SEISMIC TOMOGRAPHY: A GIANT INVERSE PROBLEM

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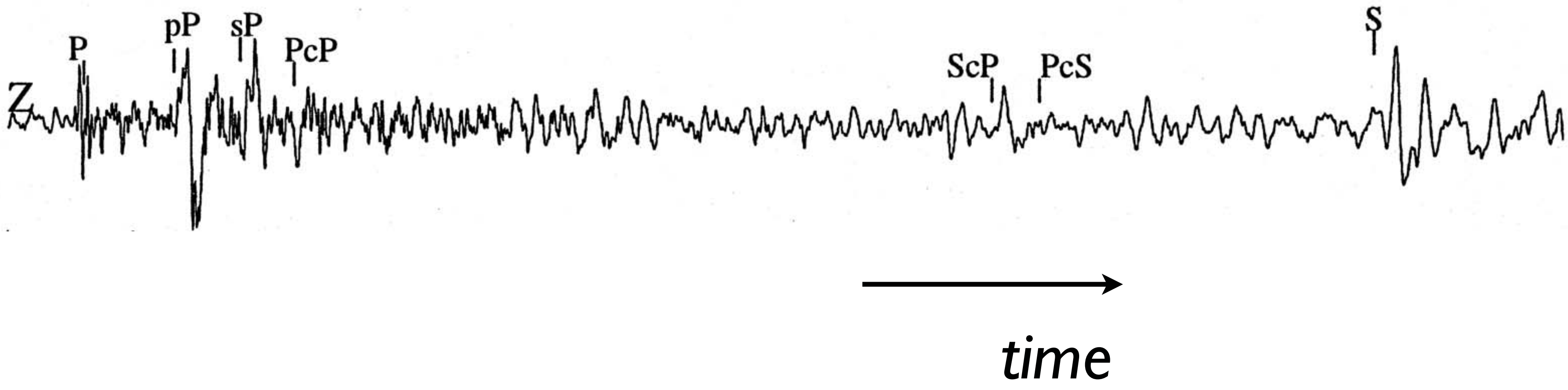
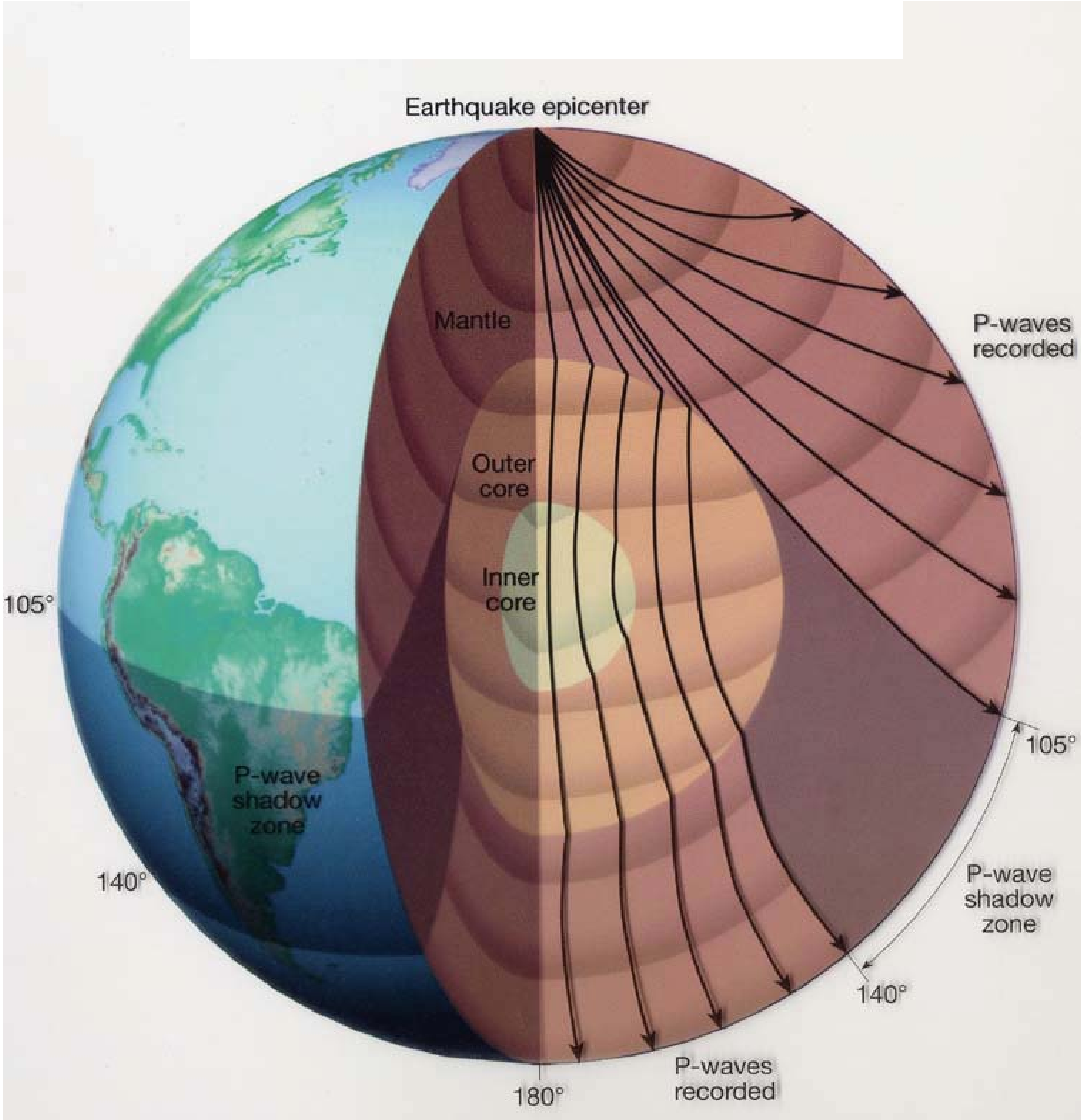
Thanks to: Jean Charlety, Dylan Mikesell, Sergey Voronin

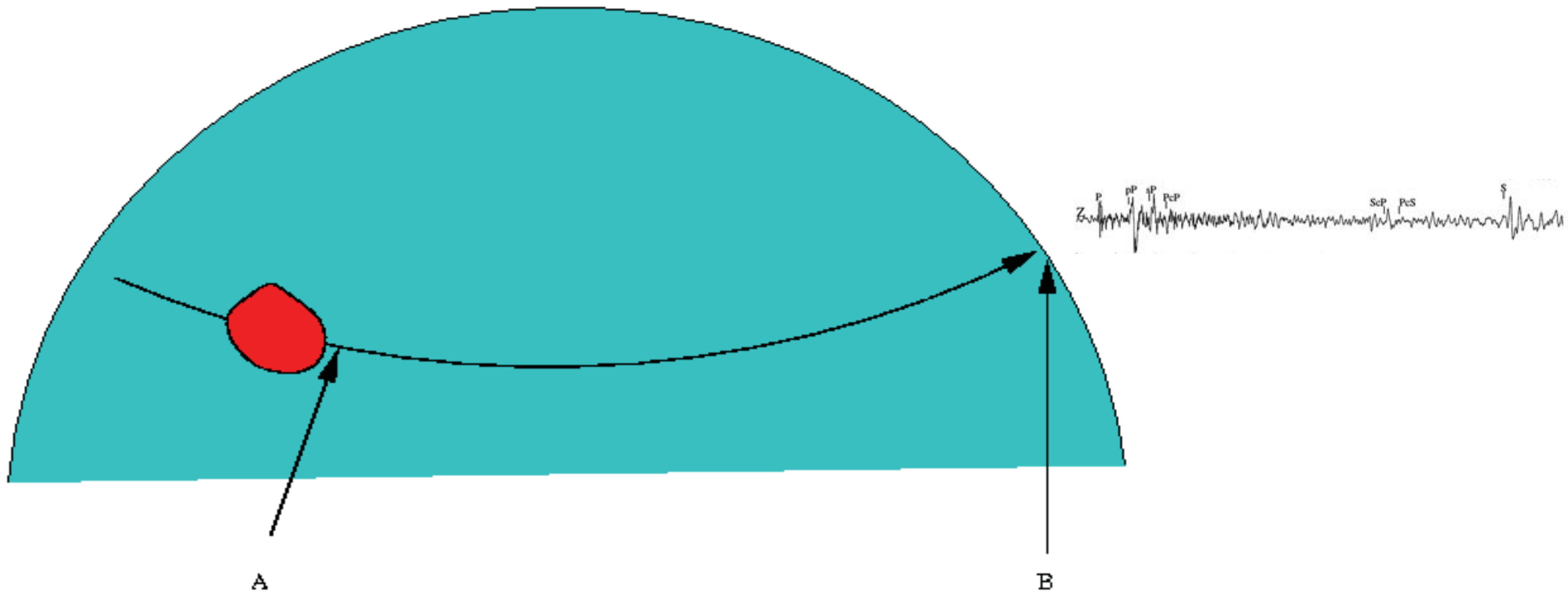
Seismic waves: *travel times*



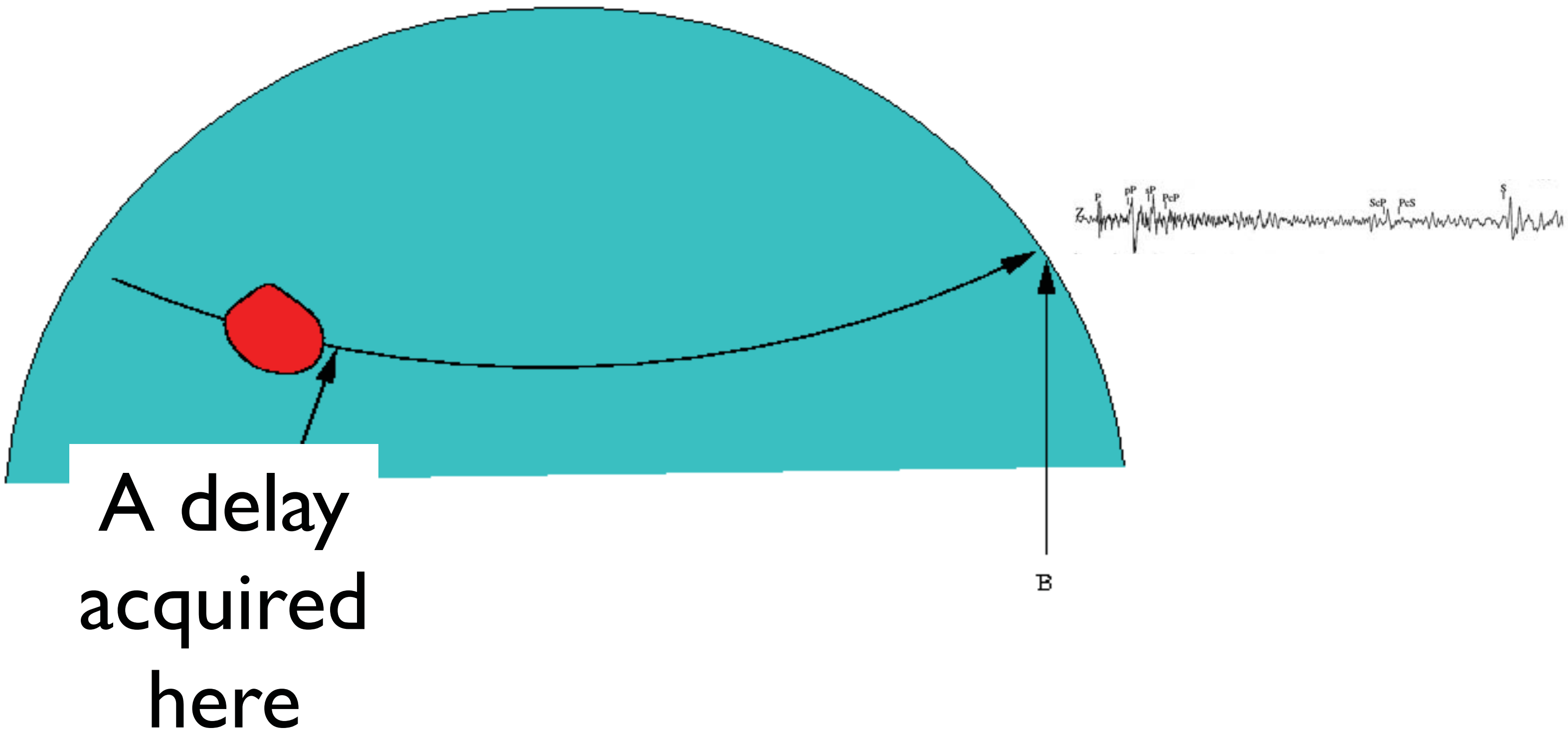
$$T \approx \int_{P_0} \frac{\mathrm{d}s}{c(\boldsymbol{r})}\,.$$


Earthquake epicenter
Mantle
P-waves recorded
Outer core
Inner core
105°
P-wave shadow zone
140°
105°
P-wave shadow zone
140°
180°
P-waves recorded

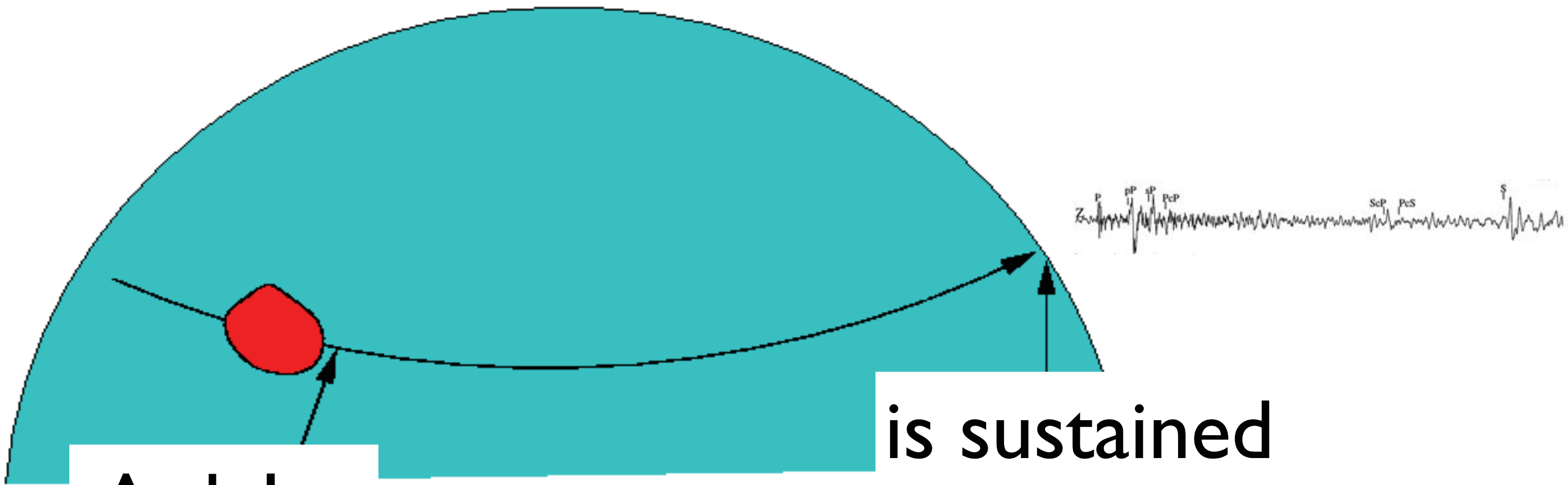
The tomography paradigm



The tomography paradigm



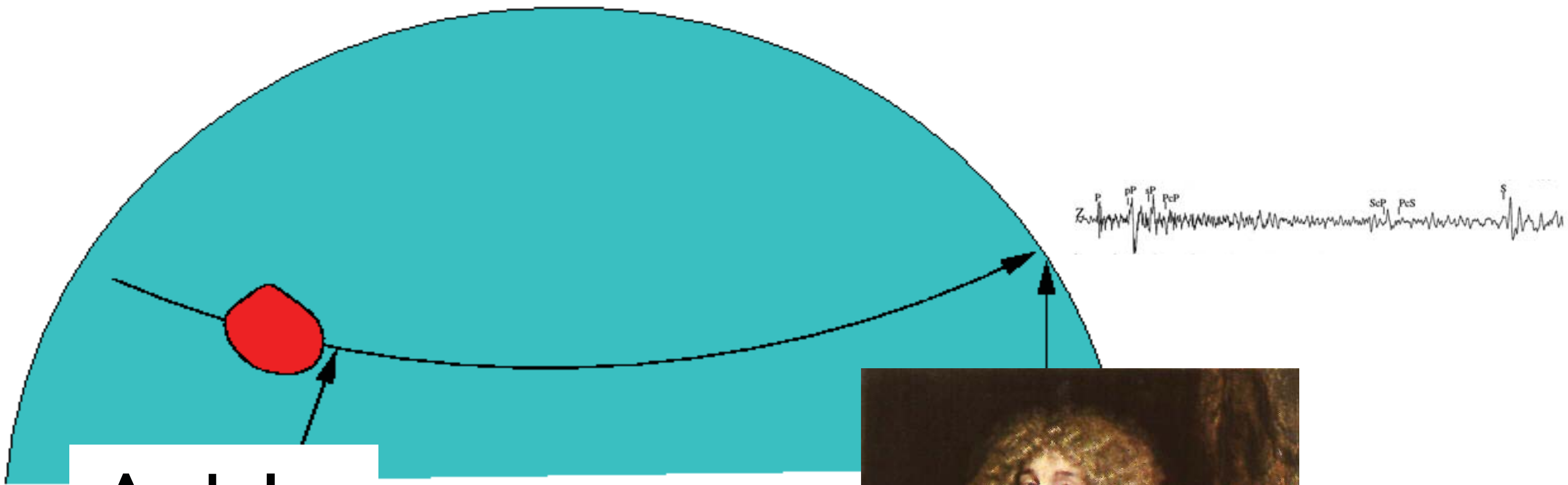
The tomography paradigm



A delay acquired here

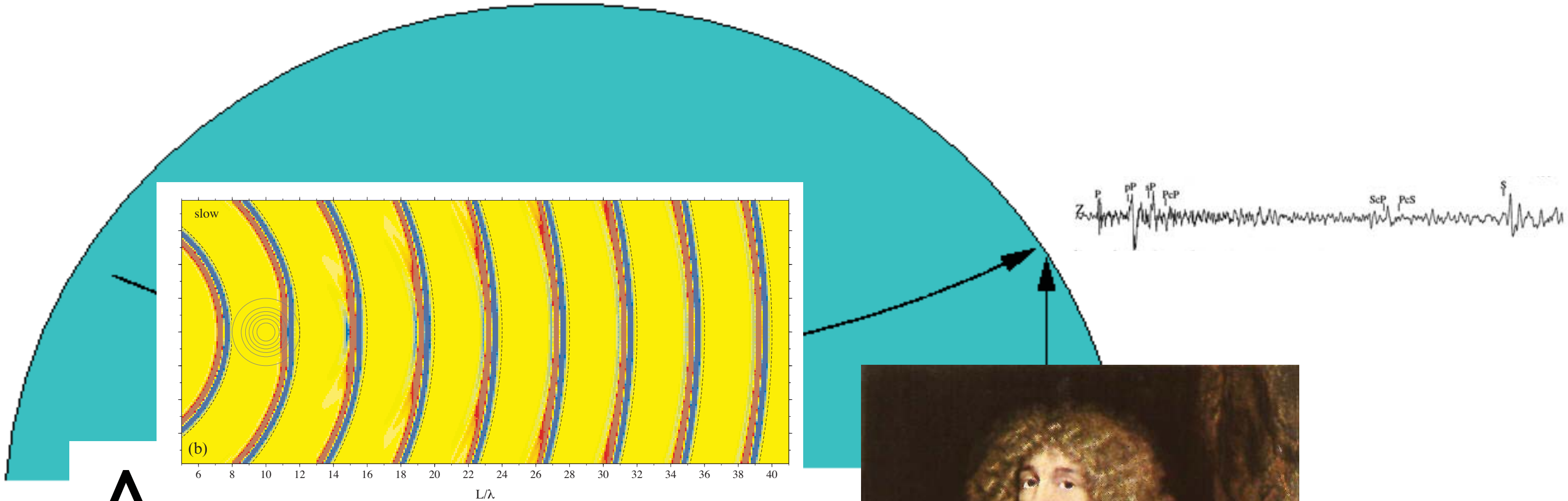
is sustained over the wavepath

The tomography paradigm



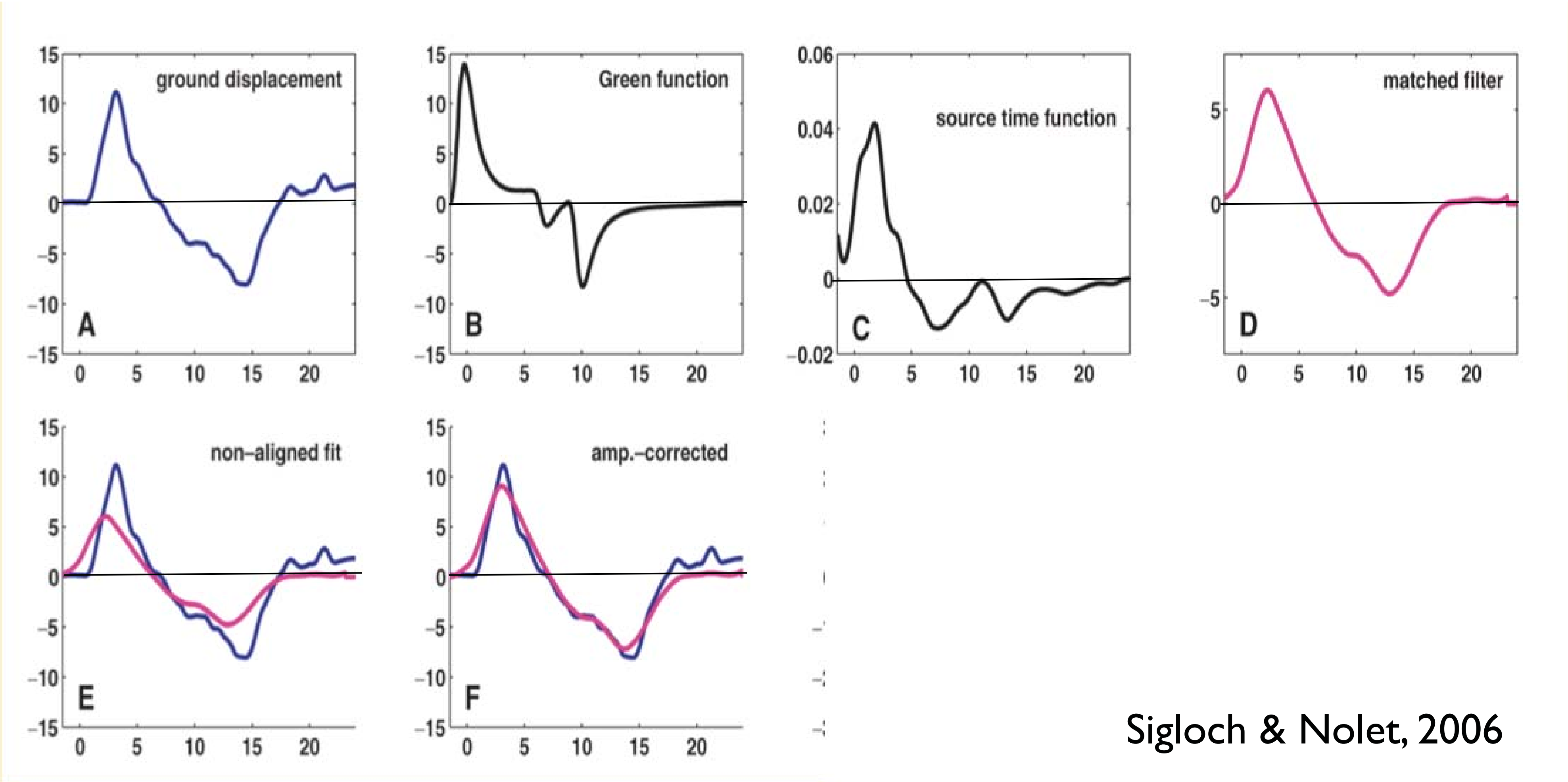
A delay acquired here

The tomography paradigm



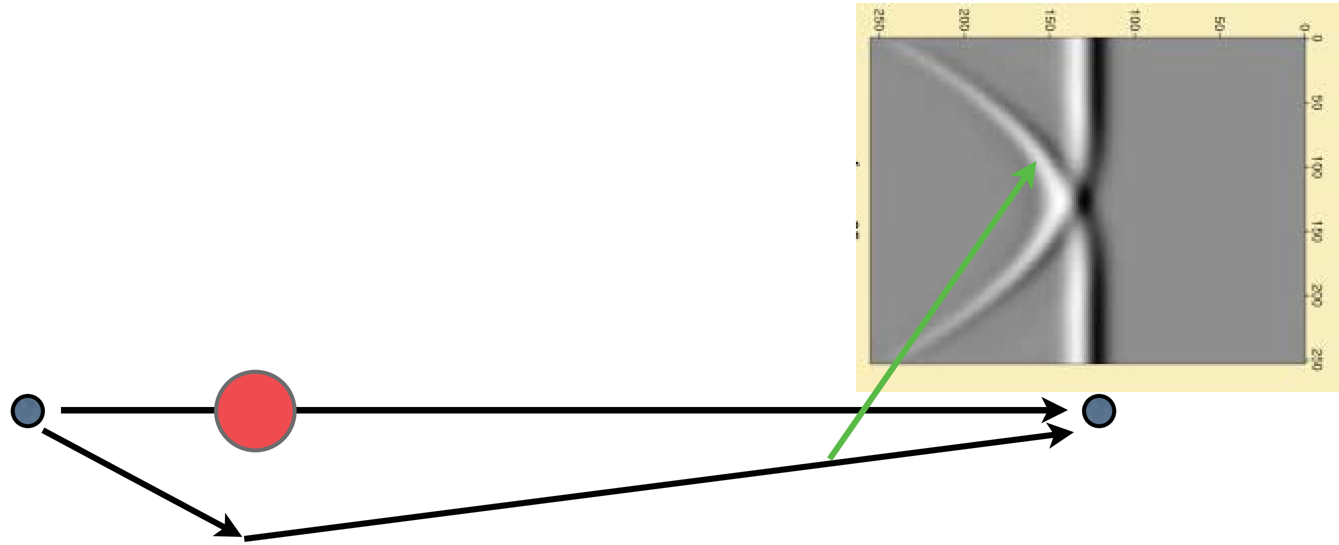
A delay
acquired
here

Cross-correlation



Sigloch & Nolet, 2006

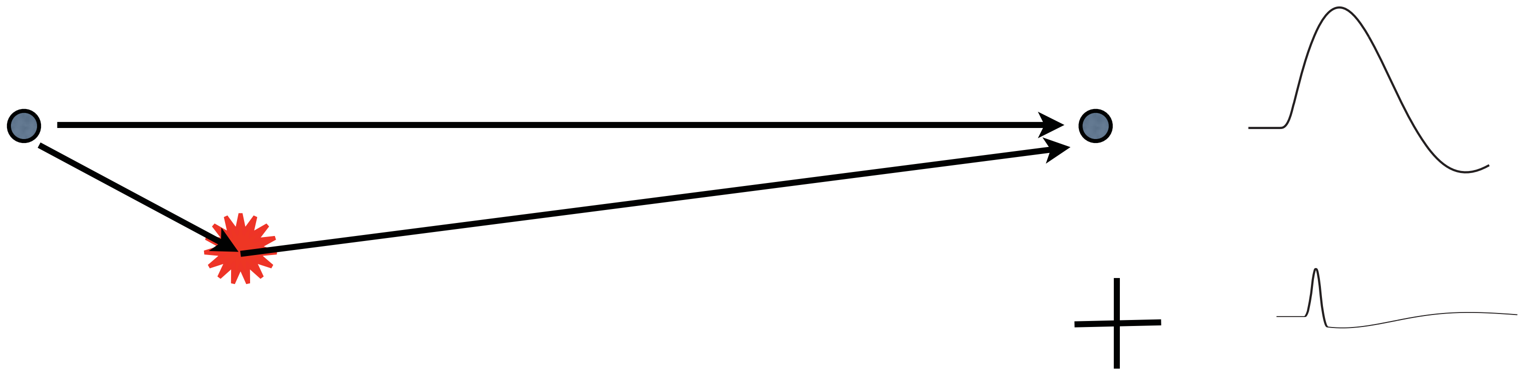
Interpretation



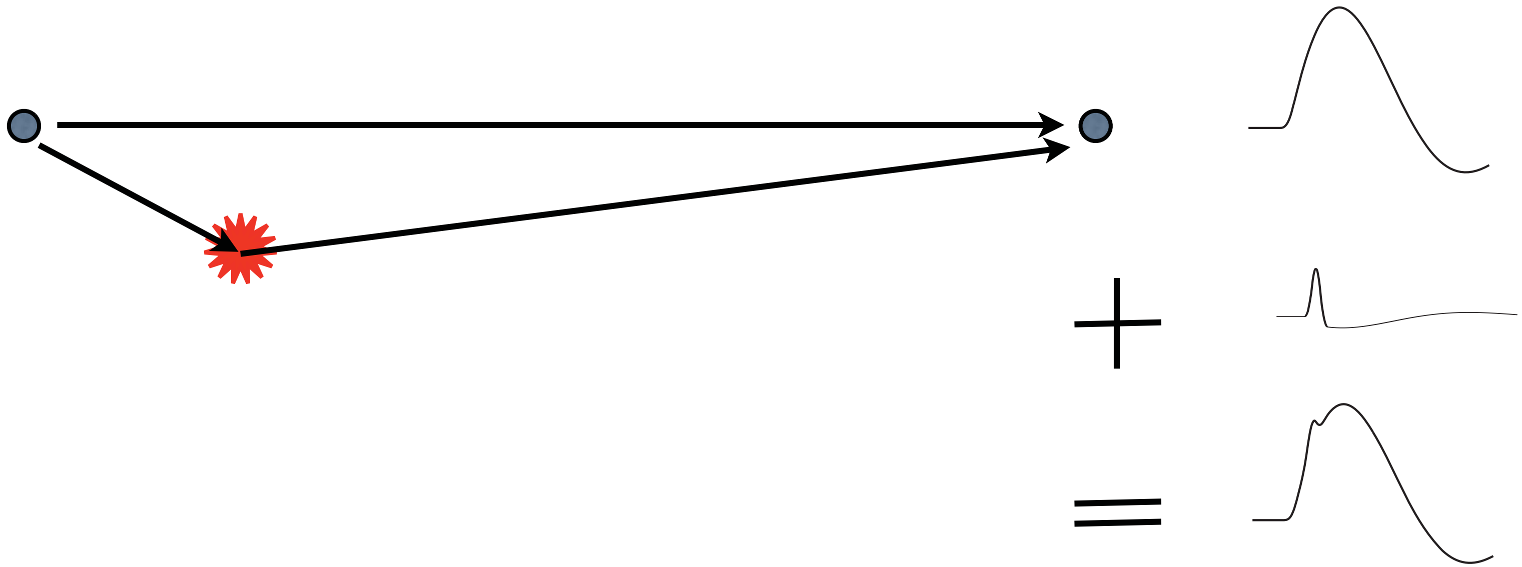
A wavefront heals because energy diffracts *around* the anomaly. Can we correct for it?

Born approximation

Born approximation



Born approximation



scattered wave perturbs cross-correlation time

Born

$$\delta T = -\frac{\delta\dot{\gamma}(0)}{\ddot{\gamma}(0)} = -\frac{\int_{-\infty}^{\infty} \dot{u}(t')\boxed{\delta u(t')}\mathrm{d}t'}{\int_{-\infty}^{\infty} \ddot{u}(t')u(t')\mathrm{d}t'} \,.$$

scattered wave perturbs cross-correlation time

Born

$$\delta T = -\frac{\delta\dot{\gamma}(0)}{\ddot{\gamma}(0)} = -\frac{\int_{-\infty}^{\infty} \dot{u}(t')\,\delta u(t')\,\mathrm{d}t'}{\int_{-\infty}^{\infty} \ddot{u}(t')\,u(t')\,\mathrm{d}t'}$$

$$\delta\bar{u}^{\mathrm{PP}}(\omega) = \frac{\omega^2}{4\pi r V_{\mathrm{P}}^2}\left[\frac{\delta\rho}{\rho} - \frac{\delta\lambda}{\lambda+2\mu} - 2\frac{\delta\mu}{\lambda+2\mu}\right]\mathrm{e}^{\mathrm{i}k_{\mathrm{P}}r}\,\mathrm{d}V$$

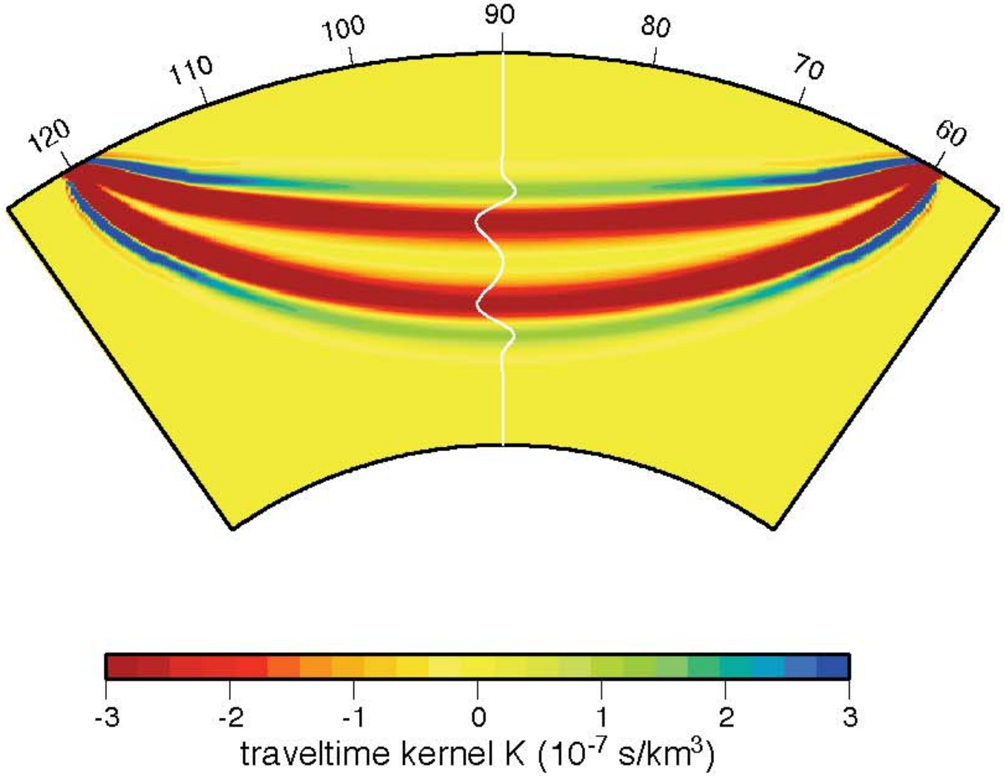
Kernels and volume integrals

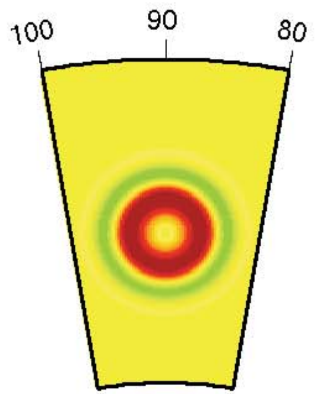
$$\delta T = \int K_{\mathrm{P}}(\boldsymbol{r}_{\mathrm{x}}) \frac{\delta V_{\mathrm{P}}}{V_{\mathrm{P}}} \mathrm{d}^3 \boldsymbol{r}_{\mathrm{x}}$$

Replace line integrals:

$$T \approx \int_{P_0} \frac{\mathrm{d}s}{c(\boldsymbol{r})} \, .$$

What kernels look like





$$\delta T = \int K_{\mathrm{P}}(\boldsymbol{r}_{\mathrm{X}}) \frac{\delta V_{\mathrm{P}}}{V_{\mathrm{P}}} \mathrm{d}^3 \boldsymbol{r}_{\mathrm{X}}$$

Computational effort for volume integrals

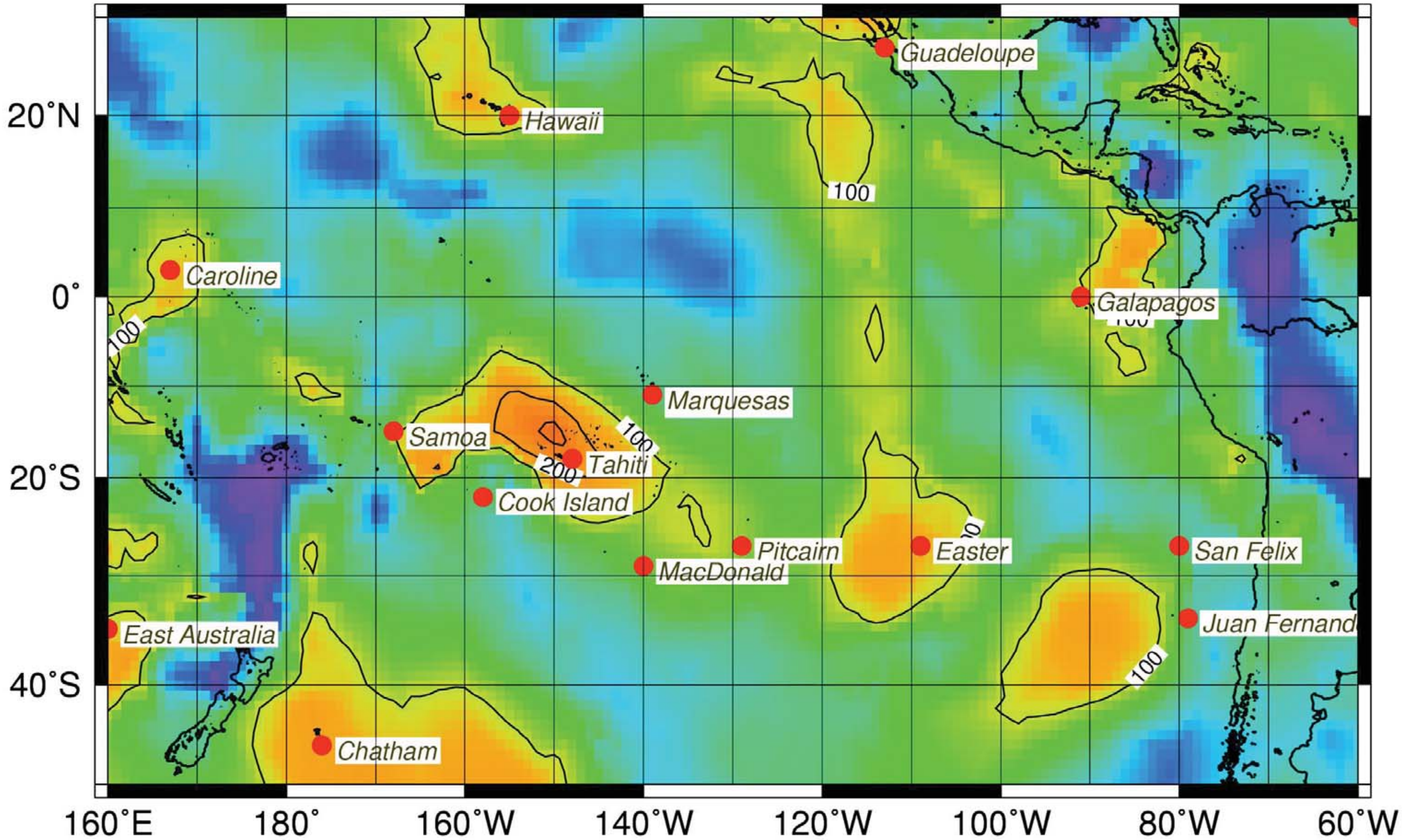
$$\propto n^3$$

Compare to line integrals:

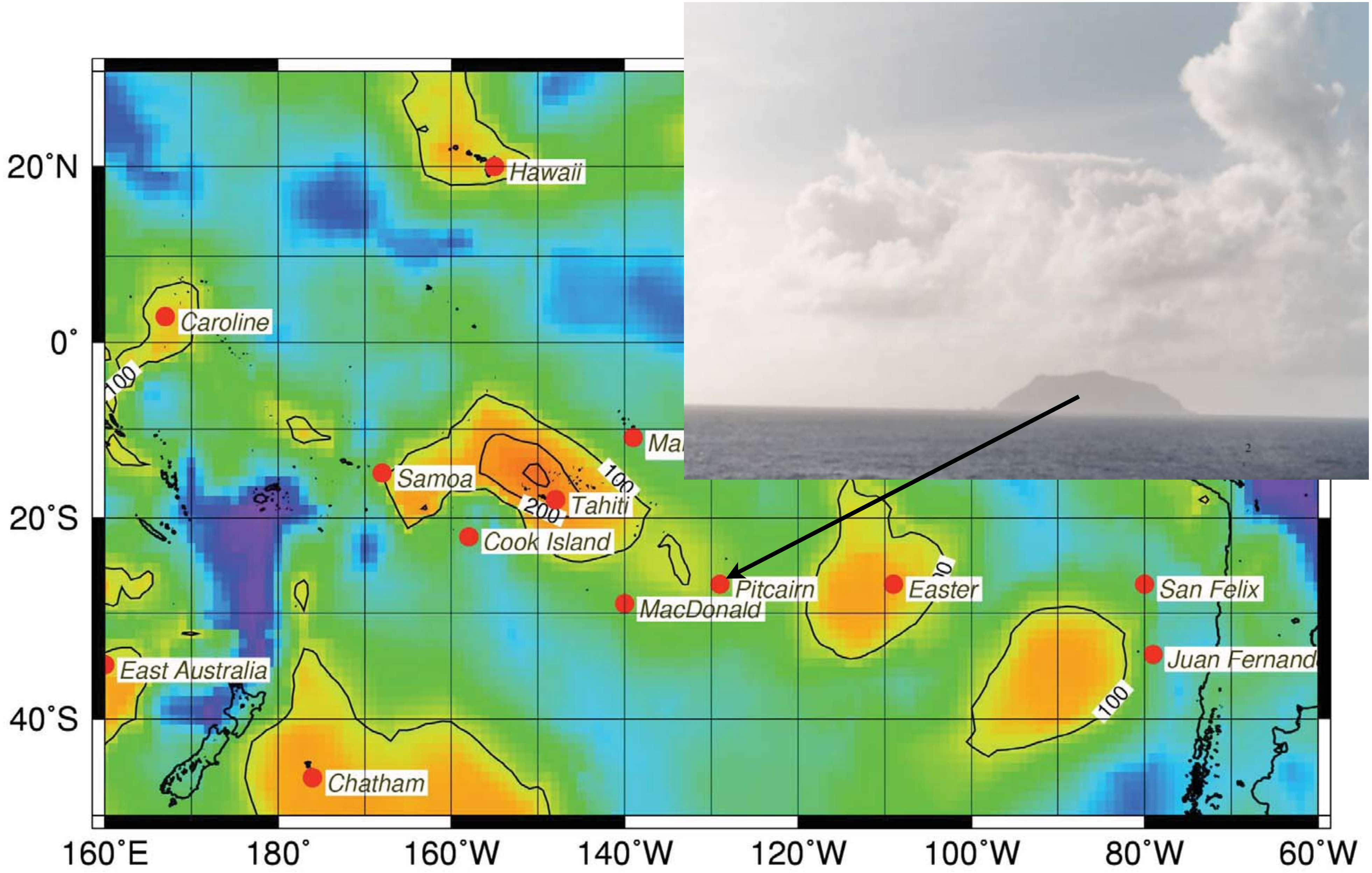
$$\propto n$$

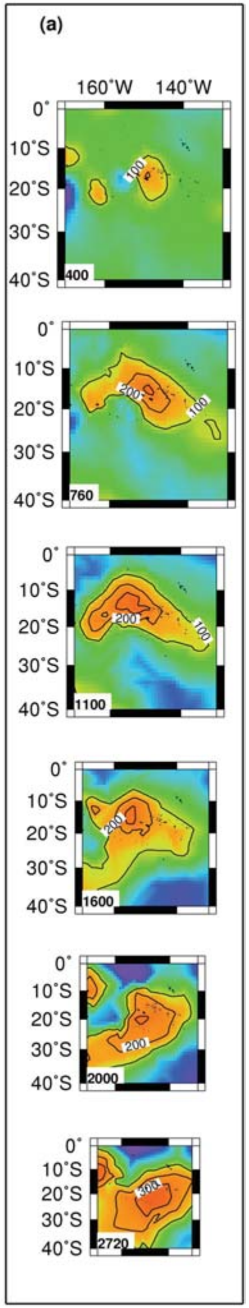
Typically *n*=100 voxels

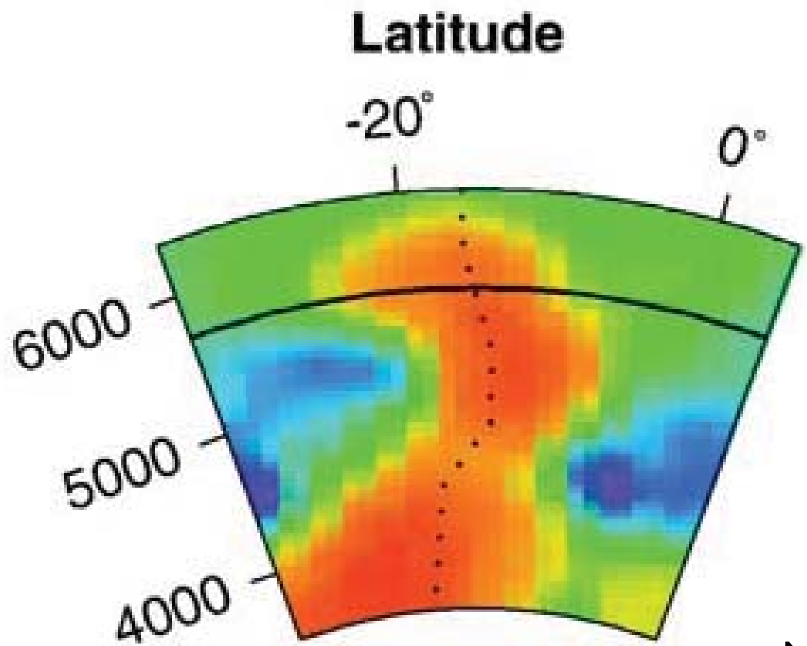
Inferred temperature at 800 km depth



Inferred temperature at 800 km depth



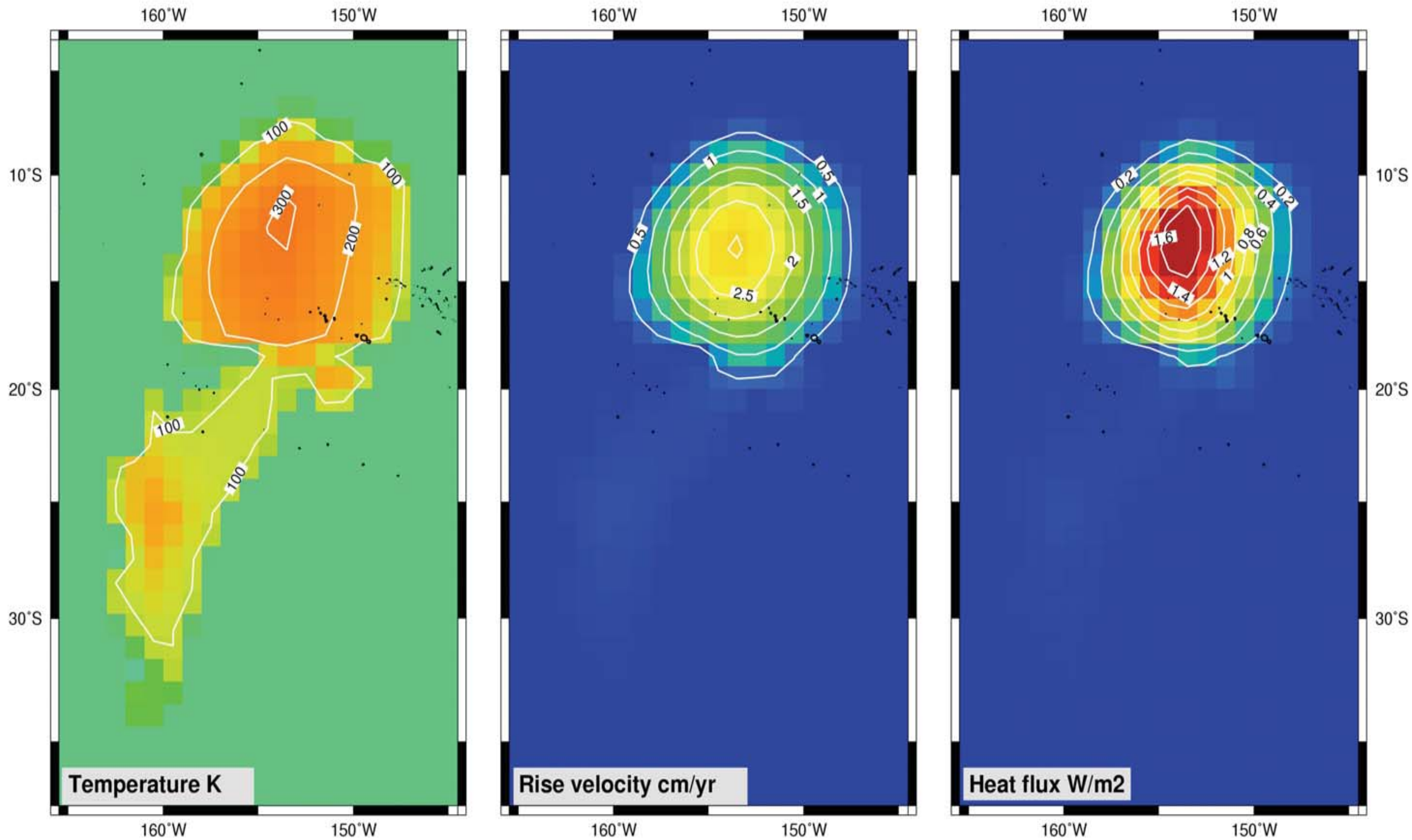




Nolet et al. 2006

Inferred heat flux

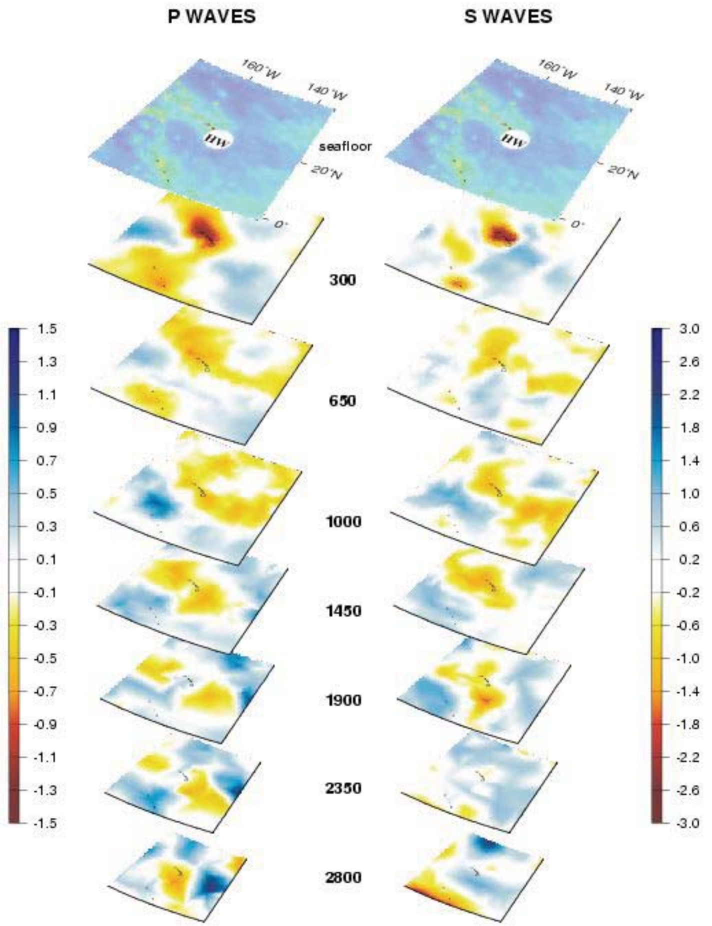
Nolet et al., 2006

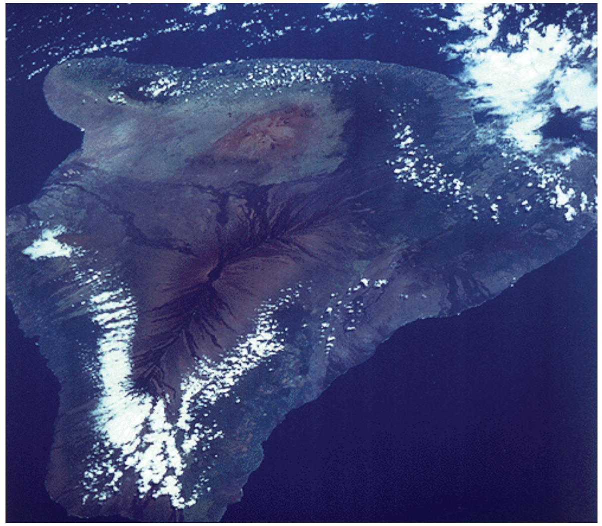


Tahiti, 1600 km depth

Early success: proof of thermal plumes in the lower mantle

Montelli et al. 2004, 2006

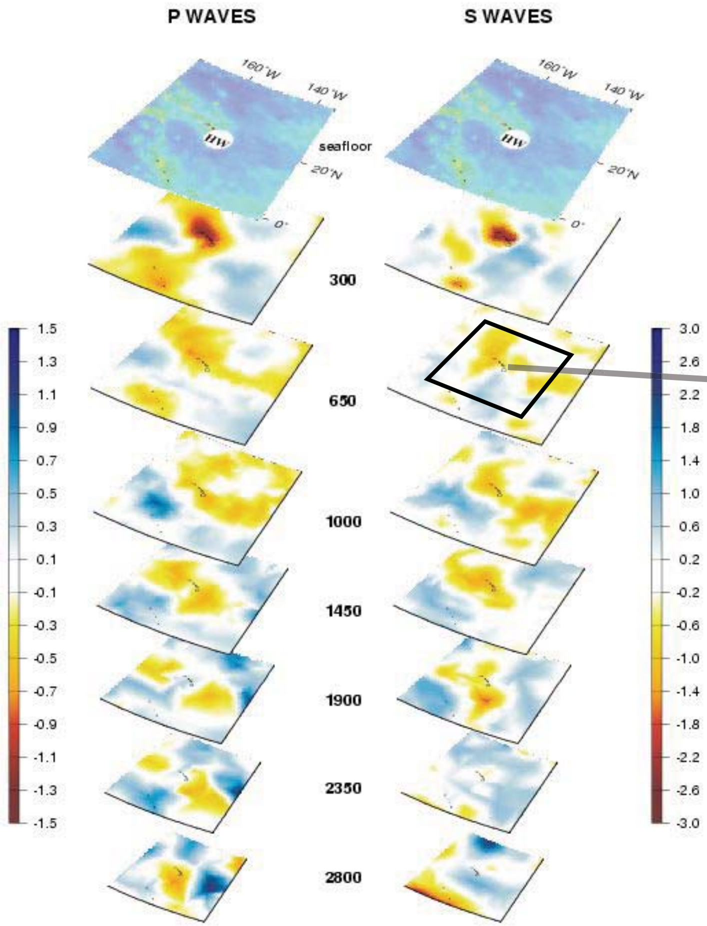


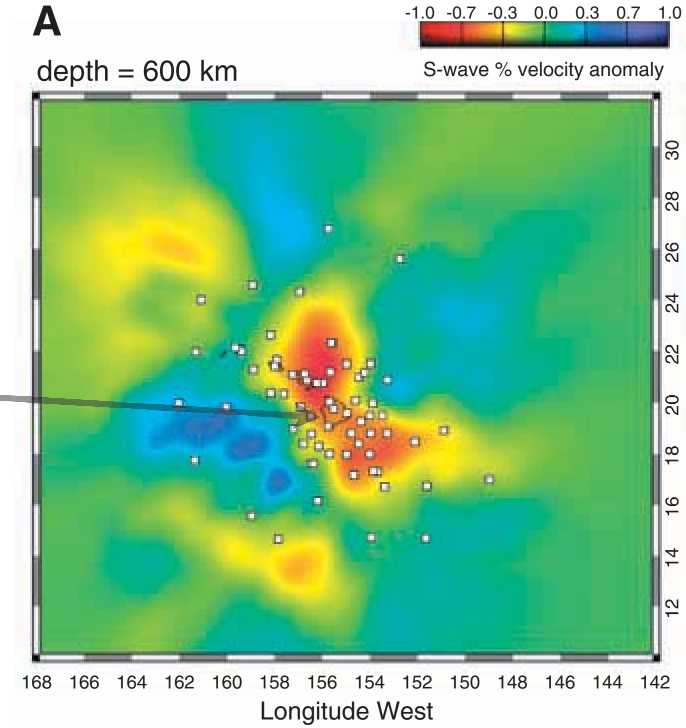


Hawaii

Early success: proof of thermal plumes in the lower mantle

Montelli et al. 2004, 2006





Wolfe et al. 2009

Hawaii

Regularization options: among all the models that fit the data, we prefer either the

- Minimum norm model
- Smoothest model
- Most compact model
- Fewest wavelets

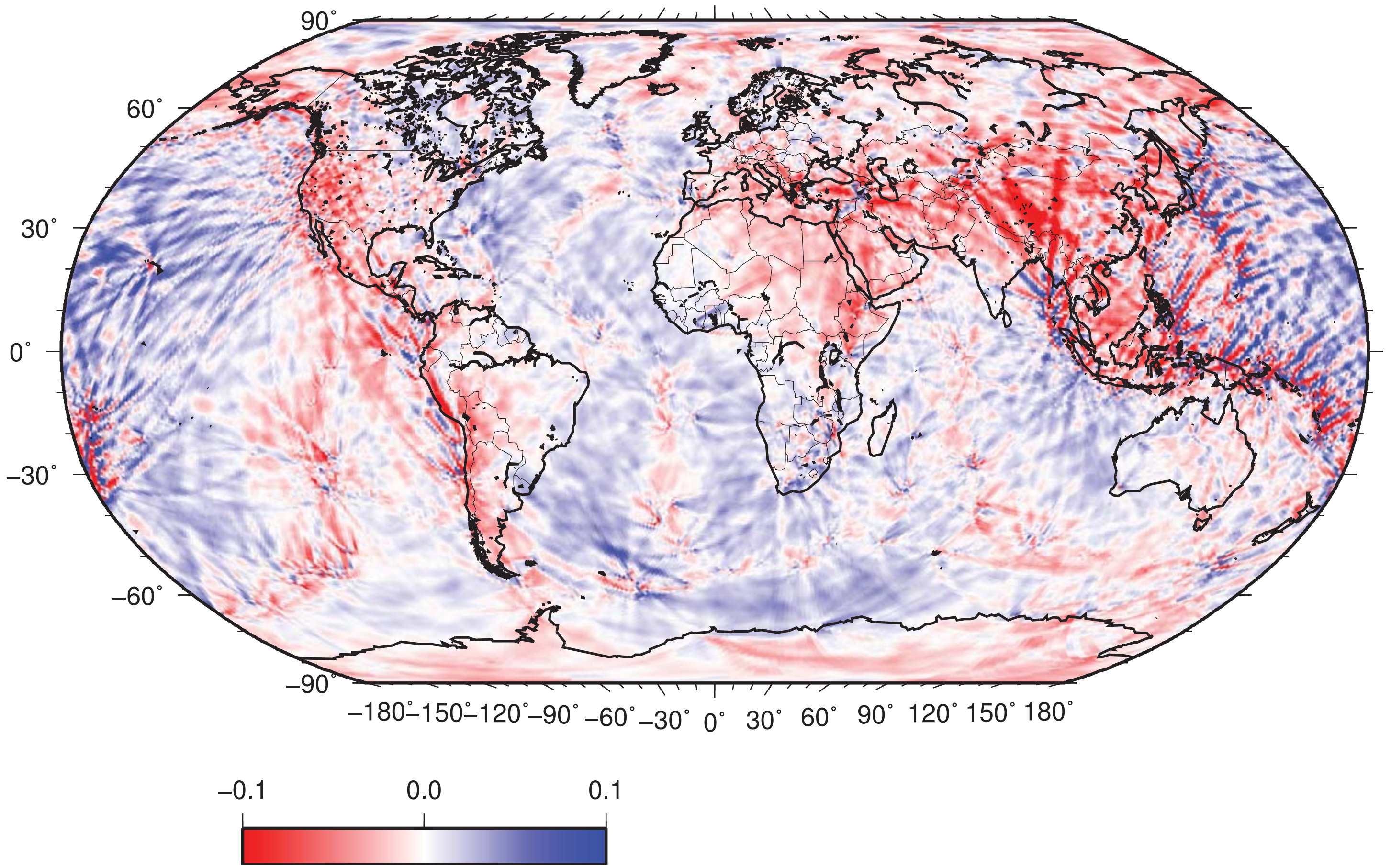
Minimum norm models

$$\text{Min } \|\boldsymbol{Am} - \boldsymbol{d}\|_2 + \epsilon\|\boldsymbol{m}\|_2$$

Smooth models

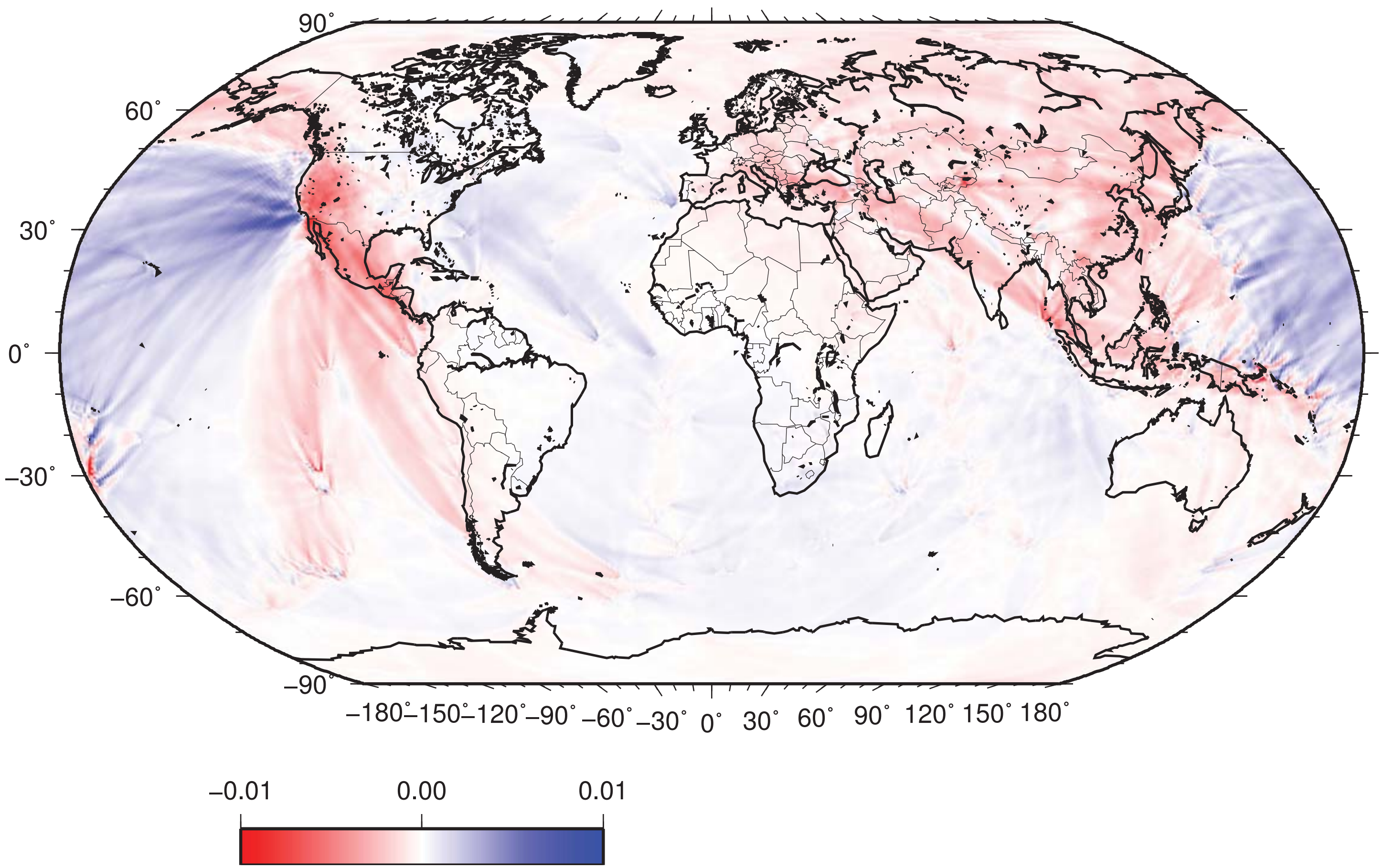
$$\text{Min } \|\boldsymbol{Am} - \boldsymbol{d}\|_2 + \epsilon\|\nabla^2\boldsymbol{m}\|_2$$

100km6



Dylan Mikesell

100km3



Dylan Mikesell

Sparse models

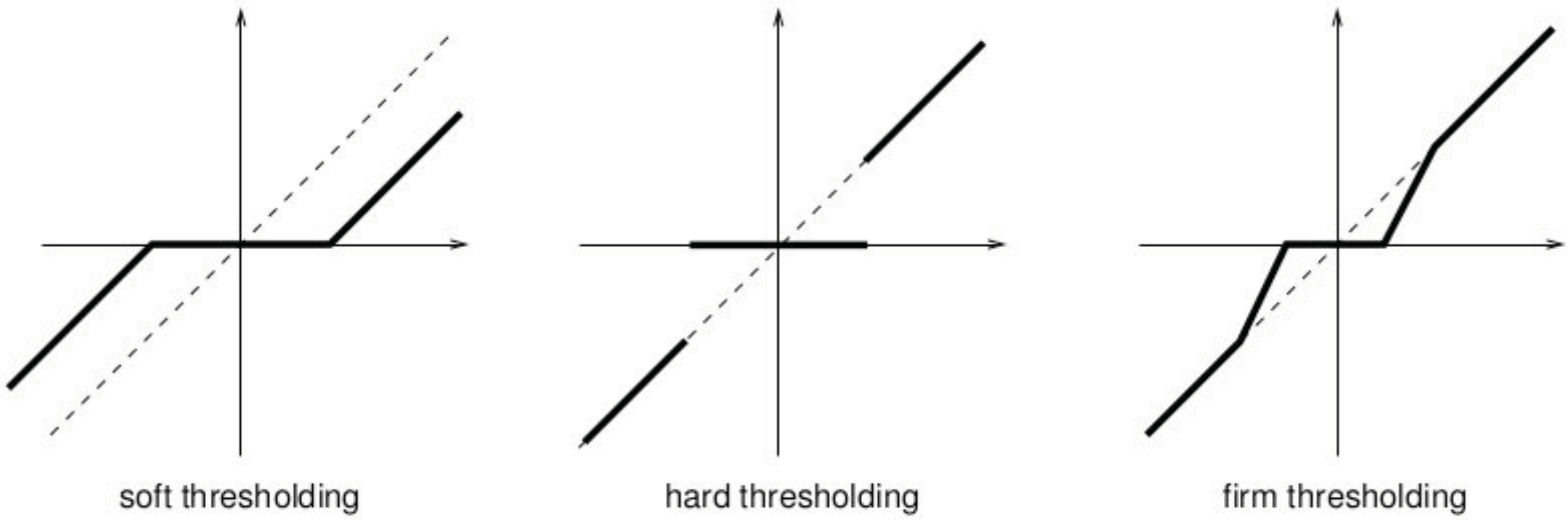
Using a wavelet transform:

$$\boldsymbol{w} = \boldsymbol{W}\boldsymbol{m}$$

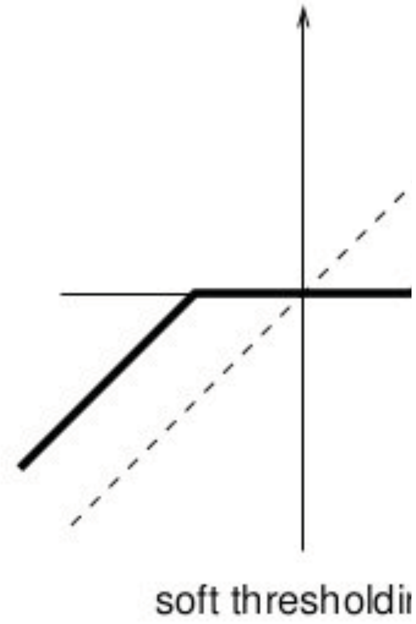
$$\text{Min } \|\boldsymbol{A}\boldsymbol{W}^{-1}\boldsymbol{w} - \boldsymbol{d}\|_2 + \epsilon\|\boldsymbol{w}\|_1$$

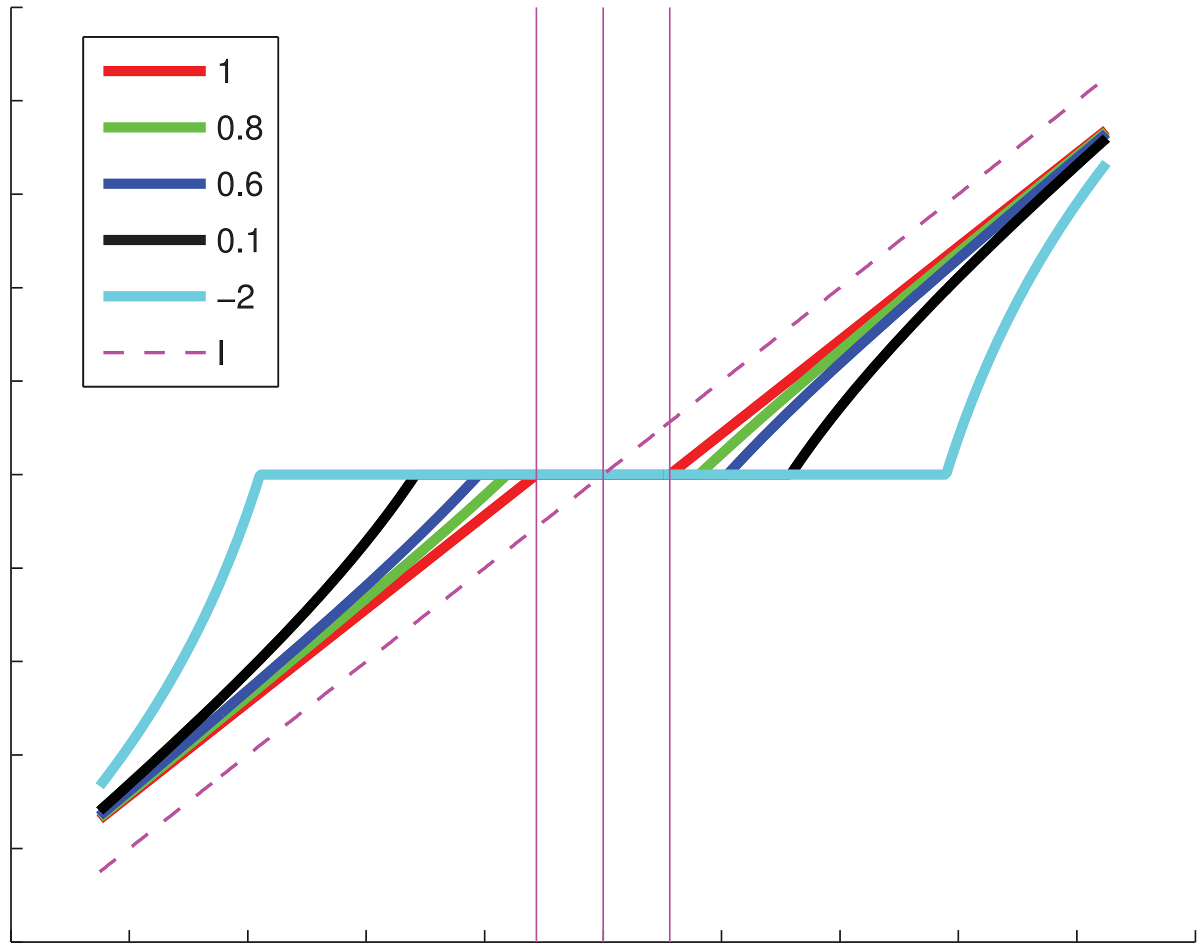
$$\boldsymbol{w}^{n+1} = \mathbb{S}_\lambda(\boldsymbol{w}^n + \boldsymbol{W}^{-T}\boldsymbol{A}^T\boldsymbol{d} - \boldsymbol{W}^{-T}\boldsymbol{A}^T\boldsymbol{A}\boldsymbol{W}^{-1}\boldsymbol{w}^n)$$

Thresholding (Sergey Voronin)

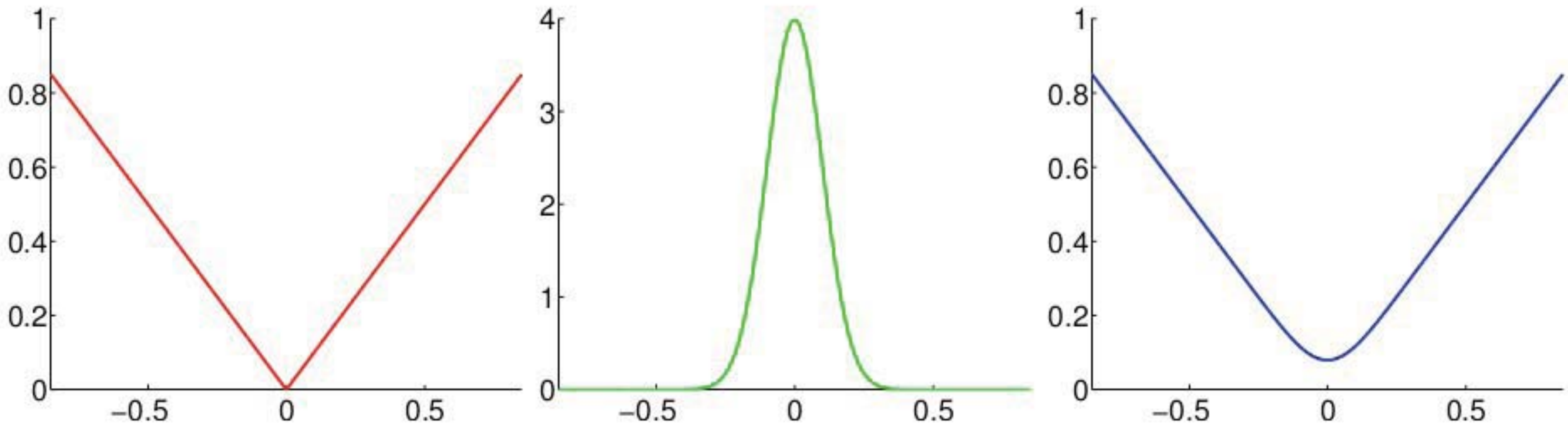


Thresholding (Sergey Voronin)



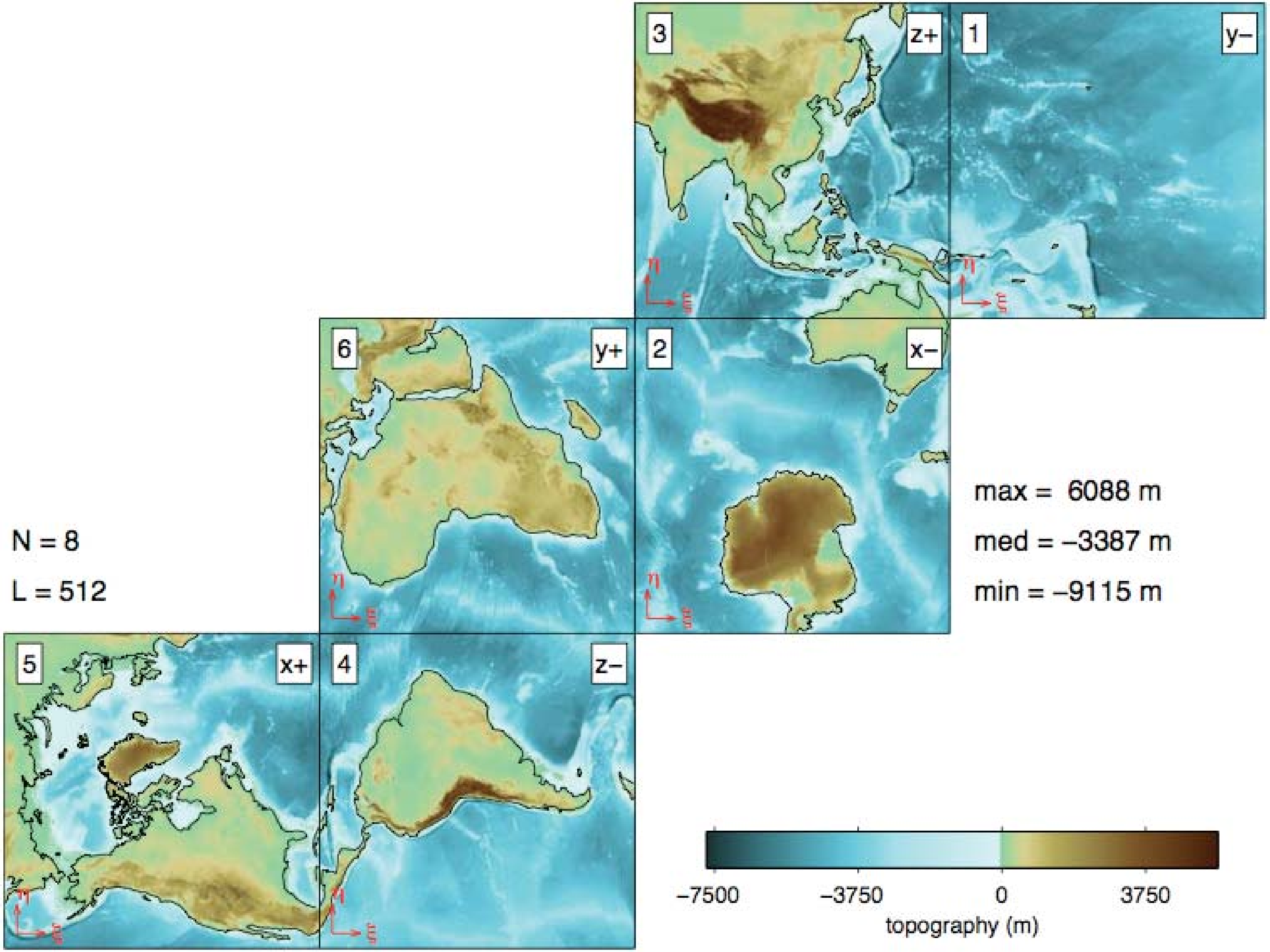


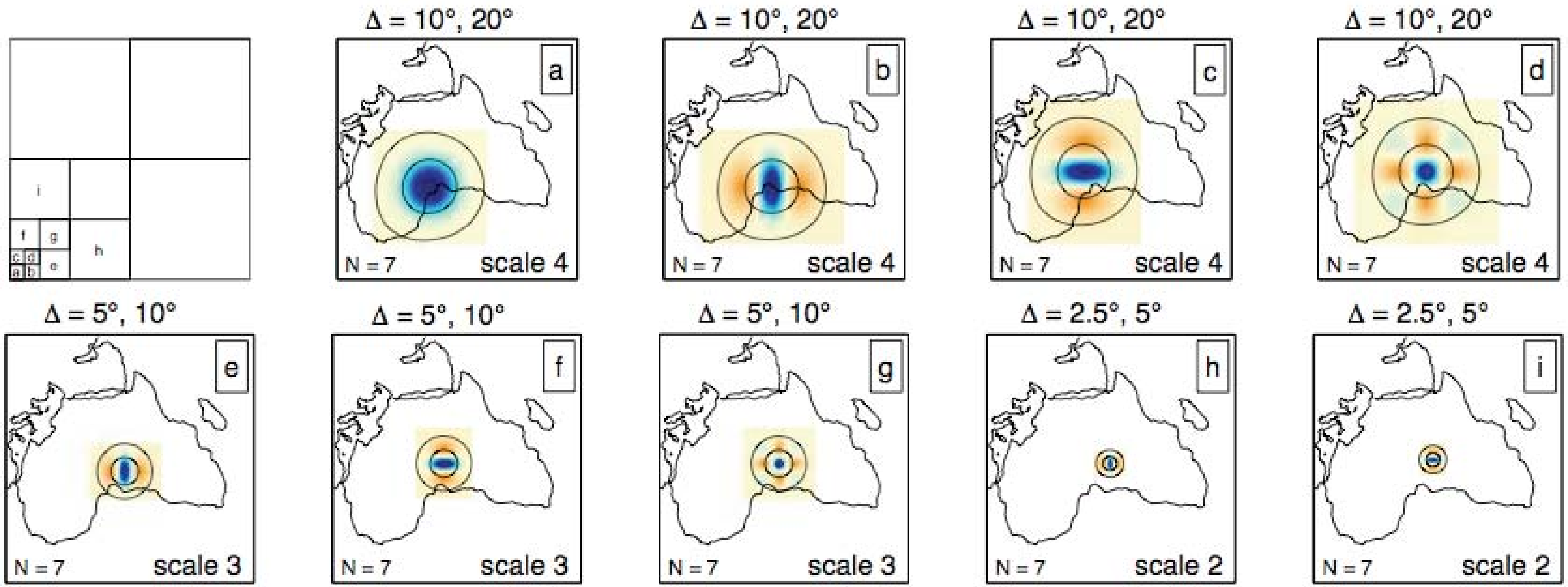
Smooth approximations of non-smooth penalties (Sergey Voronin)

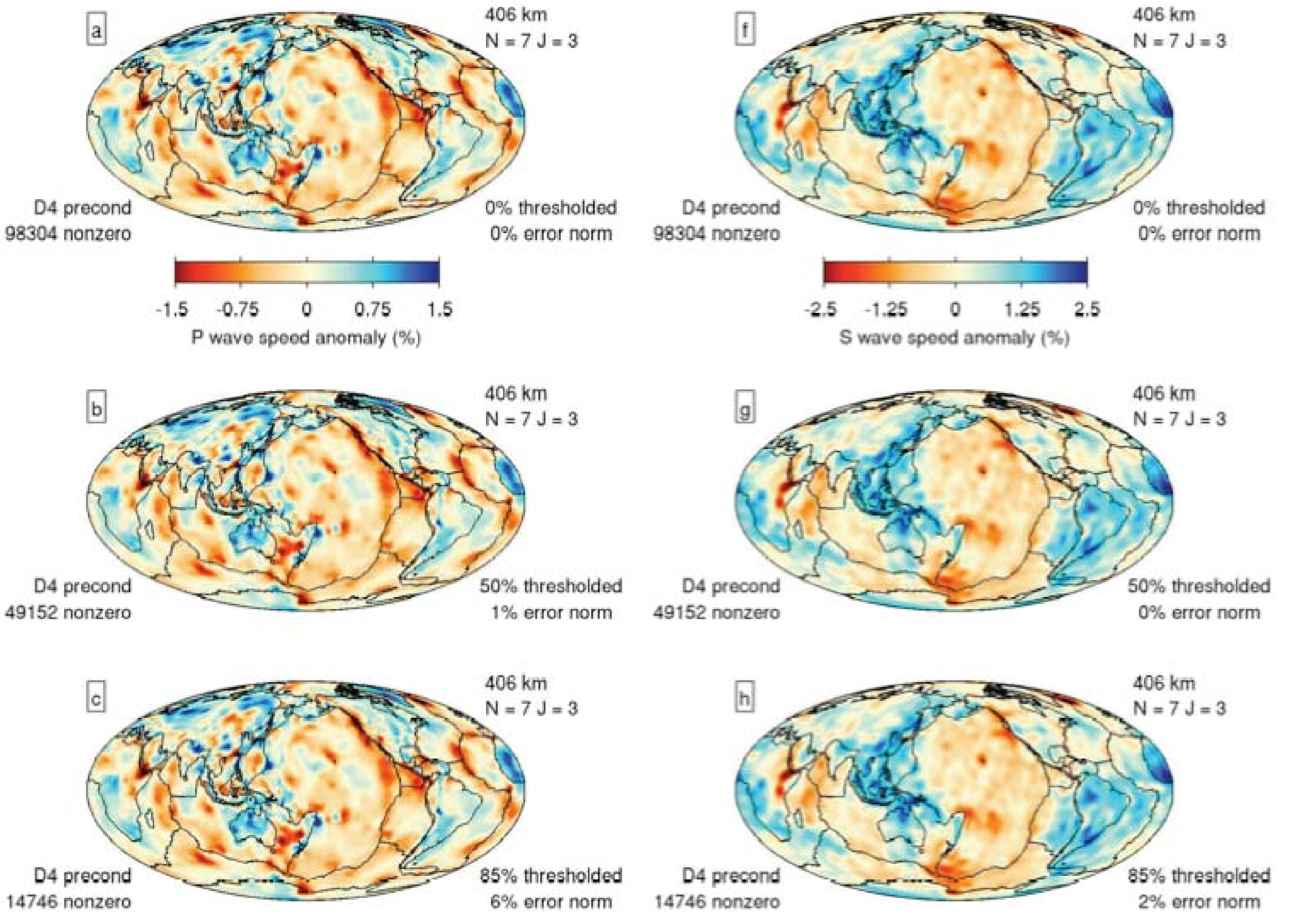


$$||x||_1 = \sum_{k=1}^{N} |x_k|$$


+X

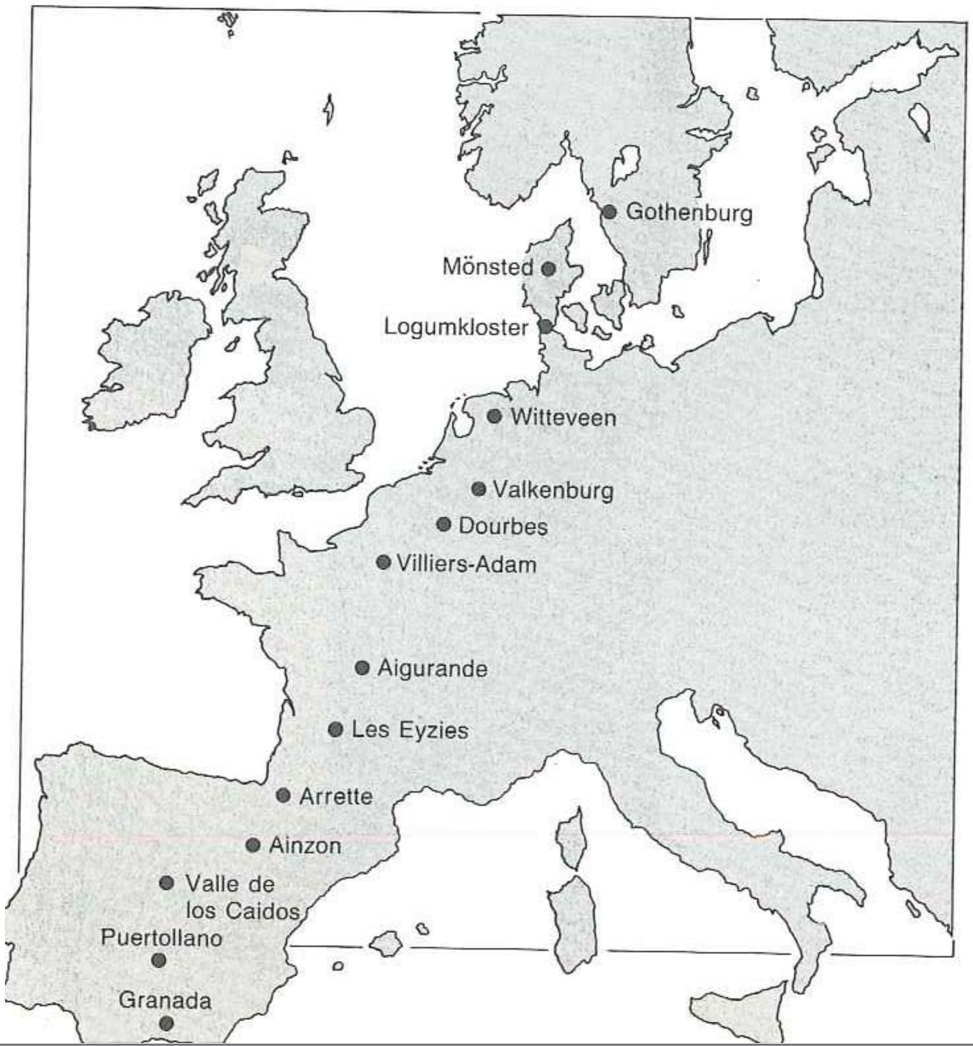

3
z+
1
y−
6
y+
2
x−
5
x+
4
z−
η
ξ
N = 8
L = 512
max = 6088 m
med = −3387 m
min = −9115 m
−7500
−3750
0
3750
topography (m)


Δ = 10°, 20°
a
N = 7
scale 4
Δ = 10°, 20°
b
N = 7
scale 4
Δ = 10°, 20°
c
N = 7
scale 4
Δ = 10°, 20°
d
N = 7
scale 4
Δ = 5°, 10°
e
N = 7
scale 3
Δ = 5°, 10°
f
N = 7
scale 3
Δ = 5°, 10°
g
N = 7
scale 3
Δ = 2.5°, 5°
h
N = 7
scale 2
Δ = 2.5°, 5°
i
N = 7
scale 2



Simons et al, 2011

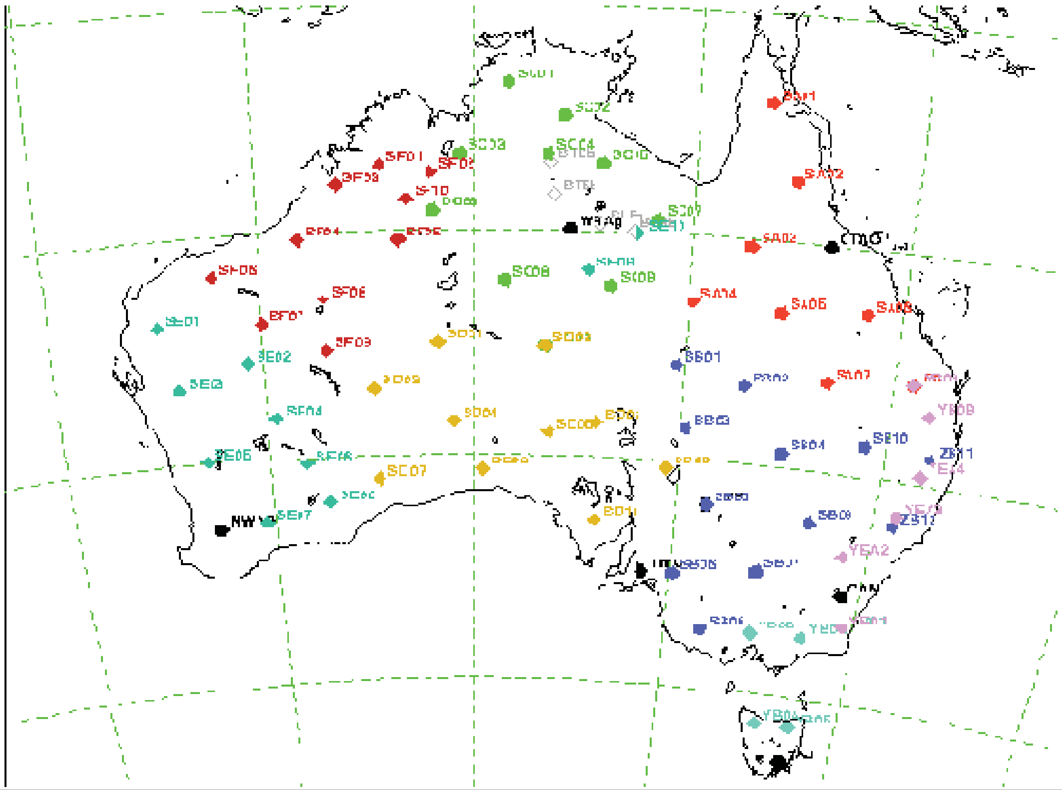
Ever more data: Super-Arrays



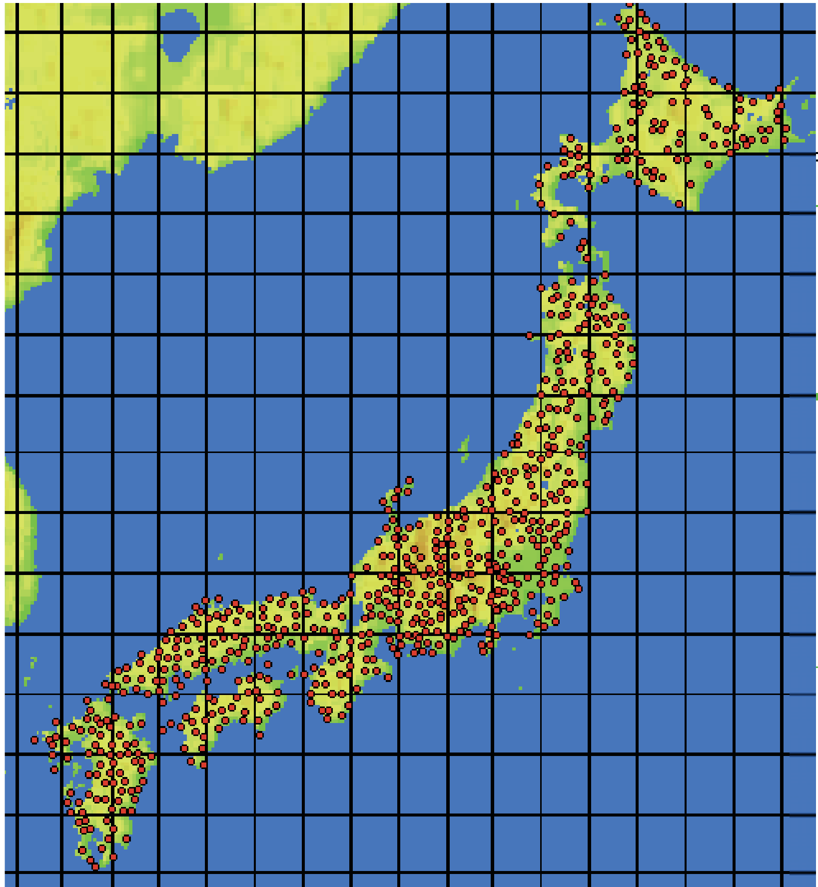
Ever more data: Super-Arrays

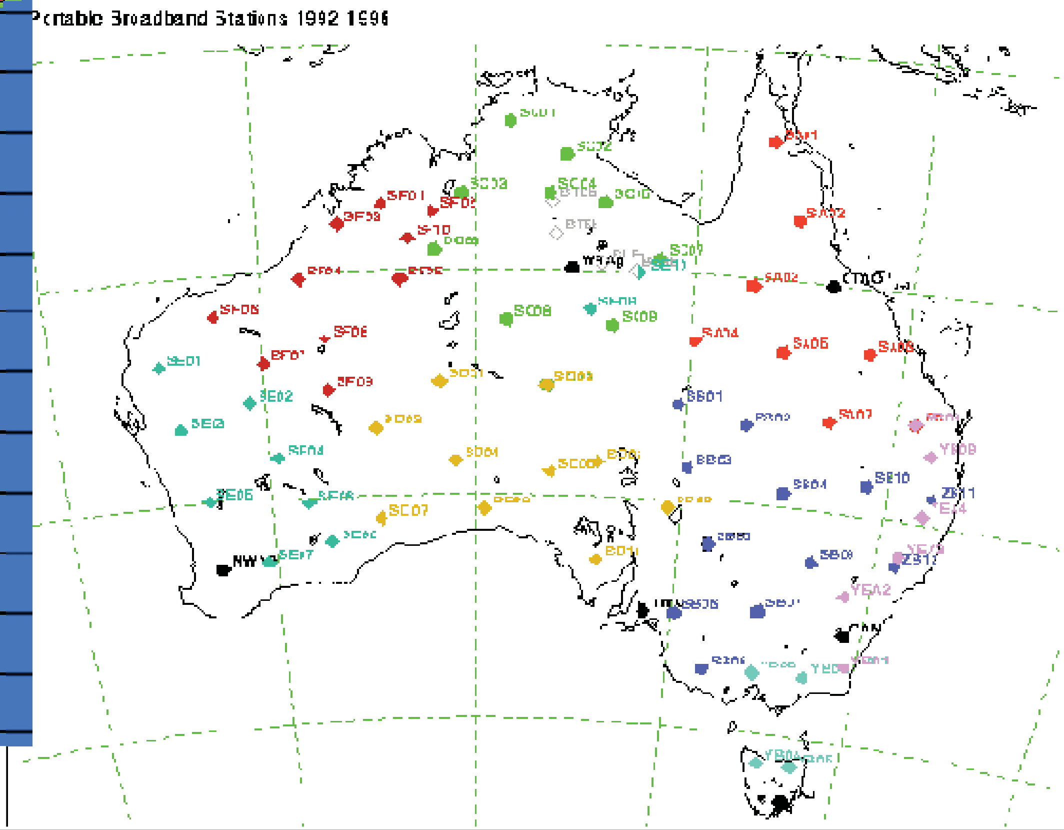
Ever more data: Super-Arrays

Portable Broadband Stations 1992 1996

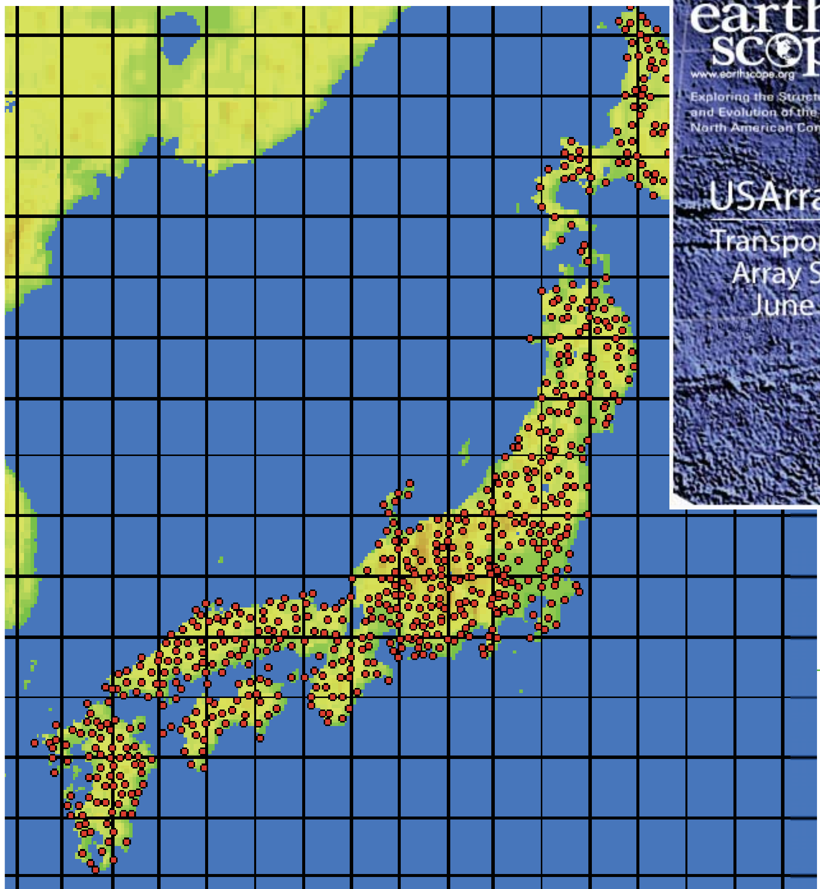


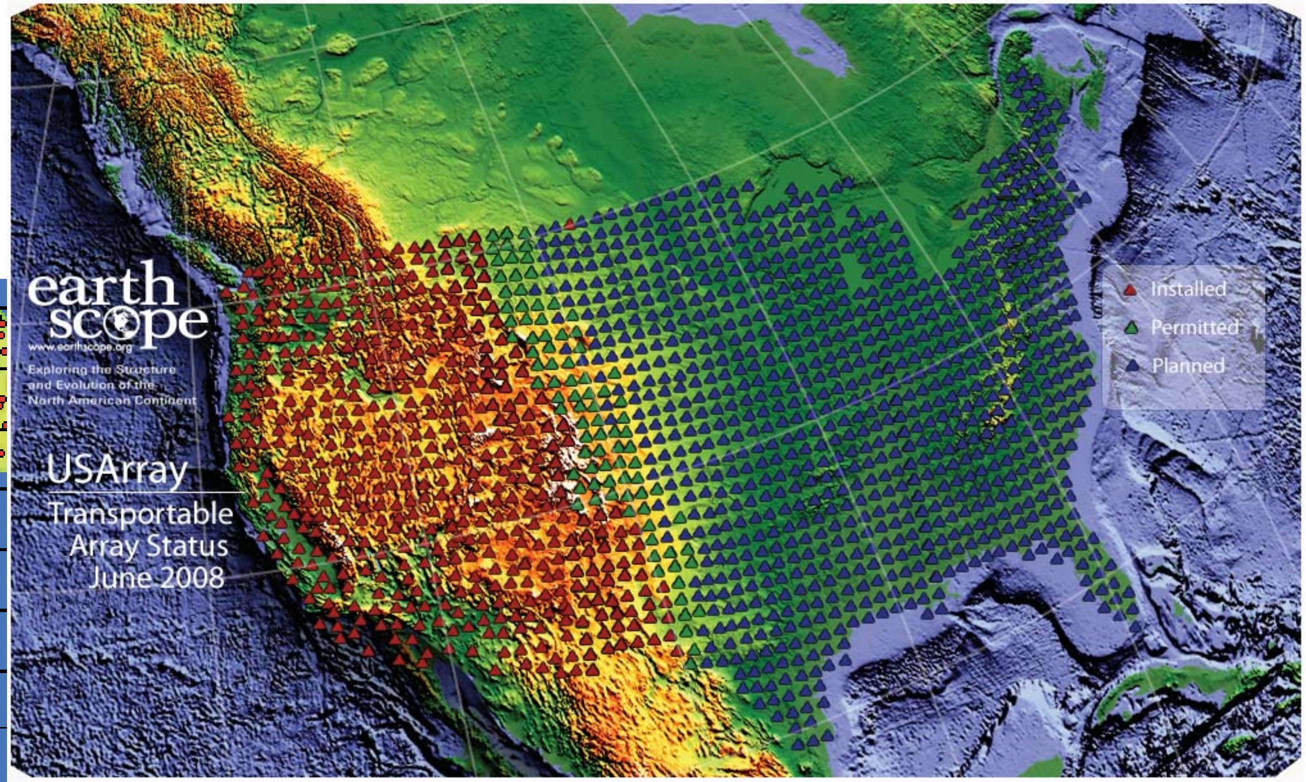
Ever more data: Super-Arrays

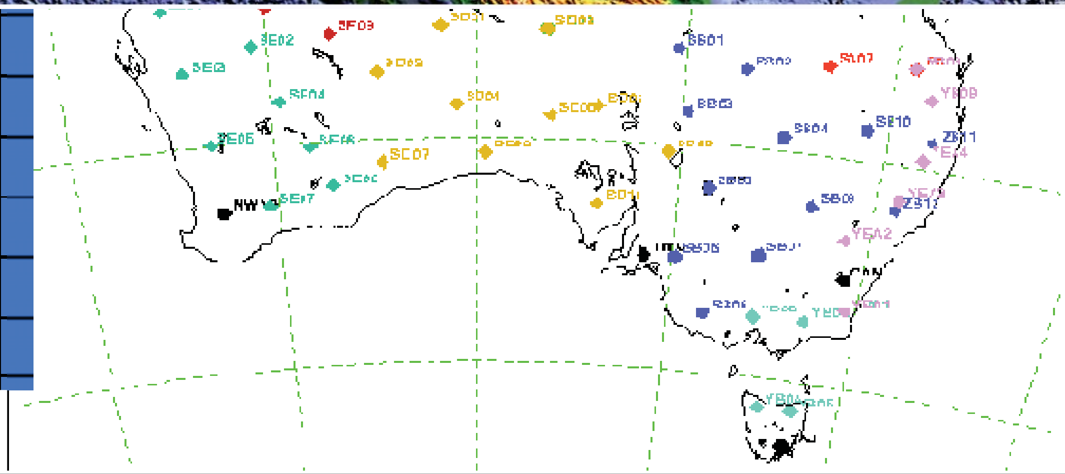




Ever more data: Super-Arrays

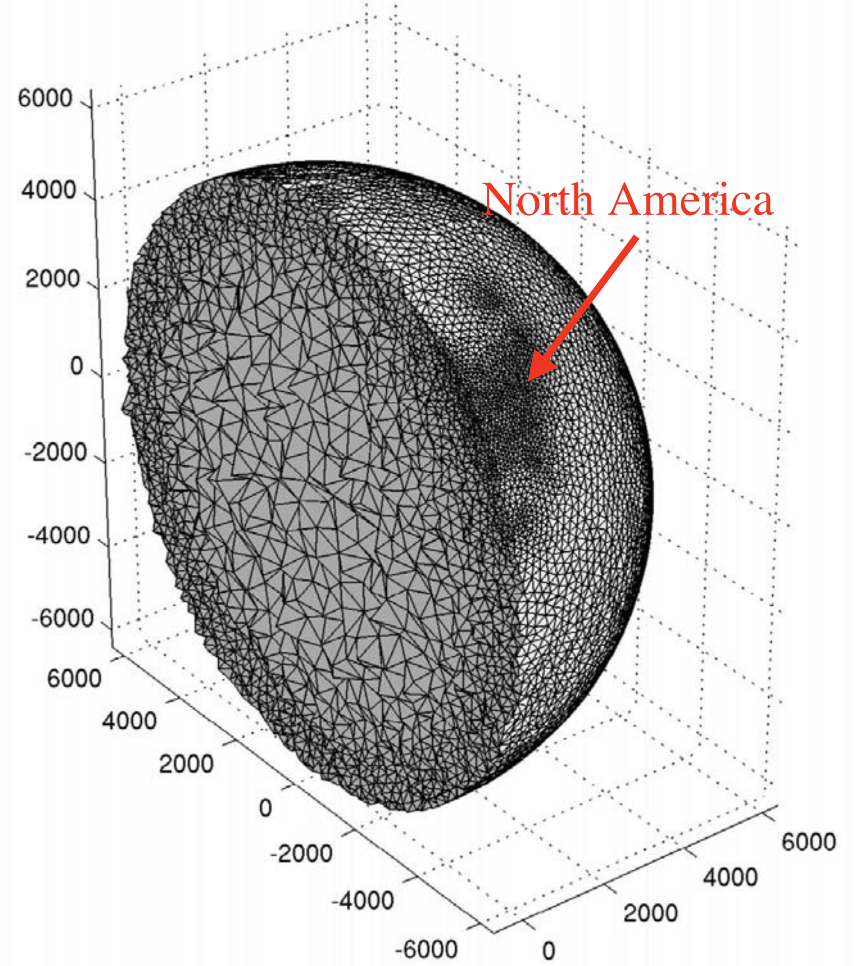


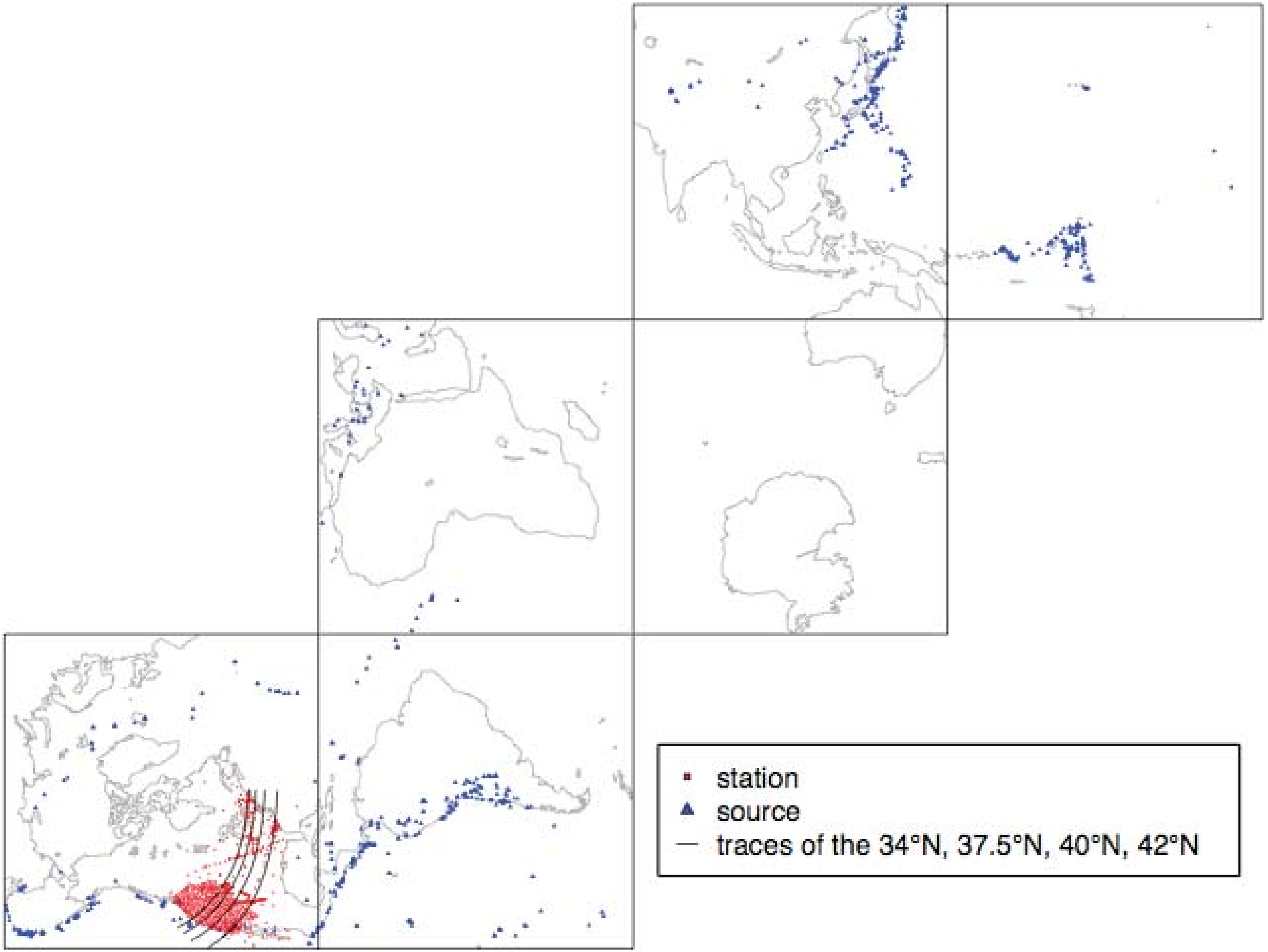




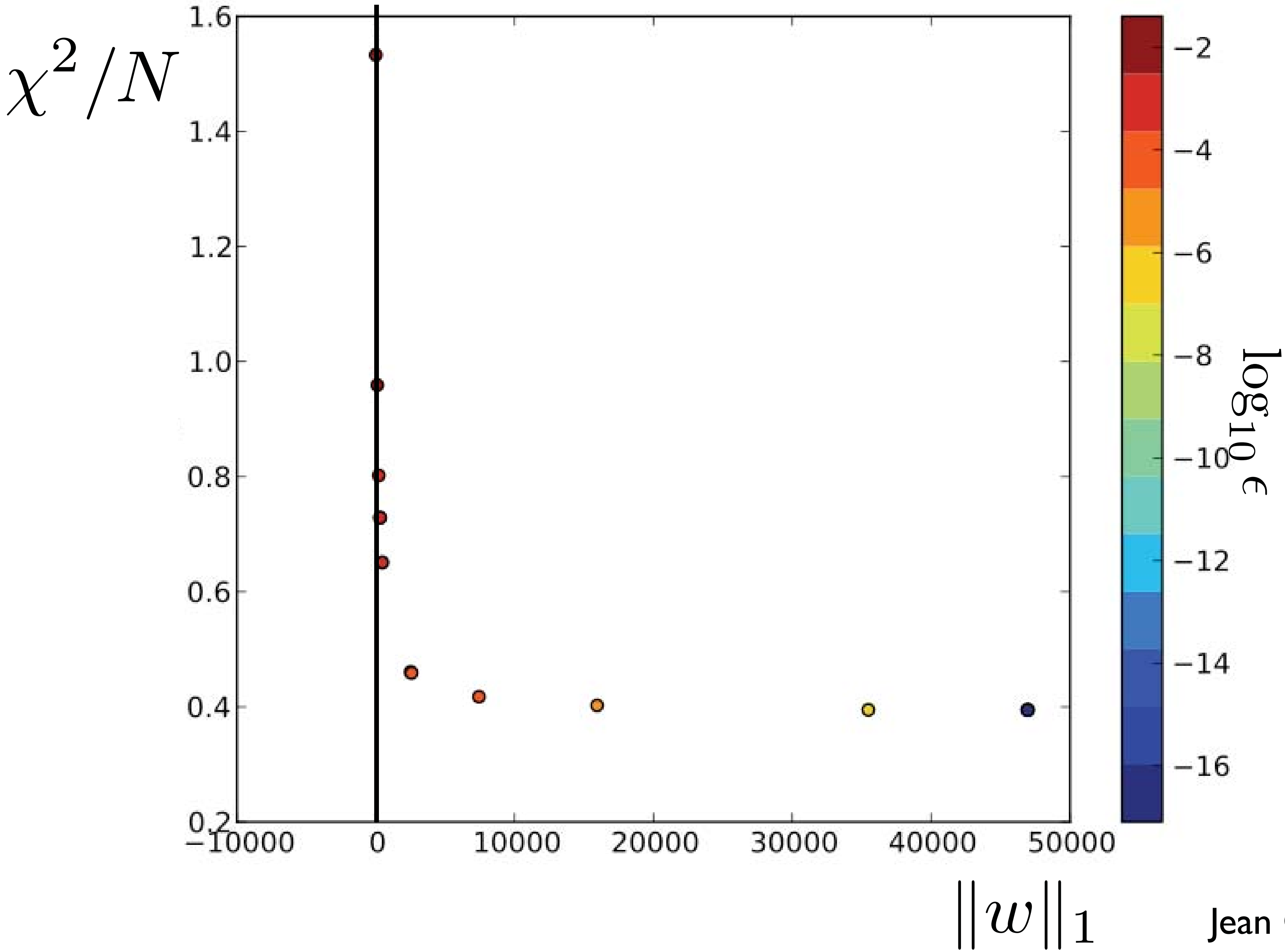
Finite-frequency inversion on a global grid

- Irregular tetrahedral mesh
- Joint inversion of ~400,000 P-wave traveltimes
- Using 7 frequency passbands between 25 s and 2 s period.

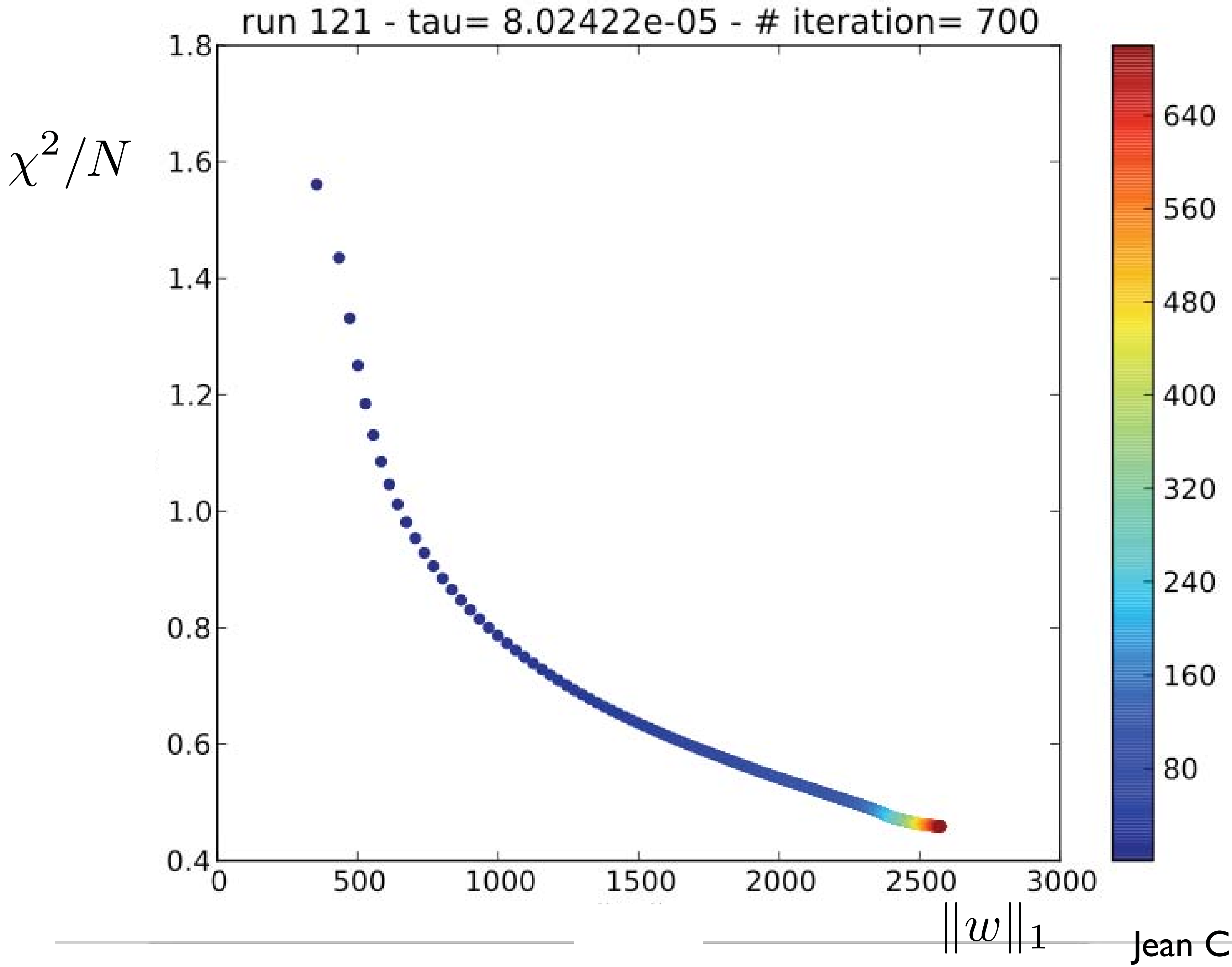




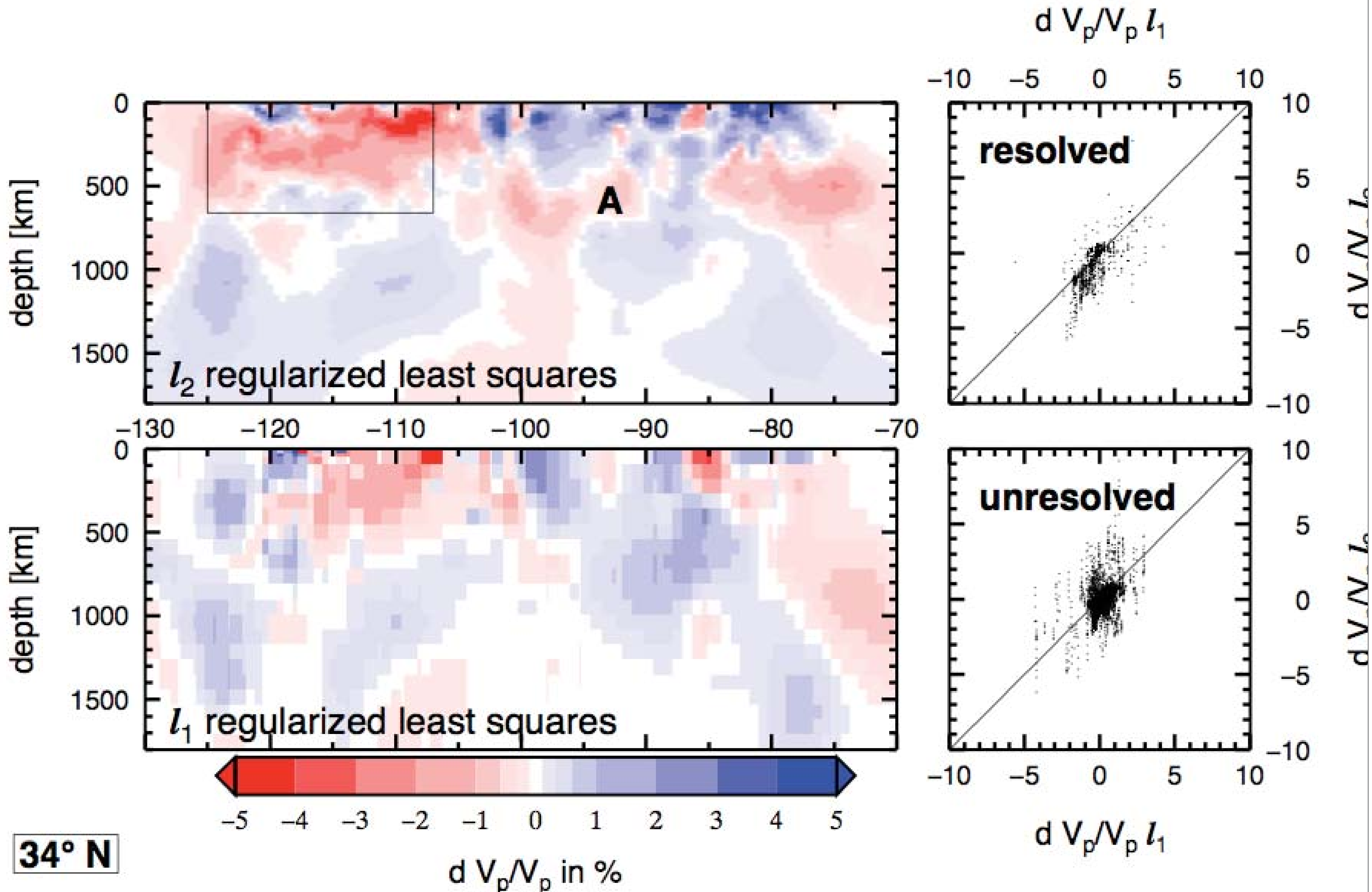
Jean Charlety



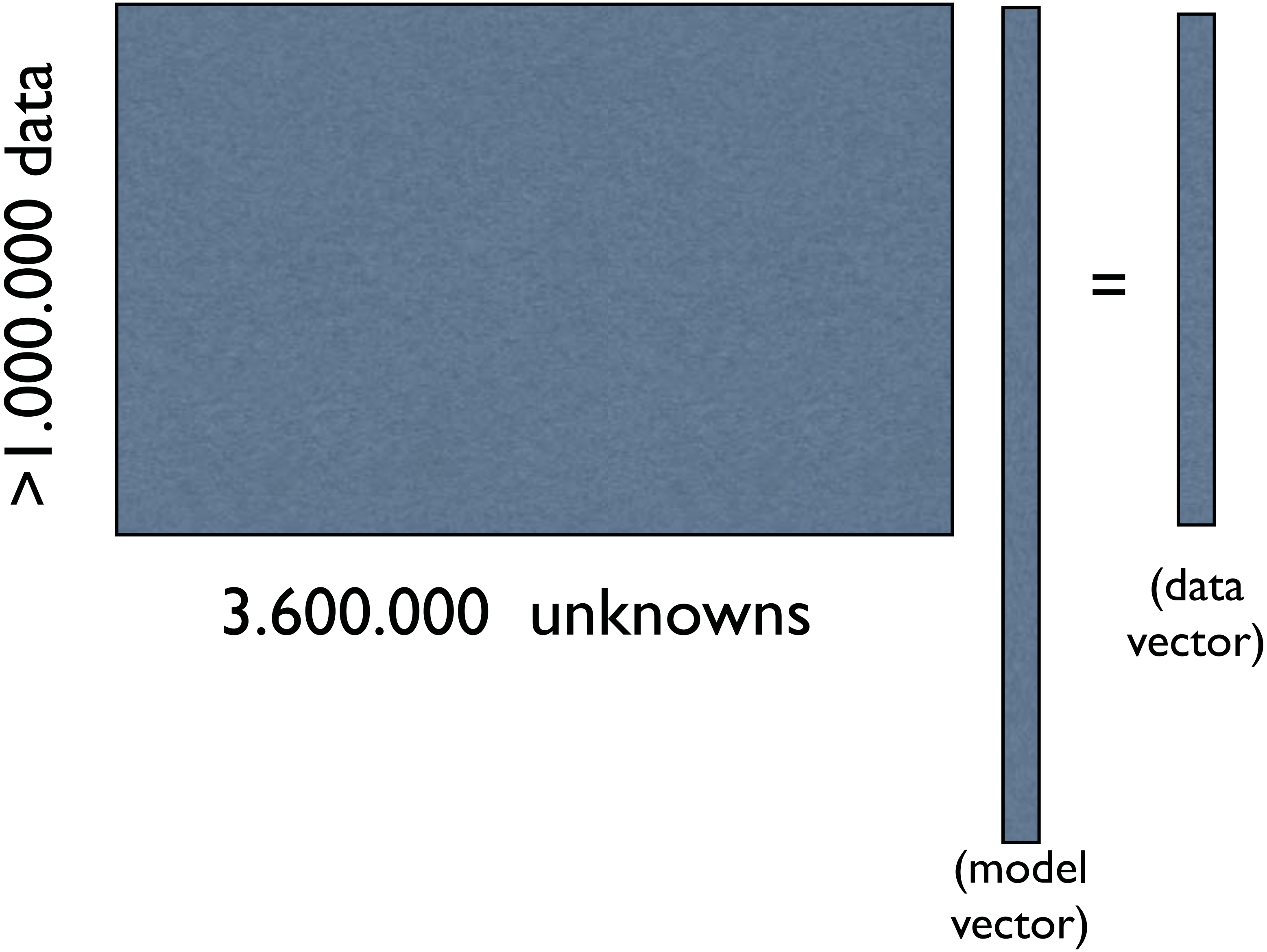
Jean Charlety



Jean Charlety

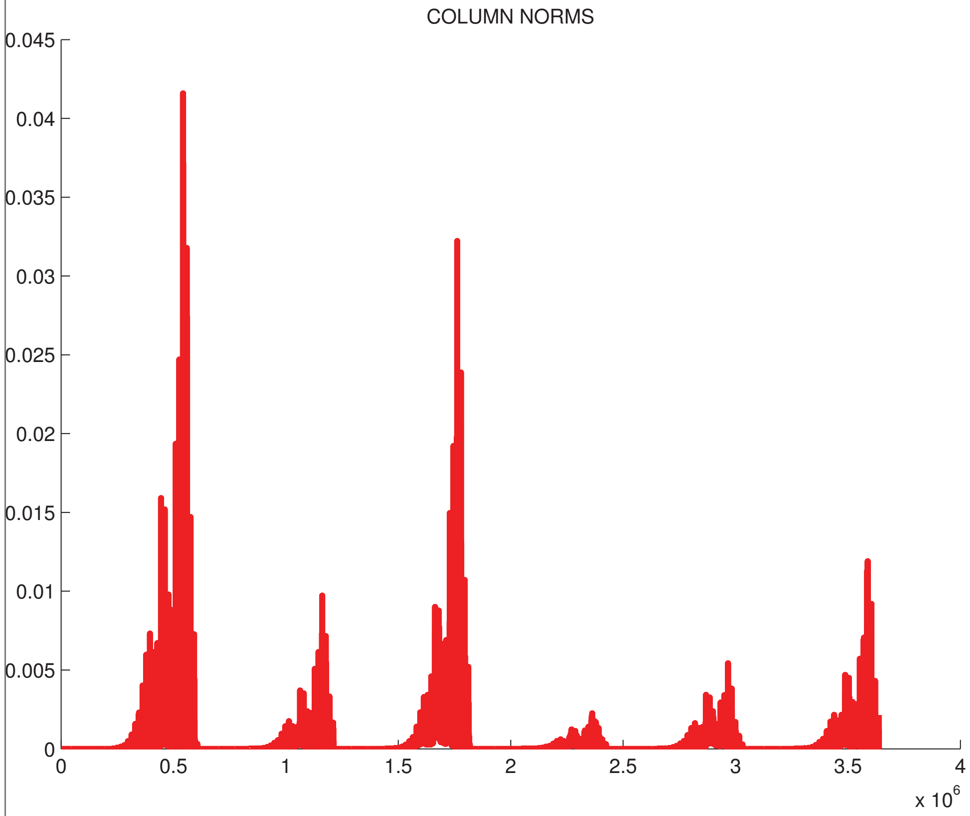


Jean Charlety


>1.000.000 data
3.600.000 unknowns
=
(model vector)
(data vector)

Option 1: Singular value decomposition using randomized algorithms

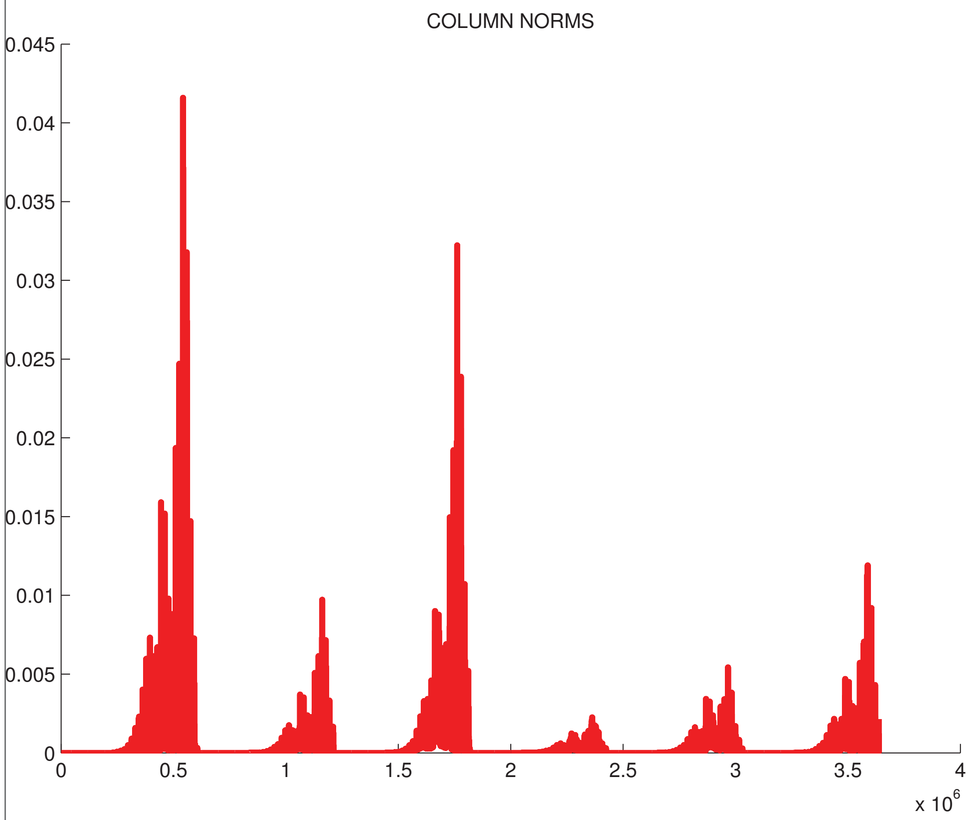
$$A \approx U_k \Sigma_k V_k^T$$

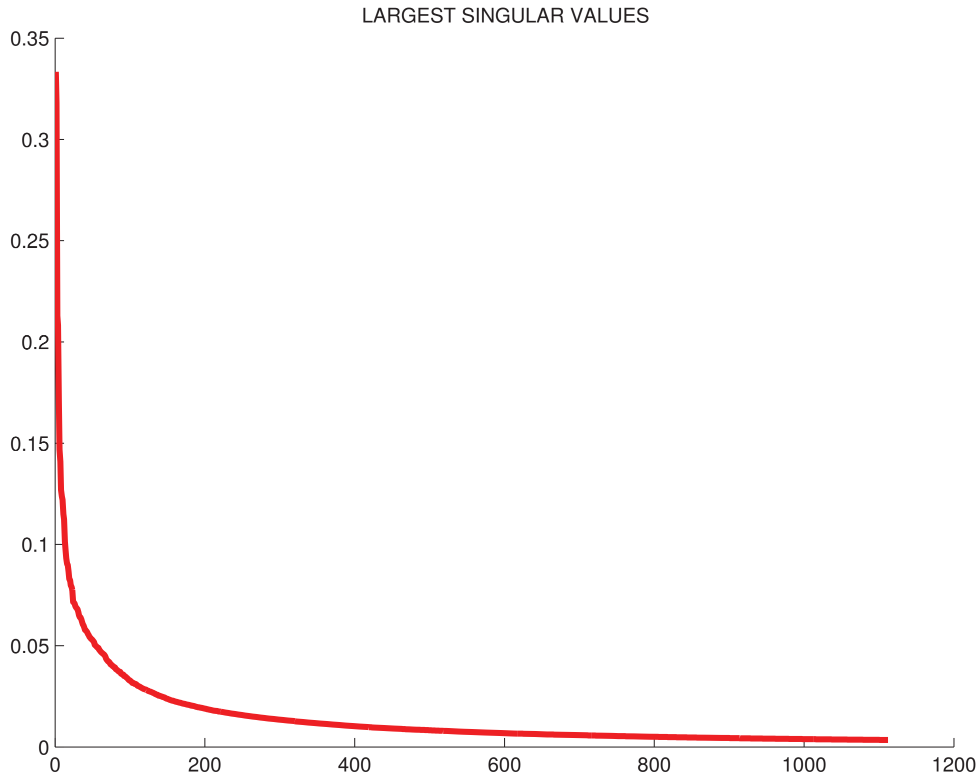


Sergey Voronin

Option 1: Singular value decomposition using randomized algorithms

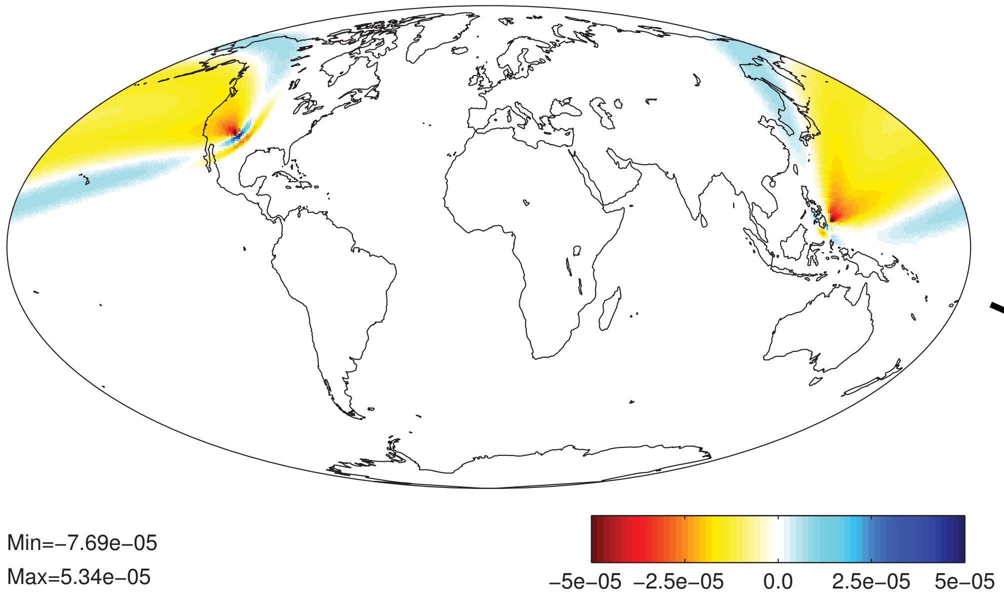
$$A \approx U_k \Sigma_k V_k^T$$



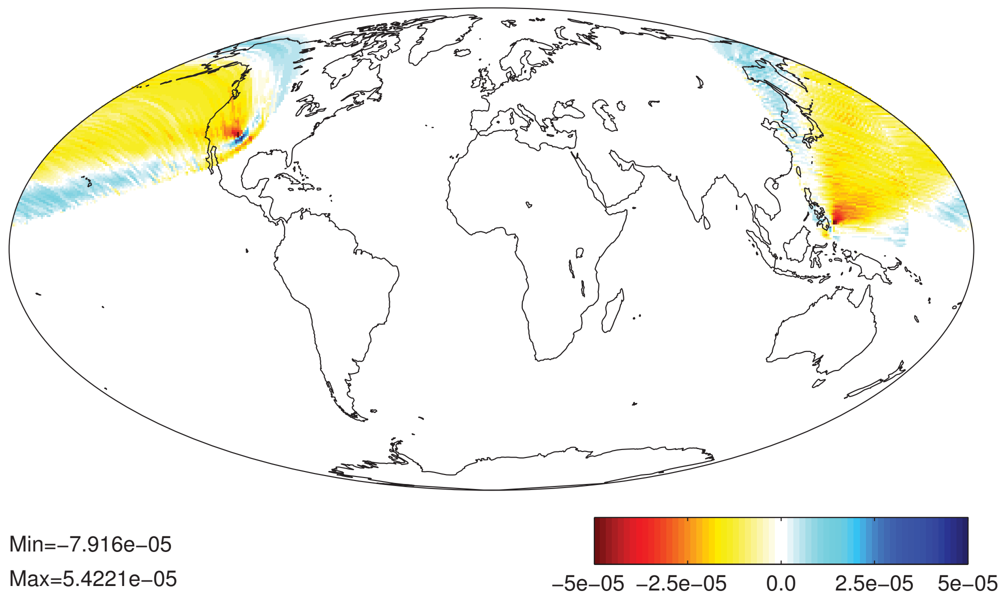


Sergey Voronin

Option II: compressing the kernels (=matrix rows)



Compressed to 2%



Sergei Voronin

A statistical conundrum (1)

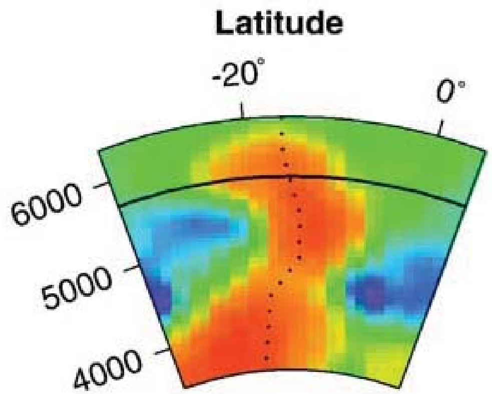
We assume normally distributed errors, define

$$\chi^2 = \sum_i \frac{|\sum_j A_{ij} m_j - d_i|^2}{\sigma_i^2}$$

And accept models with $\chi^2 \approx N$

A statistical conundrum (II)

We now take an acceptable model, and remove one anomaly (e.g. the Tahiti plume):



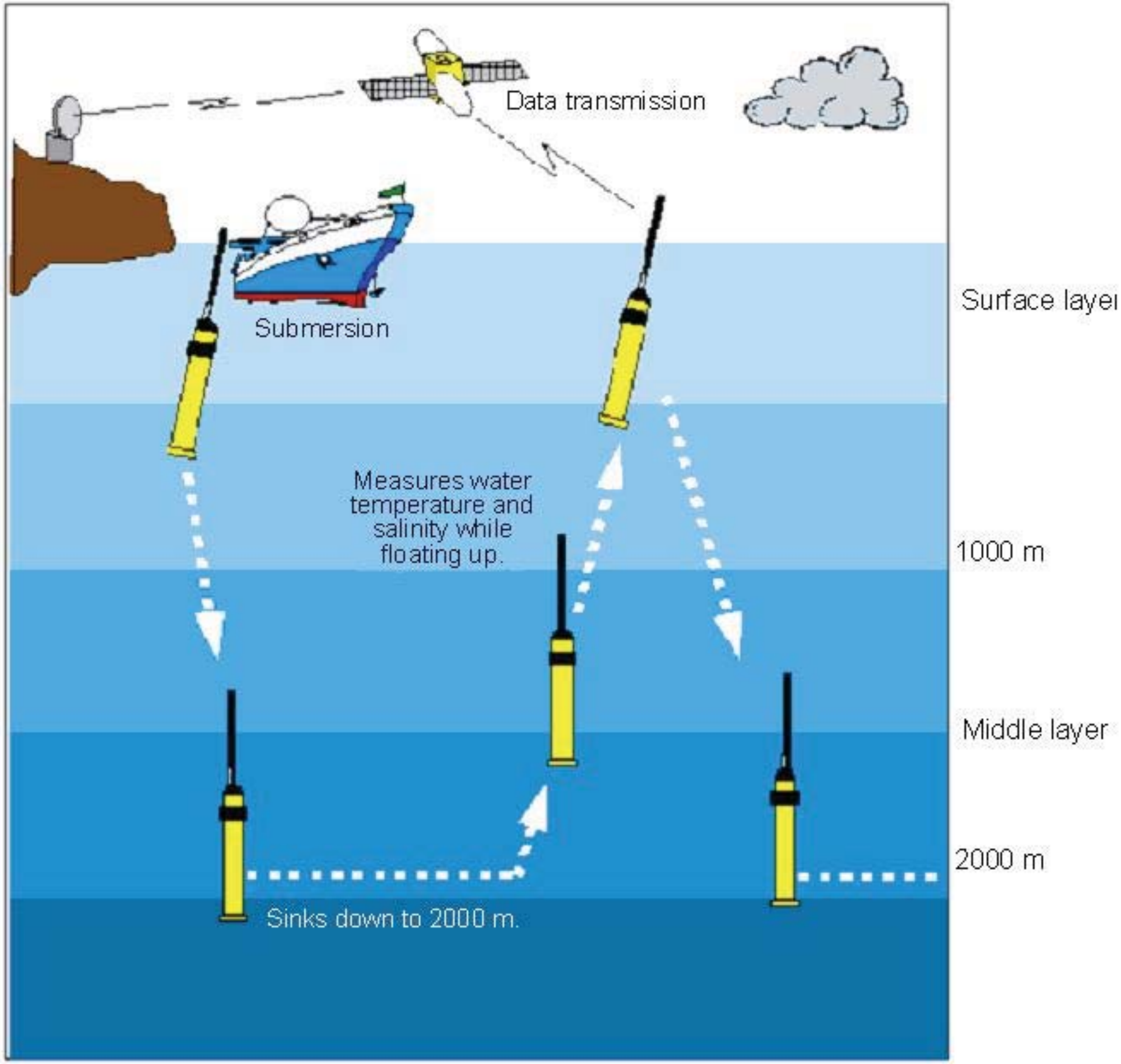
Small enough such that chi square remains acceptable

Does this mean Tahiti is not “resolved”?

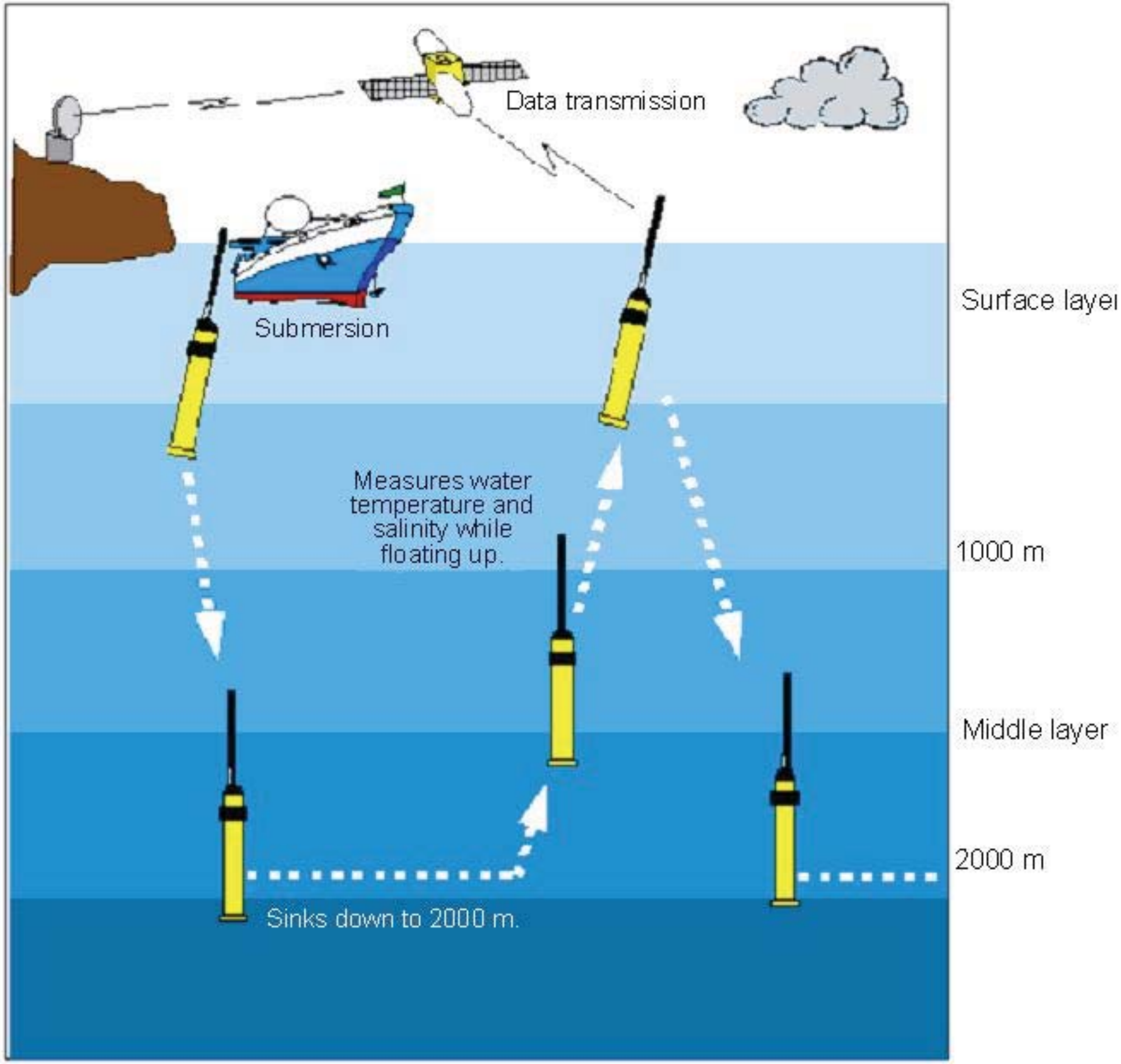
A statistical conundrum (III)

- We could do this for *any* anomaly, so *nothing* is resolved
- However, there are *subsets* of data, that violate the chi square criterion (e.g. the seismic station on Tahiti)
- Can we develop a subset criterion?

The oceans

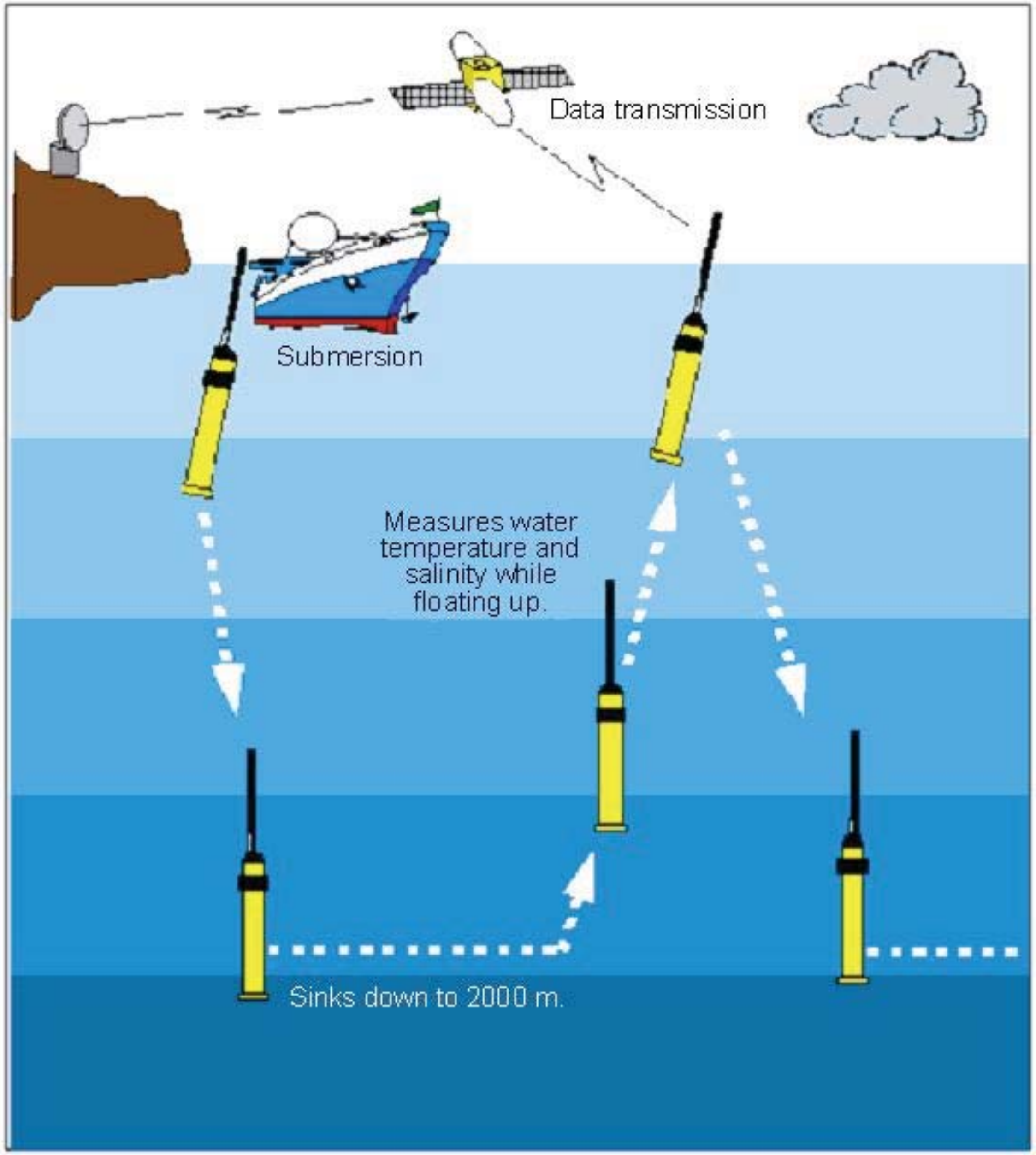

Data transmission
Submersion
Surface layer
Measures water temperature and salinity while floating up.
1000 m
Middle layer
2000 m
Sinks down to 2000 m.

The oceans



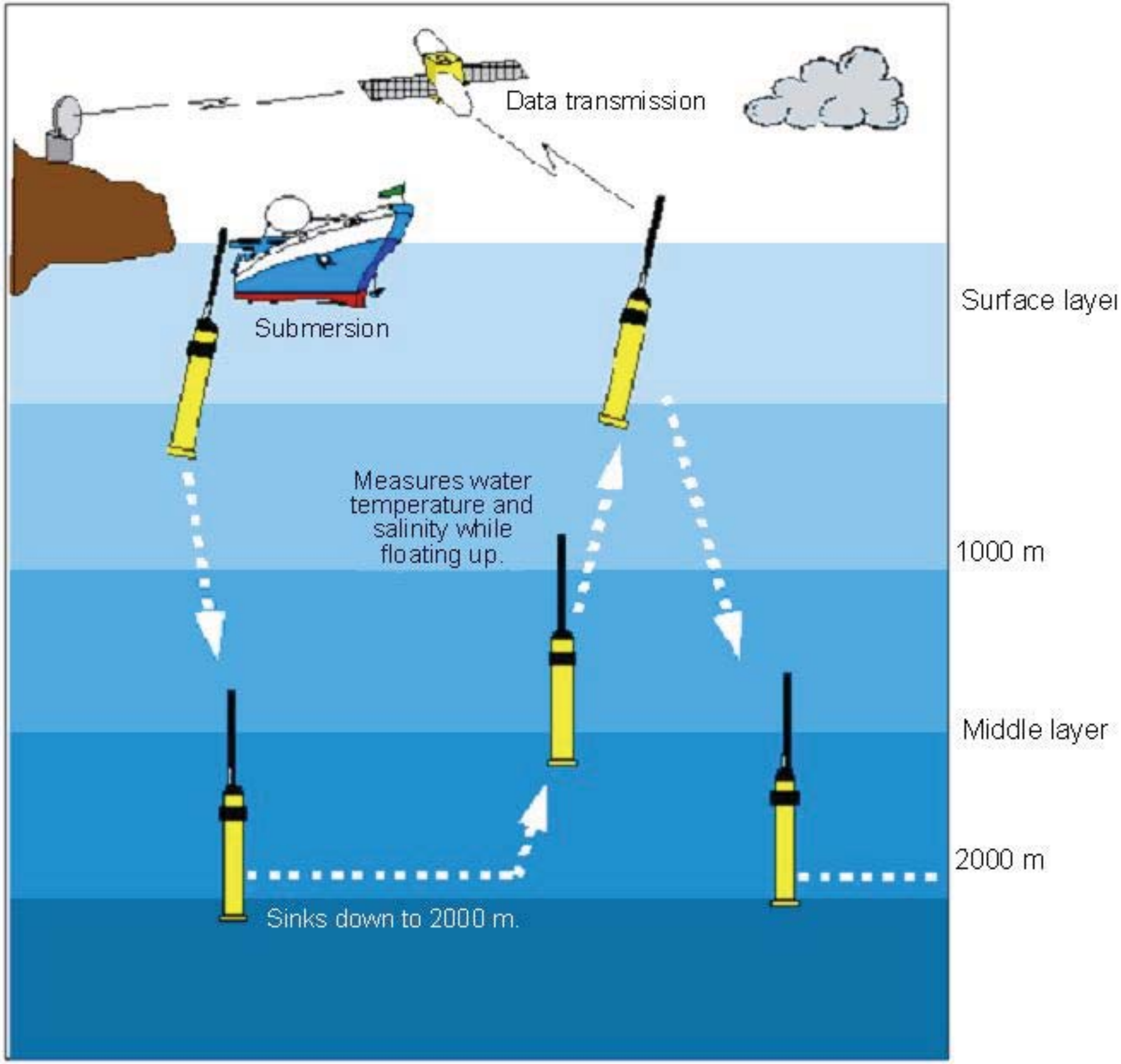
Simons et al, 2006

The oceans



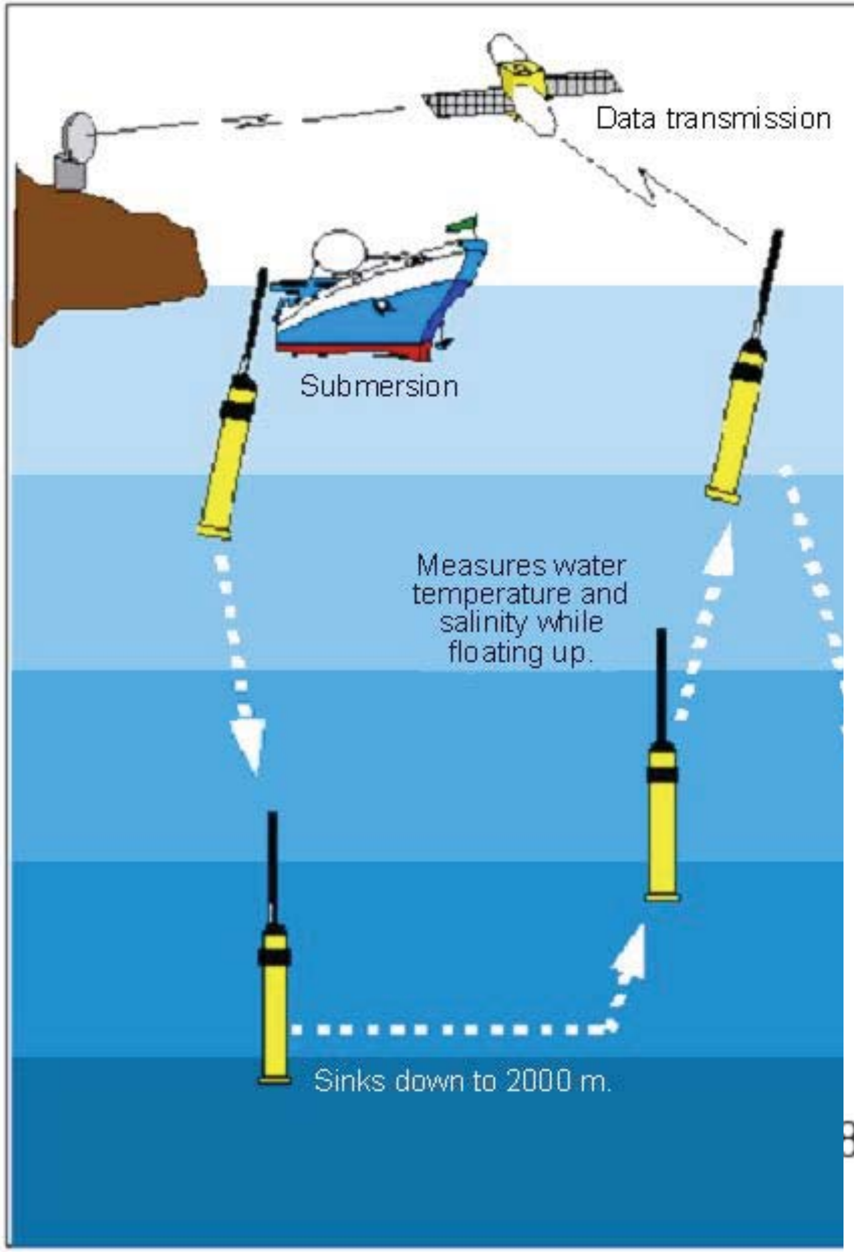
Simons et al, 2006

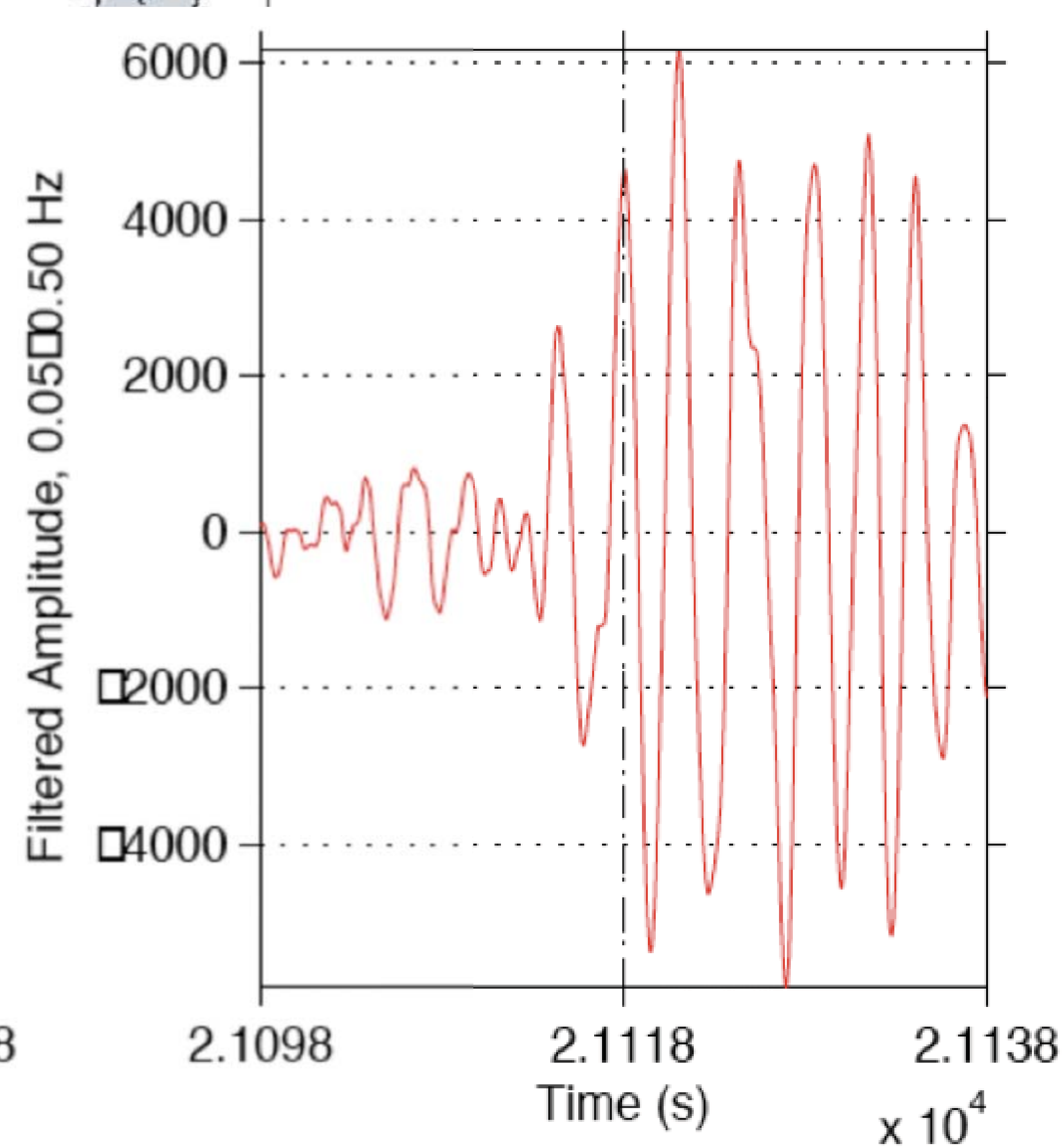
The oceans



Simons et al, 2006

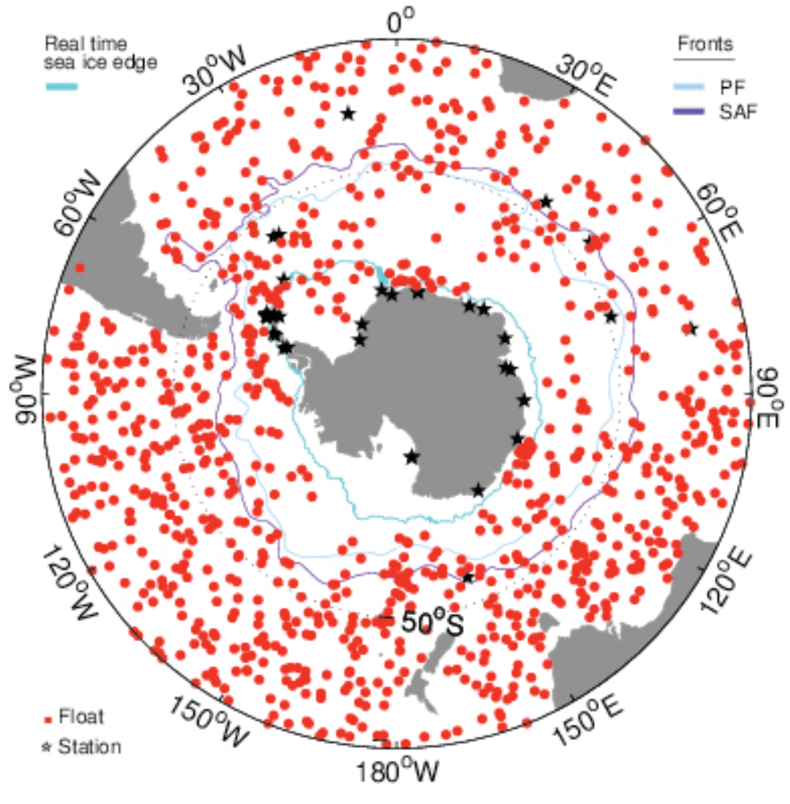
The oceans



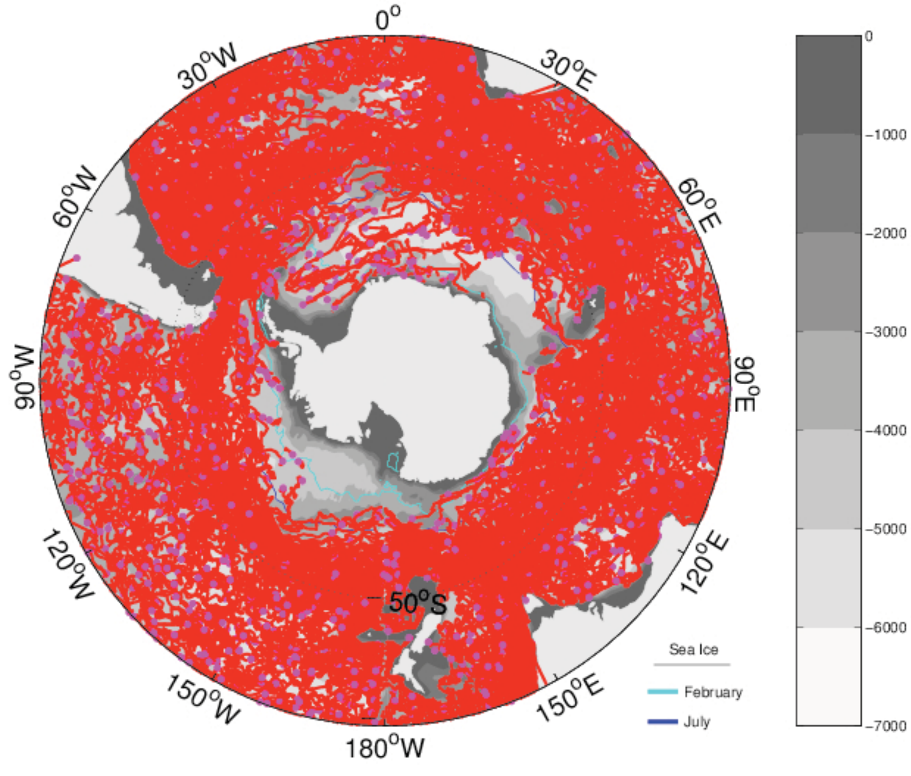


Simons et al, 2006

Wet super-arrays

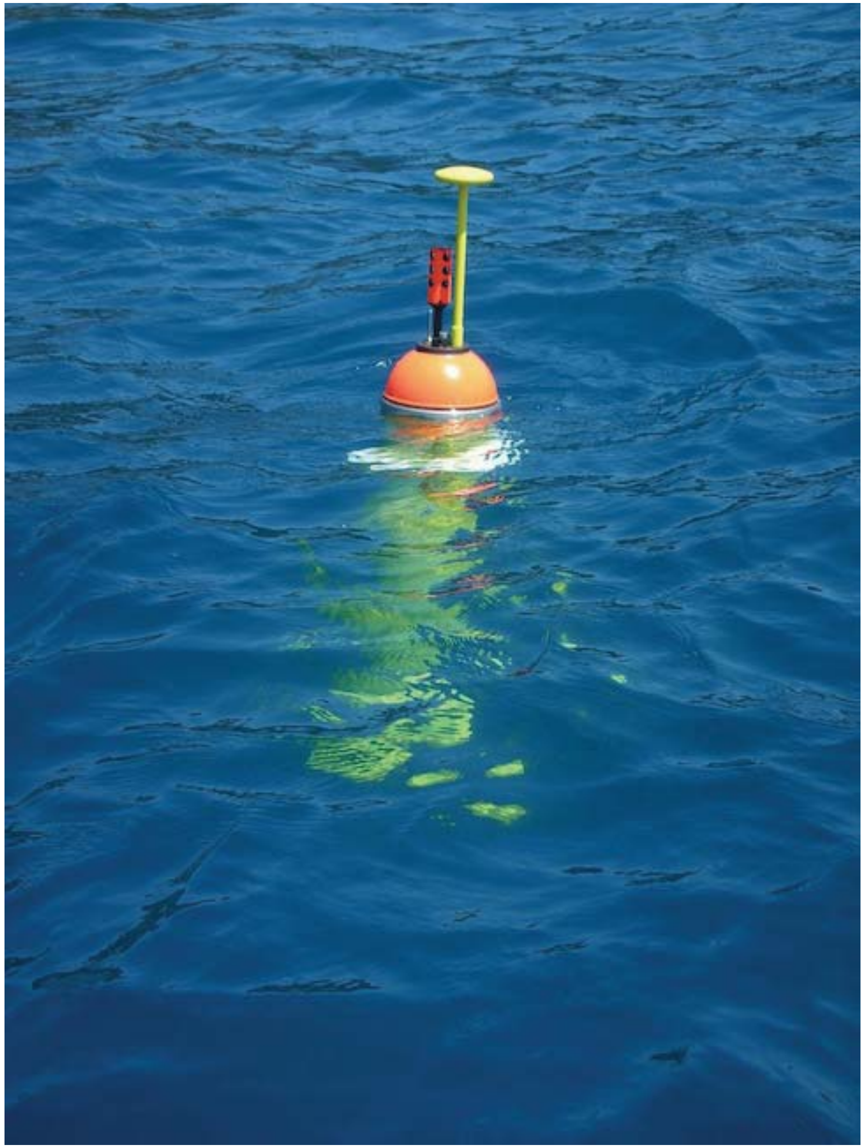


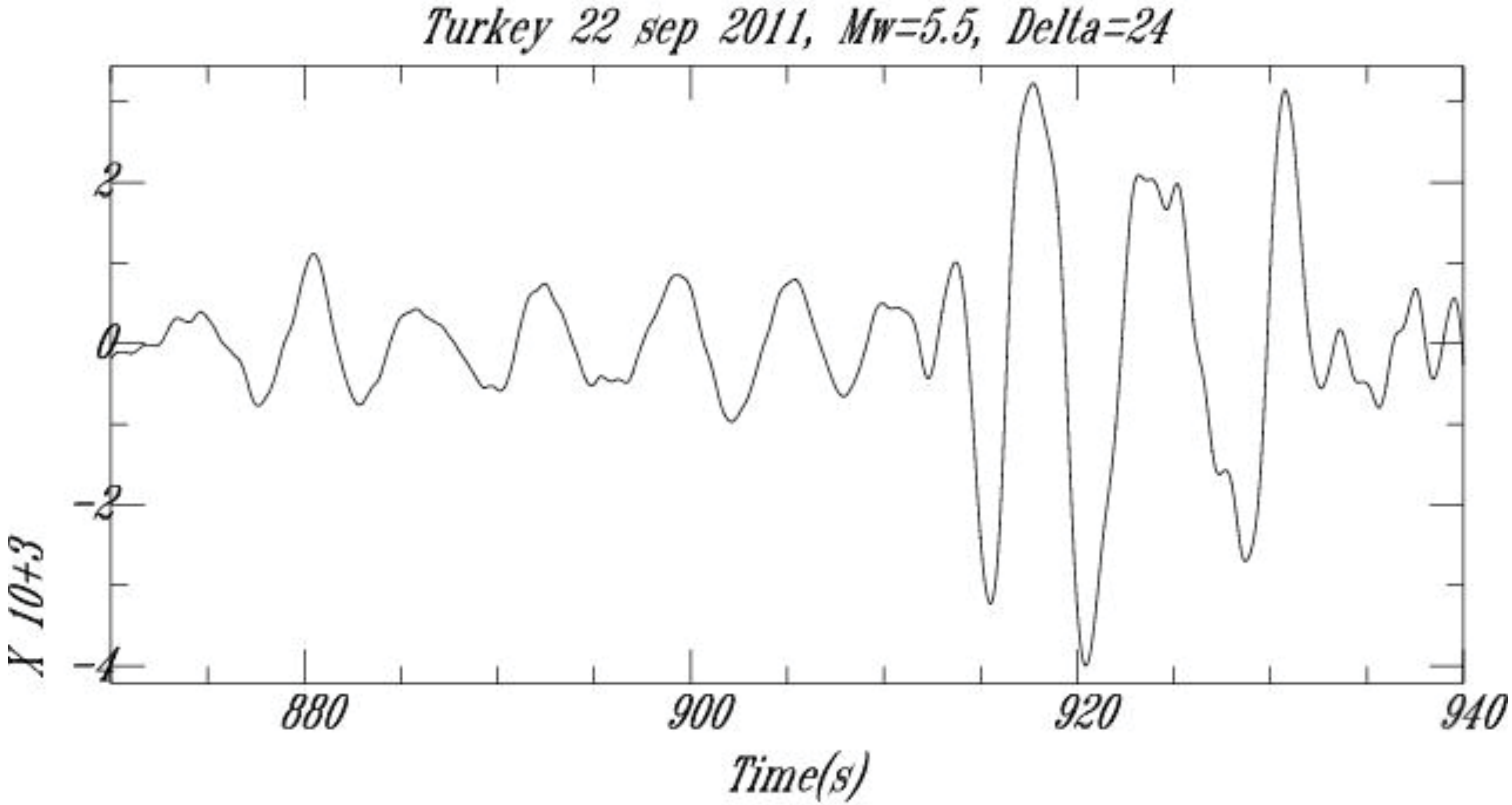
Float Positions: 02–Apr–2008



Float Positions: 02–Apr–2008

Mermaids

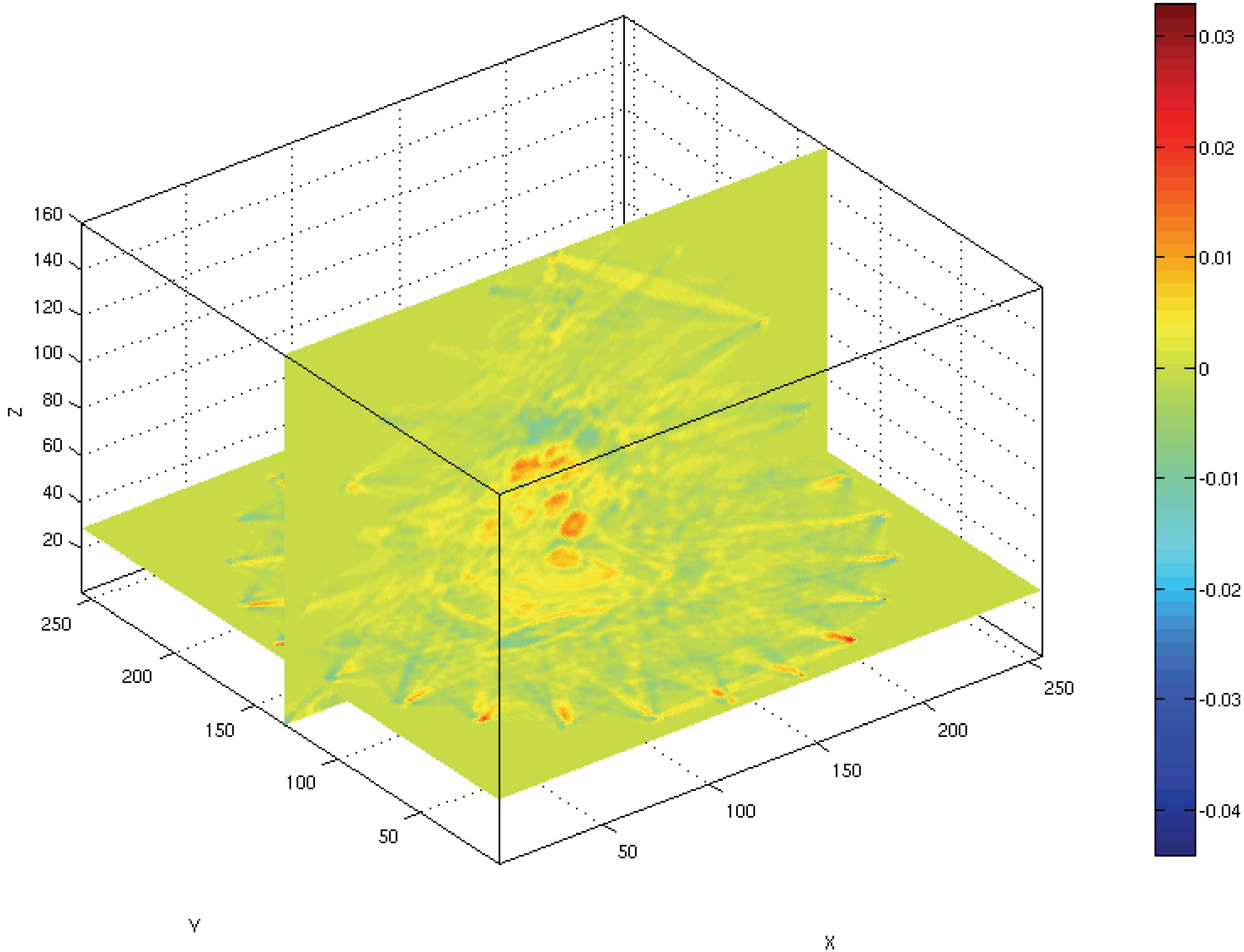




Turkey 22 sep 2011, Mw=5.5, Delta=24
2
0
-2
-4
X 10+3
880
900
920
940
Time(s)


PORT DE
6347

Spin-off (1): medical tomography



Courtesy Robin Dapp, KIT

Spin-off (2): Ocean sounding