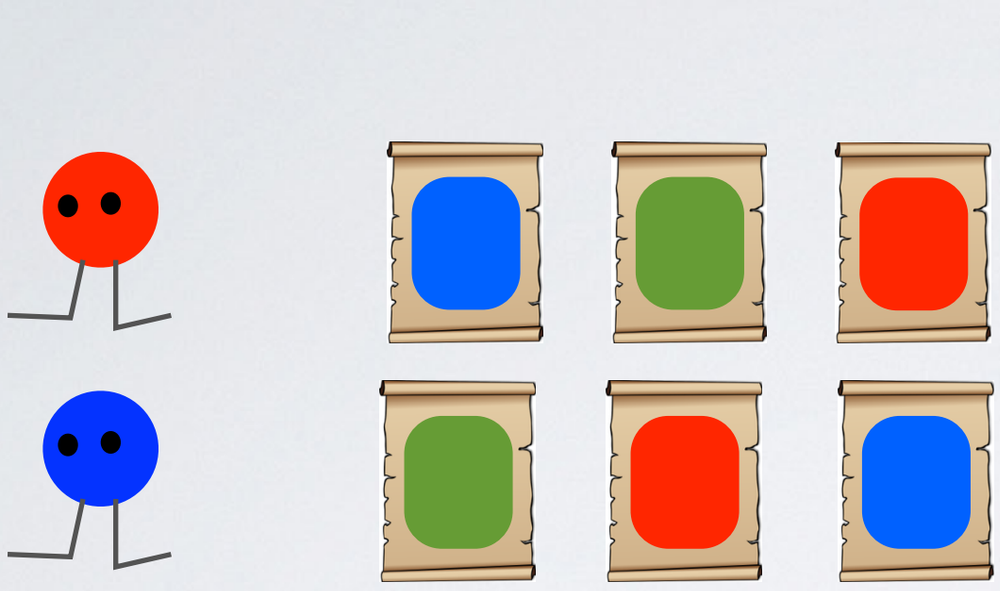

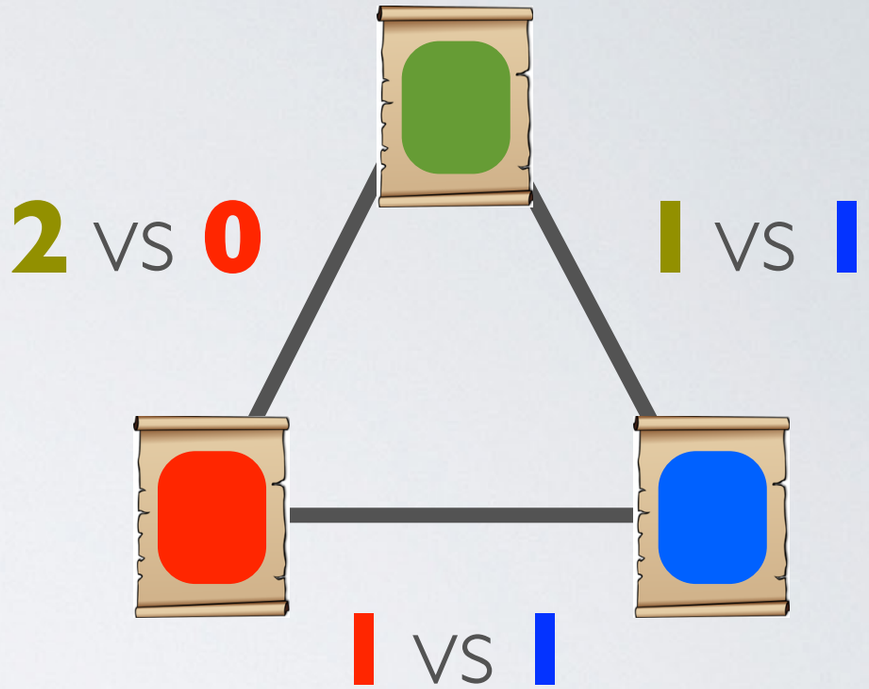

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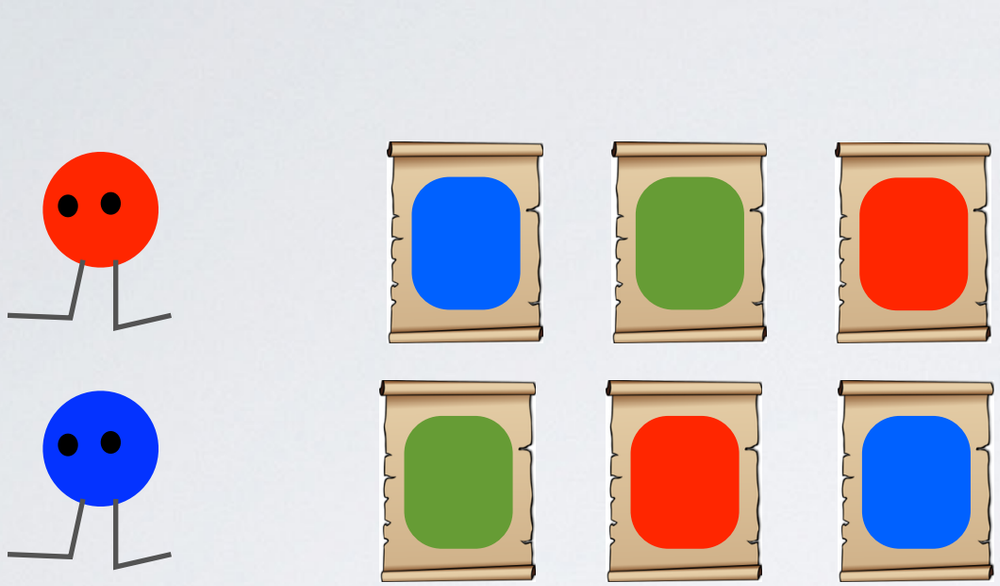

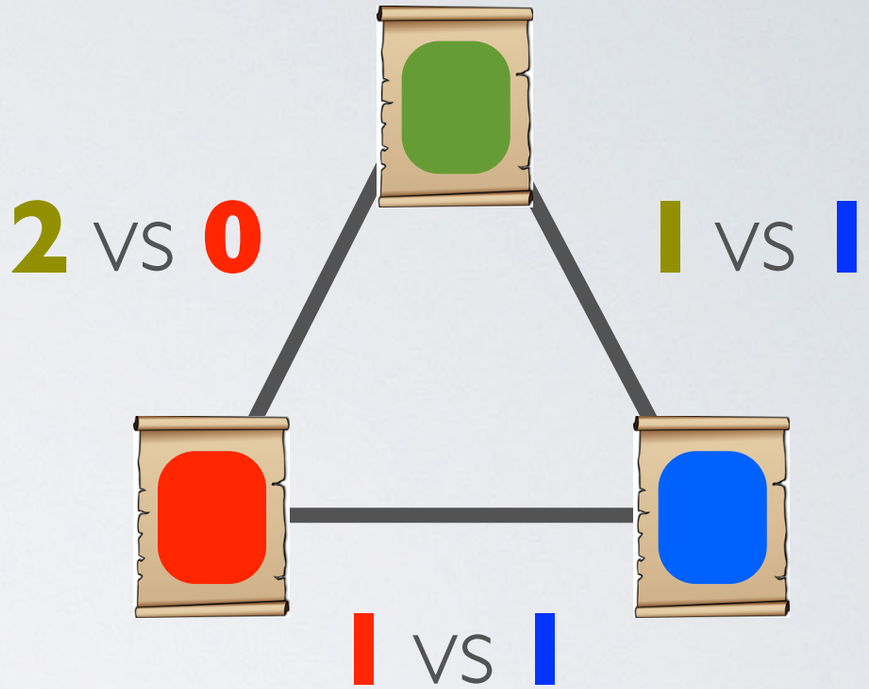

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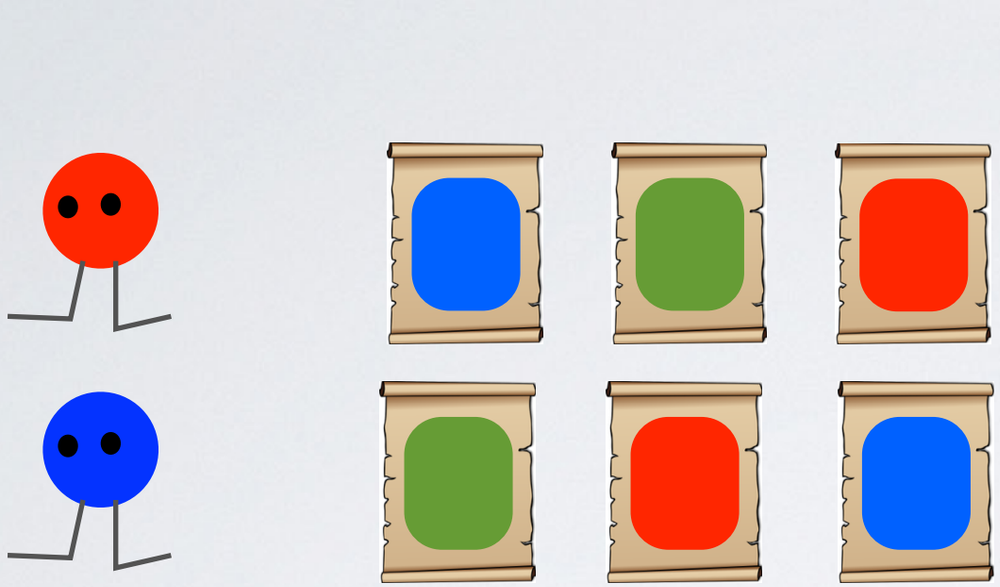

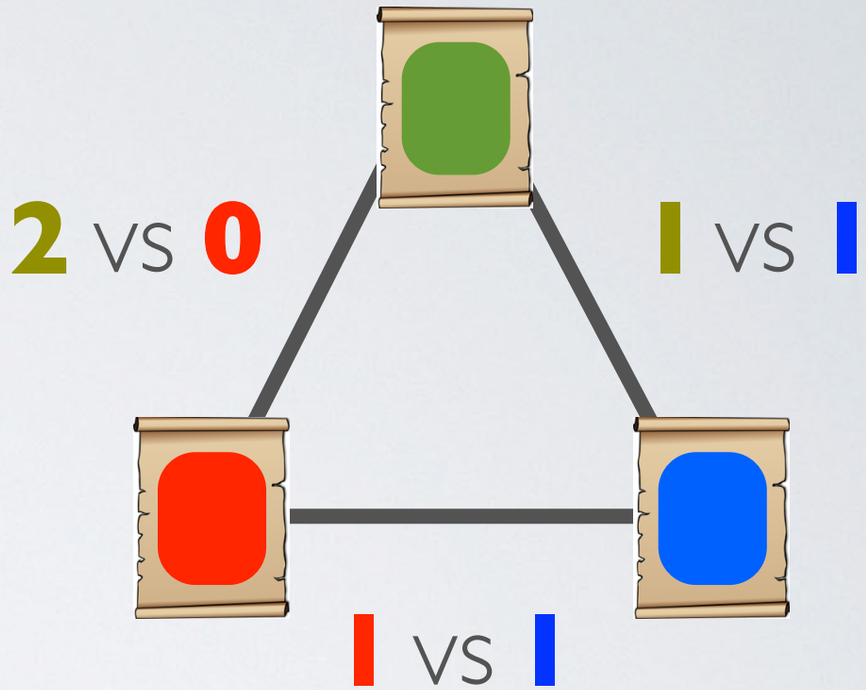

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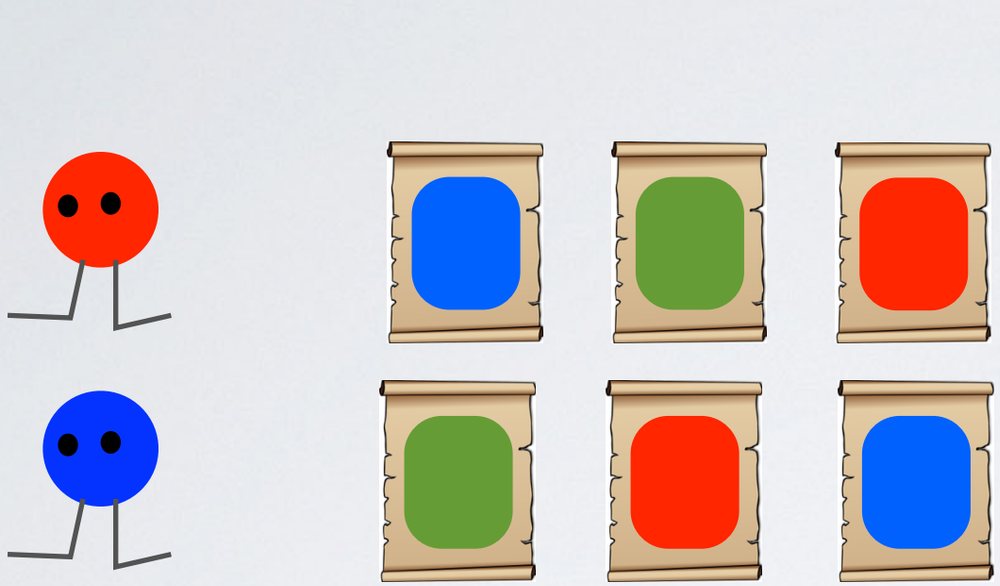

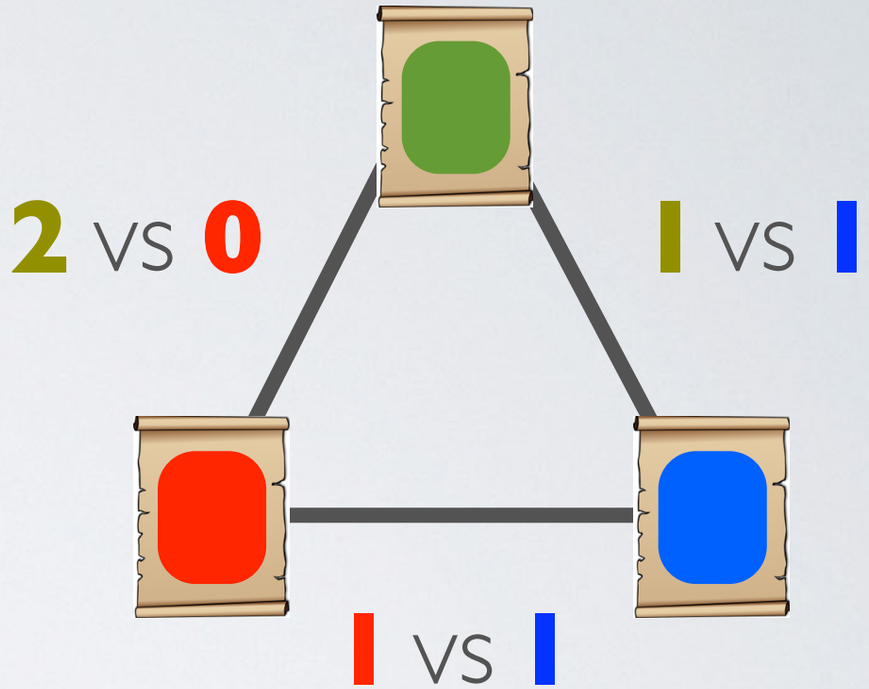

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- Prior algs work for Borda and Condorcet
- $\tilde{O}(m\sqrt{n}/\epsilon)$ suffices for all tournament social choice functions

Finland off-road traffic law

Tanja Aitamurto, Helen Landermore, David Lee

Comparing

Complete the following actions:

16 / 101 completed

Compare Instructions

Question

Location of routes. What would be the best way to situate snowmobile routes?

Ideas

Undeveloped land beyond residential areas should be made available as free-riding zones. [Background Information]

Better Idea

There should be more routes in the Eastern and Northern parts of Finland.

Better Idea

Scoring

Complete the following actions:

18 / 101 completed

Rate Instructions

Question

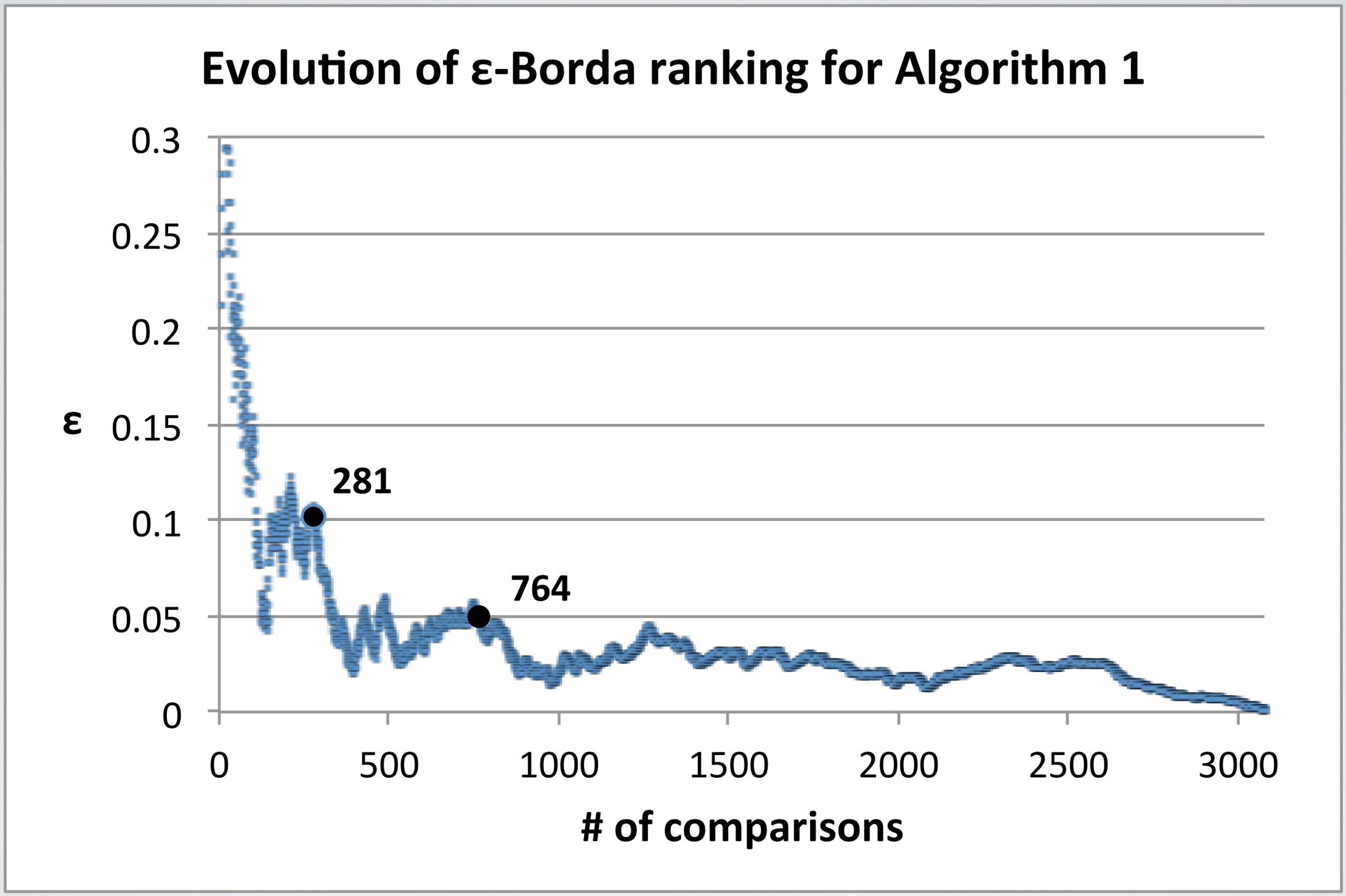
Establishing new routes. *What would be the best way to determine the time frame of route permits?*

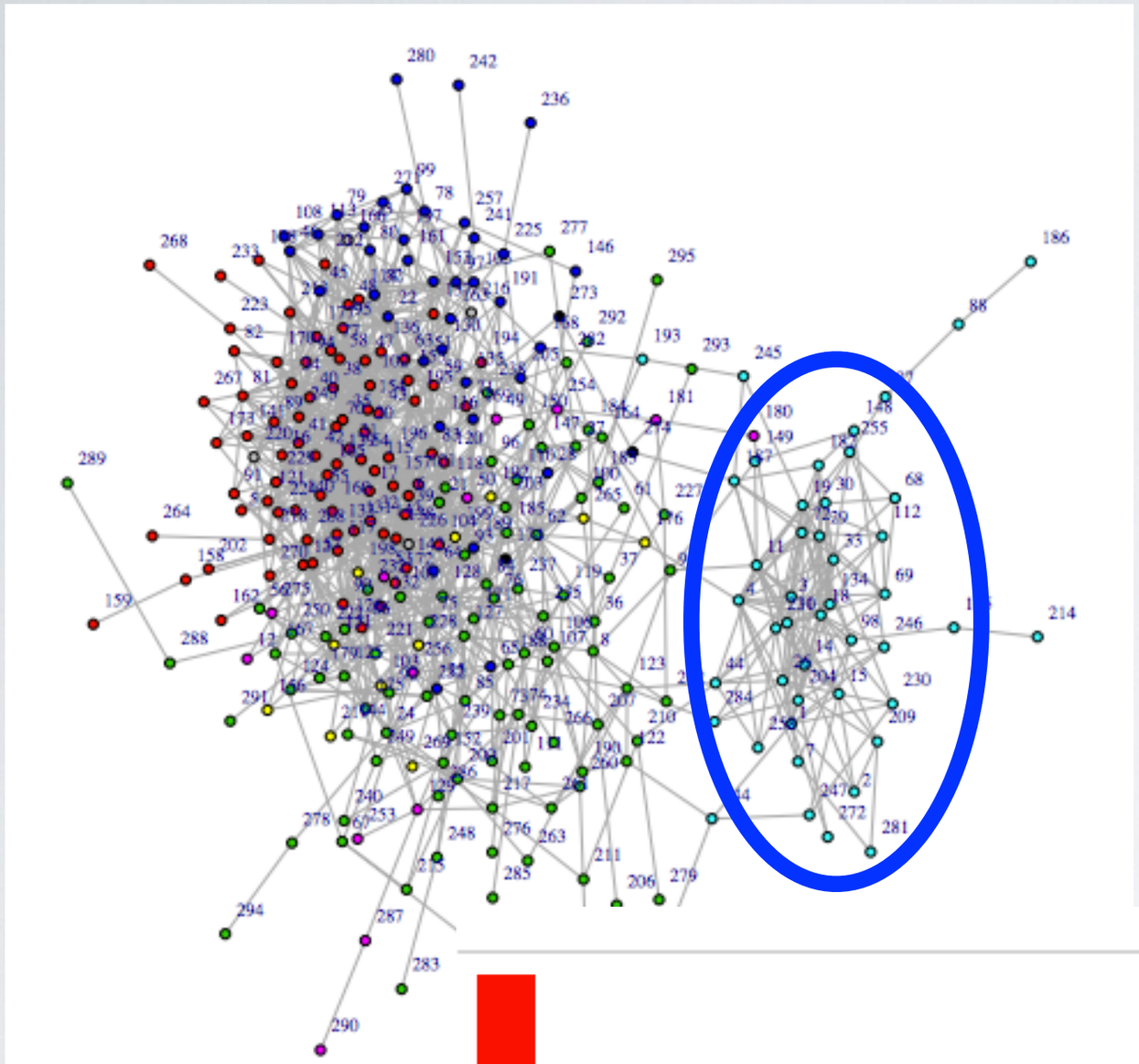
Ideas

If a route permit has been granted but the feasibility of the route plan is not inspected on a yearly basis, the route permit should be automatically annulled. [Background Information]

Arvioi idea valitsemalla sopiva määrä tähtiä.

Submit


Evolution of ε-Borda ranking for Algorithm 1
0.3
0.25
0.2
0.15
0.1
0.05
0
ε
281
764
0
500
1000
1500
2000
2500
3000
# of comparisons



Detection of a minority cluster

(ratings are key)