RÉSEAUX D'AUTOMATES

Trente ans de recherche

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Topics:

1) Neural or Threshold Networks: dynamics; energy; complexity

2) Application to Schelling Segregation Model and
bootstrap percolation complexity .

3) Regulation Networks: dynamics and Robustness.

4) Ants models and its complexity.

5) Cellular Automata Communication Problems

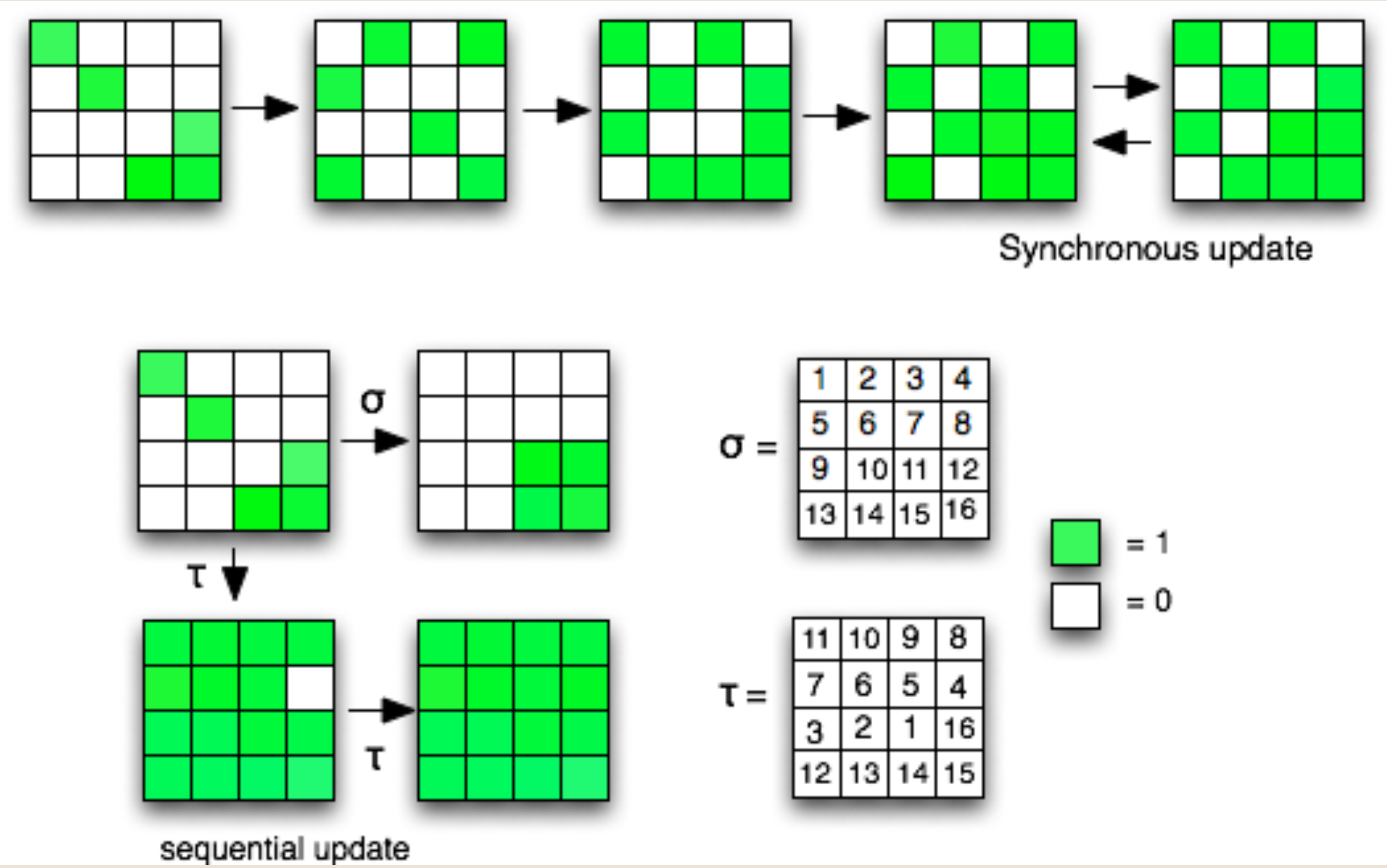
Neural or threshold Networks

We consider a 4x4 lattice with periodic conditions,
nearest interactions, states 0 or 1, and the local majority function:
If the number of ones is bigger or equal to the number of zeros then
the site takes the value 1

$$x'_{ij} = 1 \quad iff \quad x_{i\ \ 1,j} + x_{i+1,j} + x_{i,j\ \ 1} + x_{i,j+1} \geq 2$$

Situation: thésard sans sujet dans un séminaire
à l’IMAG …. Conférenciers : deux physiciens:
Maynard et Rammal (1978)

Dynamics: two cycles and fixed points; different behavior for different updates ....



Neural networks

$$x_i = s(\mathop{}_{j=1}^{n} w_{ij} x_j \quad b_i) \quad for \quad 1 \quad i \quad n$$

$W = (w_{ij})$ The weight matrix

$b = (b_i)$ The threshold vector

$s(u) = 1$ iff $u \geq 0$

0 otherwise

Given the labels of the nodes $\{1,\ldots,n\}$

Block sequential update: blocks are iterated one by one from left to right in a prescribed order:

$$(\ {}_{1},\ldots,\ {}_{n_1})(\ {}_{n_1+1},\ldots,\ {}_{n_2})\ldots(\ {}_{n_{q\ 1}+1},\ldots,\ {}_{n_q})$$

is a a permutation

The synchronous (parallel) update: $(1,2,\ldots,n)$

A sequential update: $(1)(2)\ldots(n)$

The first to remark the different iteration modes was Francois Robert , Discrete Iterations (Springer, 1986)

For arbitrary matrices W previous model may accept, iterated in parallel or block-sequentially, long period cycles and long transients … But when W is symmetric the network admits short periods and an energy: (E.G and J.Olivos, Discrete Mathematics, 1980, Discrete Applied Maths, 1981; E.G, SIAM J of Computing, 1982; E:G, F. Fogelman, Discrete Applied Maths(1985))

$$E(x(t)) = -\sum_{i=1}^{n} x_i(t) \sum_{j=1}^{n} w_{ij} x_j(t-1) + \sum_{i=1}^{n} b_i (x_i(t) + x_i(t-1))$$

Further, if diag (W) ≥ 0, any sequential update admits the energy (E.G., F. Fogelman, G. Weisbuch, Disc. Applied Maths. 1982)

$$E(x) = -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} x_i x_j + \sum_{i=1}^{n} b_i x_i$$

Which implies that:

1) for the synchronous iteration the attractors are only Fixed points or two cycles !!

2) For any sequential iteration with diag(W)≥0 there are only fixed points

3) For the parallel update $E = E(x(t)) \quad E(x(t \quad 1)) < 0$
if and only if $x(t) \, \pi \, x(t \quad 2)$

For the sequential update $E = E(x') \quad E(x) < 0$
iff $x \, \pi \, x$ So, the attractors are only fixed points.

4) In both situations transients are bounded by
α | | W | | x | | b | |

Some applications:

1) A neural equation with memory

$$x(t) = s\left(\sum_{k=1}^{n} w_k x(t - k) - b\right)$$

(la paramecie: le comportement de ce neurone unique a eté étudié par plusieurs chercheurs …..M.Cosnard, M. Tchuente., T.de Saint Pierre. and E.G … ce qui constitue un abus too much !!! ….Trop de neurones pour étudier un organisme unineuronal !!!!!!)

2) Majority functions and Bootstrap percolation models
(Pedro Montealegre, E.G (2011))

3) Schelling Segregation
(Nicolás Goles-Domic, Sergio Rica, E.G(2010-11)

Symmetry is so restrictive?

…… No …. because one may simulate any non-symmetrical neural network in linear space by a symmetric one with an specific update mode …

To give an other kind of answer I have to introduce a complexity
measure usually used in theoretical computer science.

The class P: problems which we can be decided in a serial computer in polynomial time.

The class NC: problems which can be decided in a parallel machine (say a PRAM) in poly-logarithmic time by using a polynomial number of processors.

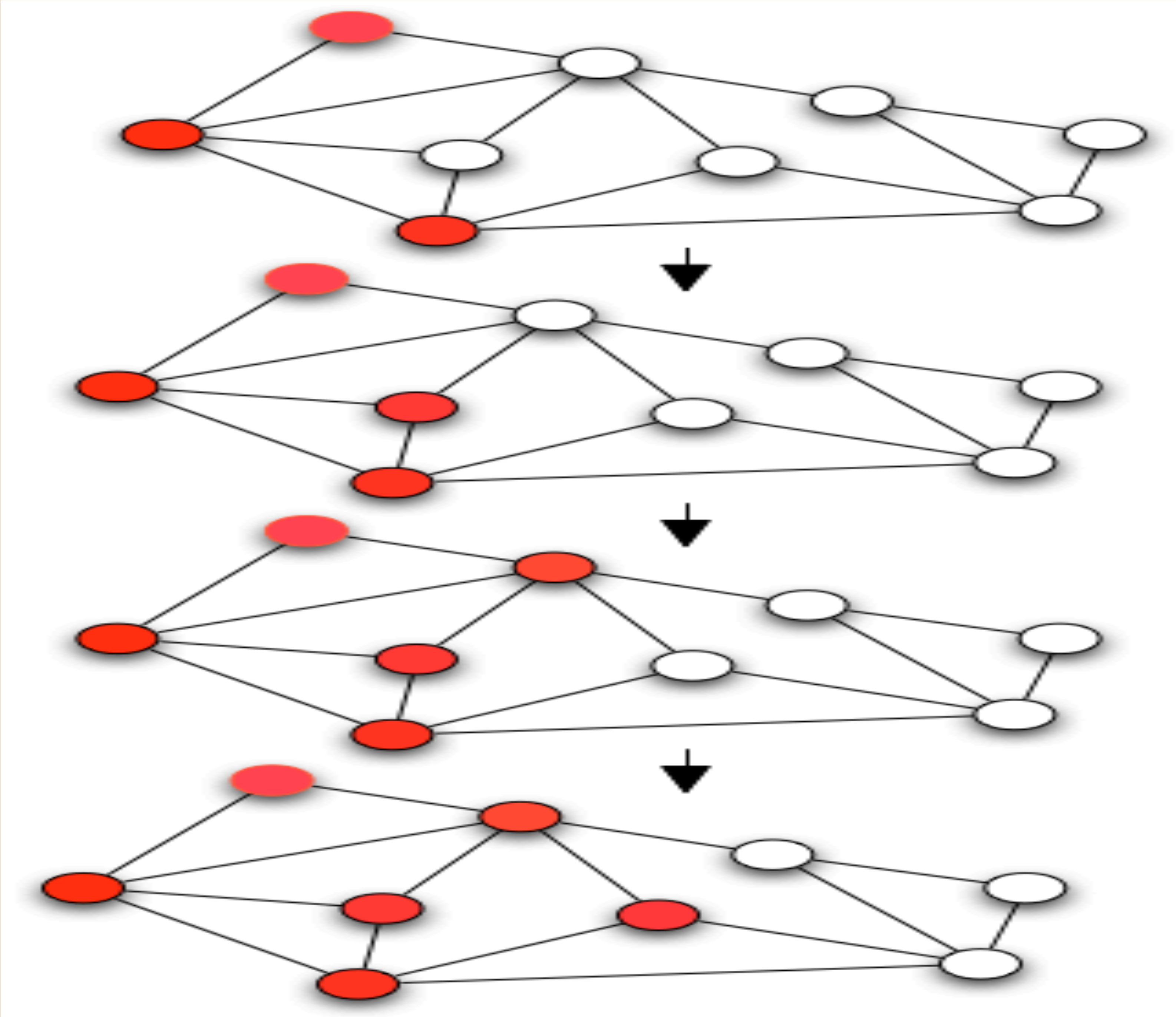
Bootstrap Percolation

Given a finite non oriented graph G=(V,E)

And an initial configuration of 0's and 1's

Consider the strict majority function operating at each node

What is the relationship between the graph and the proportion of 1's such that iterated in parallel every node will become 1?



Decision problem PER: given an initial configuration and a specific node at value 0. does there exist T>0 such that this node becomes 1?

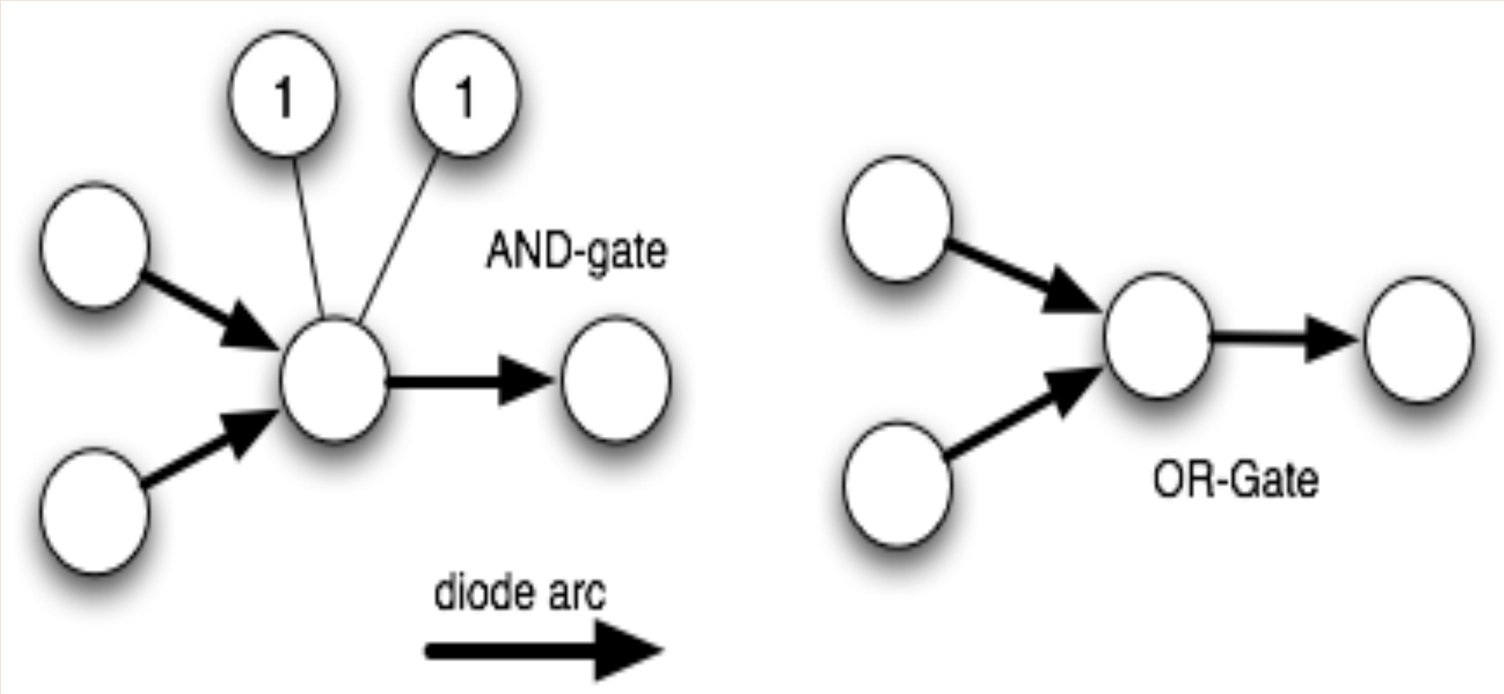
Theorem (Pedro Montealegre, E:G (1911))

If the graphs may have vertices with degree ≥ 5, PER is P-complete.

If the maximun degree ≤ 4, PER belongs to NC

Clearly PER belongs to P, because in almost O(n) steps
the dynamics arrives to the steady state.

The proof of P-Completeness consists in simulating the monotone circuits behavior inside the strict majority dynamics.


1
1
AND-gate
OR-Gate
diode arc

For the case when the maximum degree ≤ 4 one may reduce the problem to compute connected and biconnected components in the graph, which one may do in a PRAM in

$$O((\log n)^2)$$

See Jaja …………ce ne pas une blague !!!!

The Schelling Segregation model

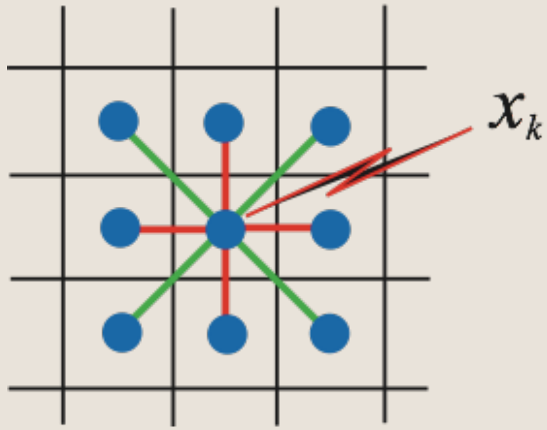
(Nicolas Goles-Domic, Sergio Rica, E.G.

PHYSICAL REVIEW E **83, 056111 (2011)**
And work in progress.

The Model of Segregation by Shelling

Thomas C. Schelling (1969 - 1972)

- Lattice one or two dimensional with periodic conditions
- State $x_k = \pm 1$



- Neighborhood Moore (green and red arrows) and von Neumann (red arrows)
- Tolerance threshold $\{1, \ldots, |V|\}$

Happiness threshold

An individual is unhappy if there are more than individuals on the other state in its neighborhood

The update rule

At each step, one lists the unhappy individuals of both species, and then randomly (for instance) one exchanges two individuals of opposite value.

Quantitative behavior

≥ 5 :the energy decreases

$$E[\{x\}] = -\frac{1}{2}\sum_{k=1}^{N} x_k \sum_{i \in V_k} x_i$$

In general, if V is the neighborhood, the energy decreases
If and only if

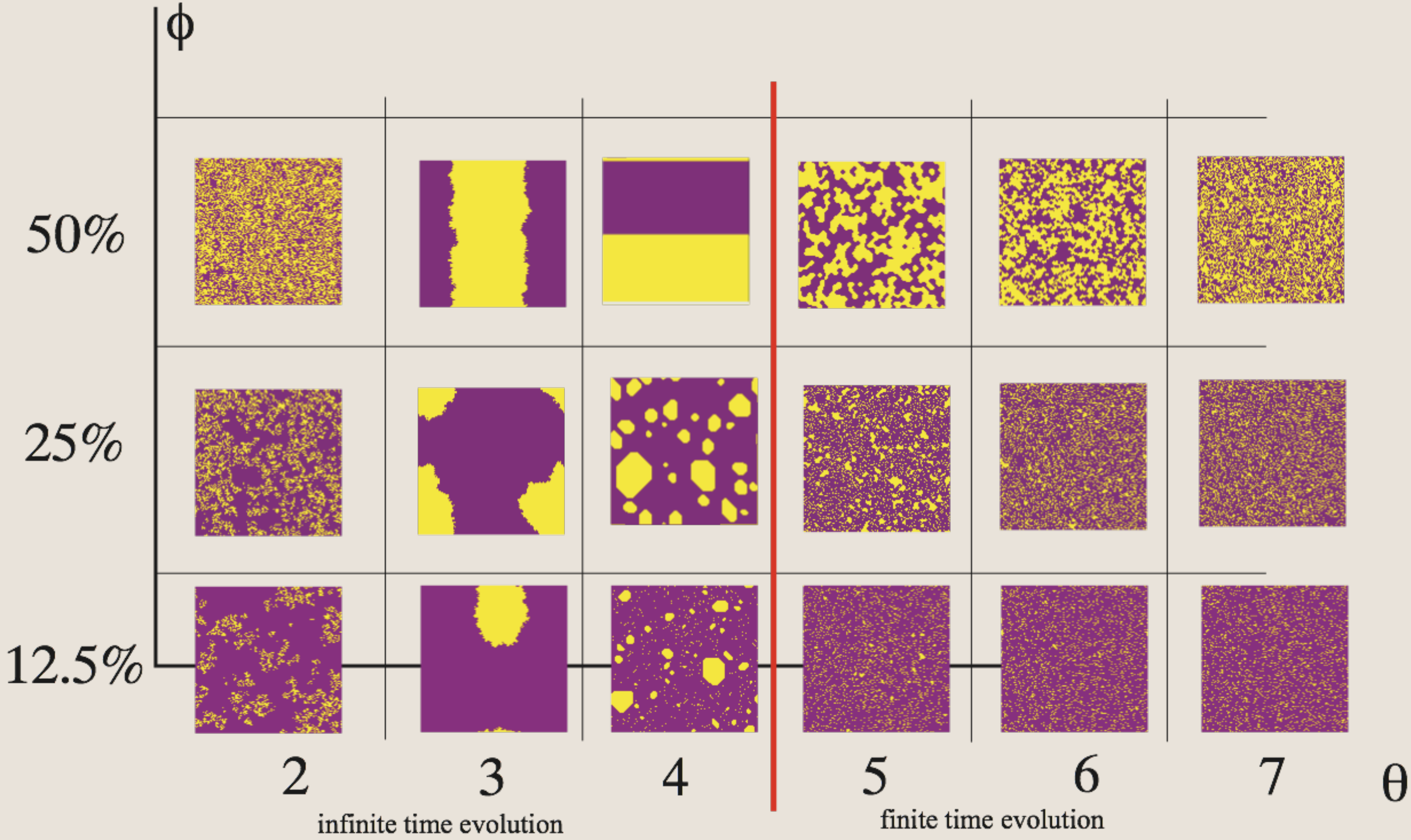
$$> \frac{|V|}{2}$$

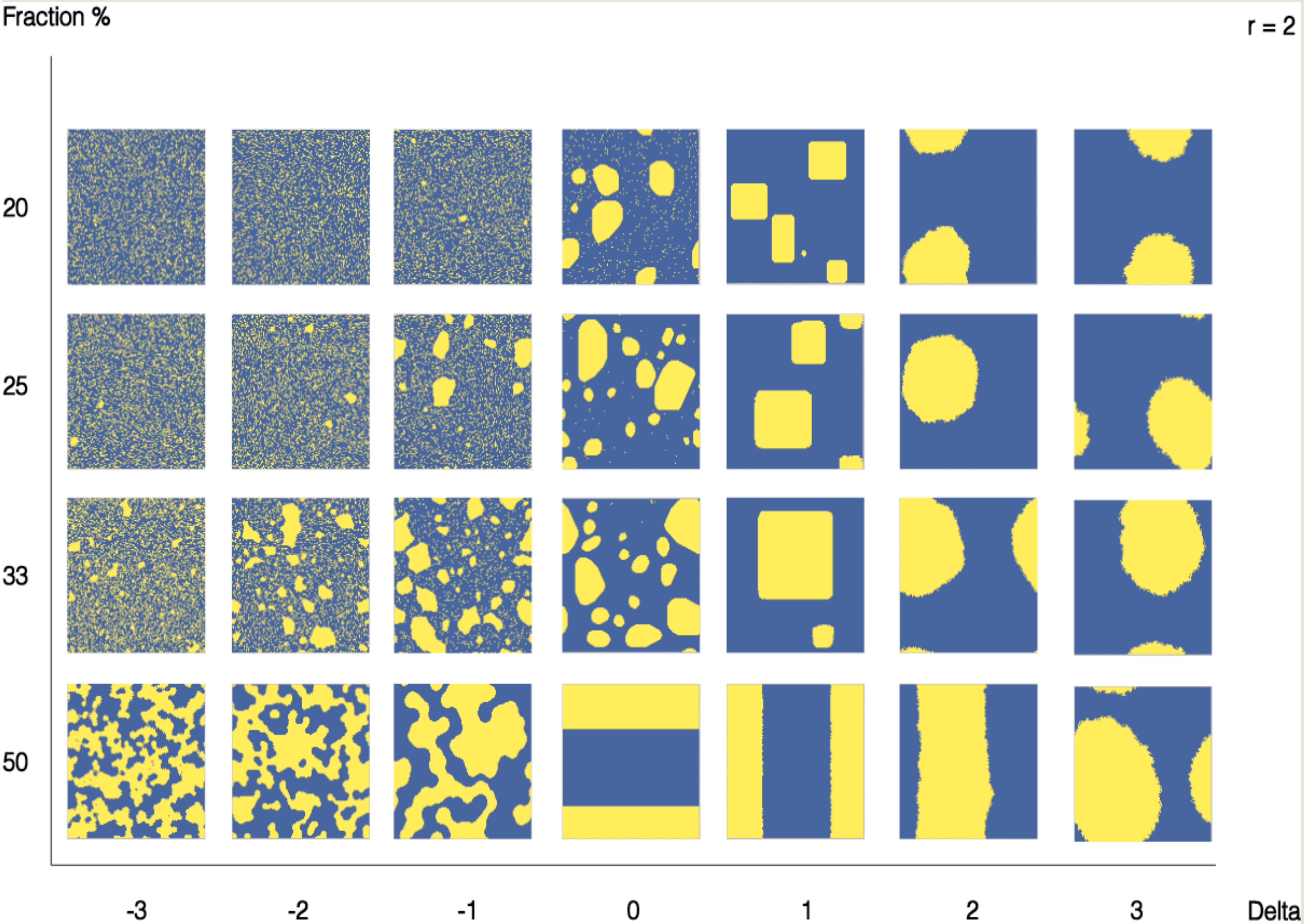
Geometrical interpretation

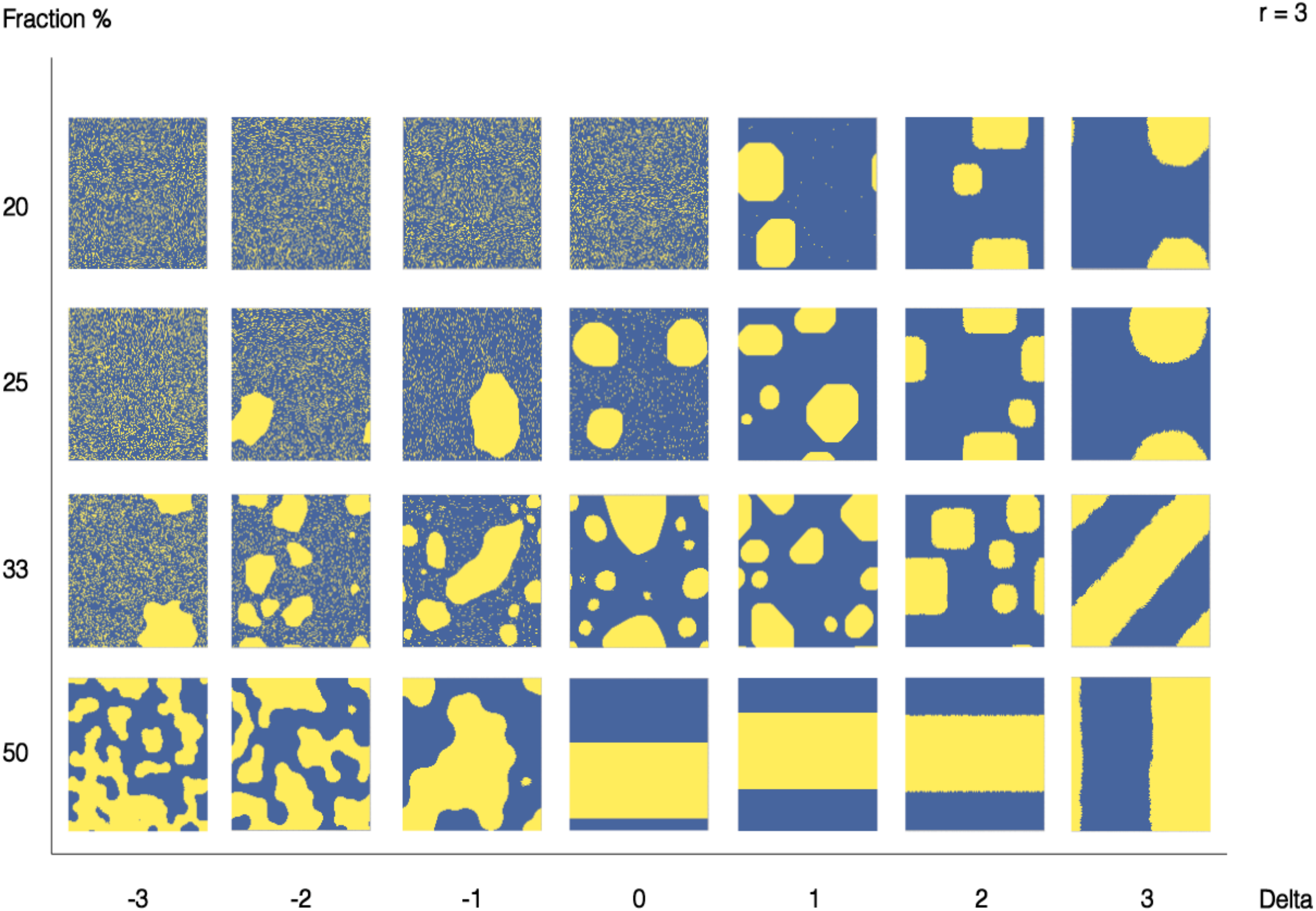
It is easy to see that the energy minimizes the perimeter of the clusters …… so the dynamics tries to do that !!

Other phase diagrams with circle-neighborhoods with different radios (Nicolás Goles-Domic Simulations):

Phase diagram
for Moore's neighborhood




Fraction %
r = 2
20
25
33
50
-3
-2
-1
0
1
2
3
Delta


Fraction %
r = 3
20
25
33
50
-3
-2
-1
0
1
2
3
Delta

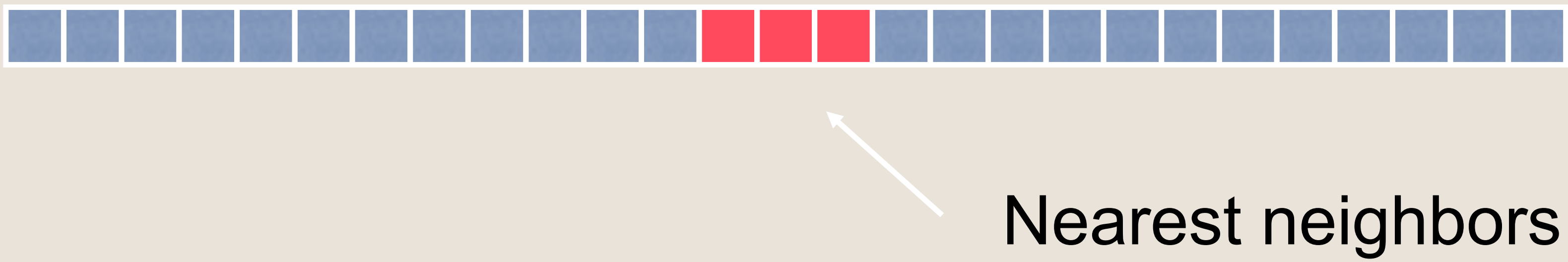
Prediction, short-cuts and Computational Complexity

The real state problem: Is it easy to know if someone would change his house?

- The Real State Prediction problem (RSP)

- Will a site *i*, such that $x_i = -1$, have a non zero probability to change its state at some step $T \geq 1$?

We will first analyse the real state problem for 1D and the von Neumann neighborhood in 2D.



For both parameters, $\theta \in \{1,2\}$, RSP is easy to solve

For $=1$ belongs to NC

$$= 1$$

-1	1

Both are unhappy: swaps for T= 1

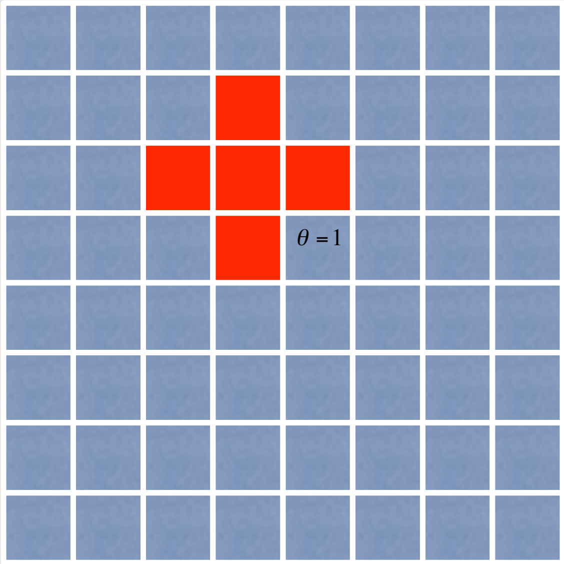
In general consider the nearest +1

-1	-1	-1	-1	1
0	1	2	3	4

So P=0 for $T<4$ else $P>0$

Two dimensions

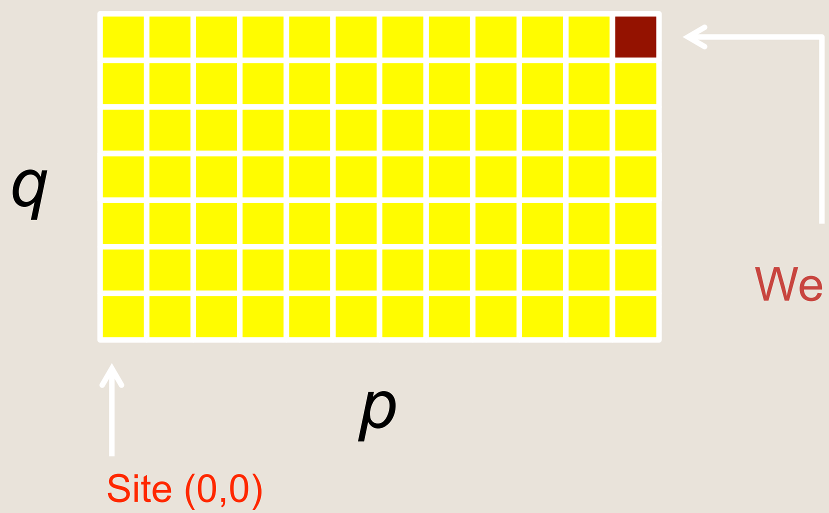
The von Neumann Neighborhood



$$\theta \in \{1,2,3,4\}$$

Case $=1$ i.e. a site is unhappy iff there exists at least one neighbor in a different state

Further, in this case two neighbors in diferent state are both unhappy !!!



= -1

=+1

We find the nearest site at different value +1

$$D = p+q$$

Clearly me may do it by a PRAM as we
did in the one dimensional case

Case $=4$

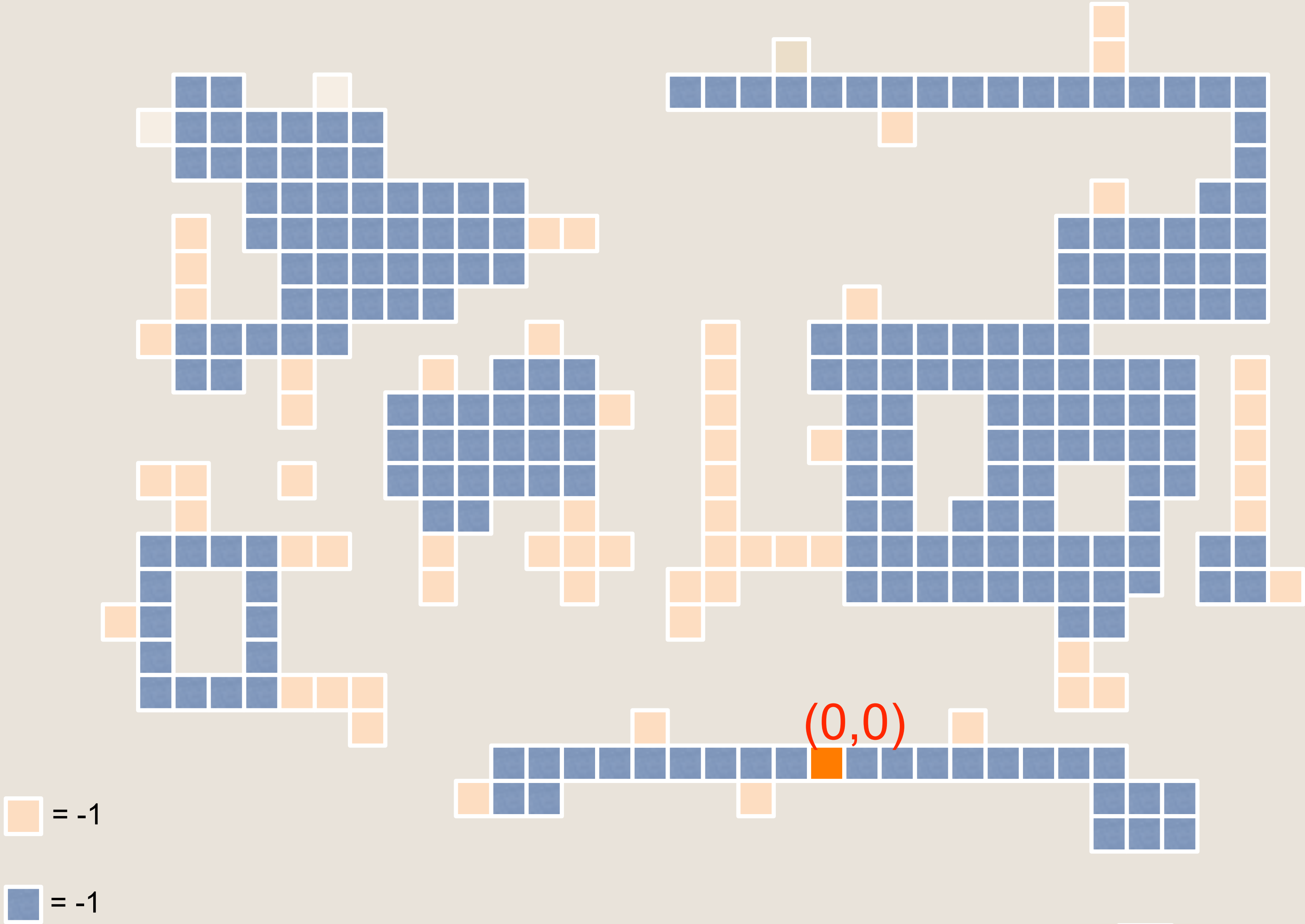
An unhappy site has to be in a very bad situation: every neighbor being in the other state

So we may now decide if there exists two unhappy people in different state in O(1)

Case $= 3$

Recall that for $\{3,4\}$ the operator E is an energy, so the dynamic converges to fixed points which are local minima of E.

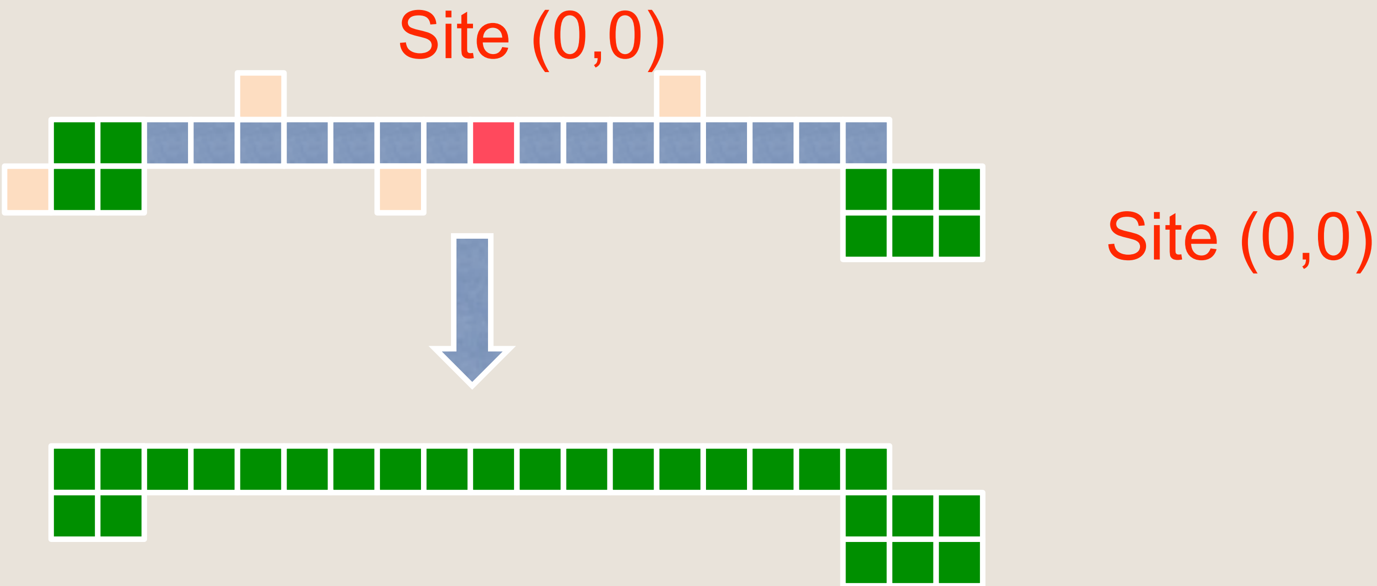
A fixed component of, say -1, is such that each element has at least two neighbors at the same state



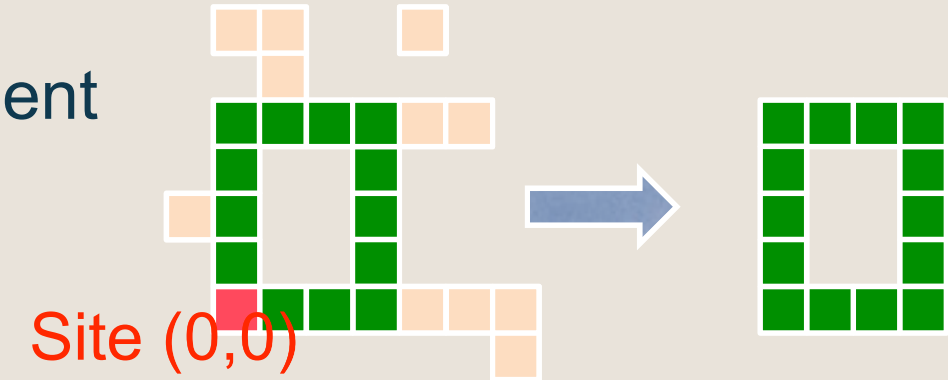
will vanish if there are enough unhappy

1

So the site (0,0) at value -1 will never change if it belongs to a connected component such that there exists two different paths to stable clusters



Or it belongs to a biconnected component



The search of the connected component of -1's where site (0,0) belongs can be done in $O\left(\left(\log(N)\right)^2\right)$ with polynomial number of processor in a PRAM

Also one may compute biconnected components in $O\left(\left(\log(N)\right)^2\right)$

See JaJa's book et ce n'est pas une blague !!!!

Finally we may compute the number of unhappy +1's in O(1) with O(N) processors

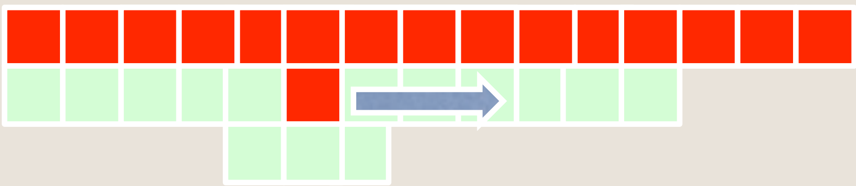
Remark: $N = n \times n$ the number of sites in the network

So the Schelling problem belongs to NC for $\{1,3\}$ and it is constant for $=4$

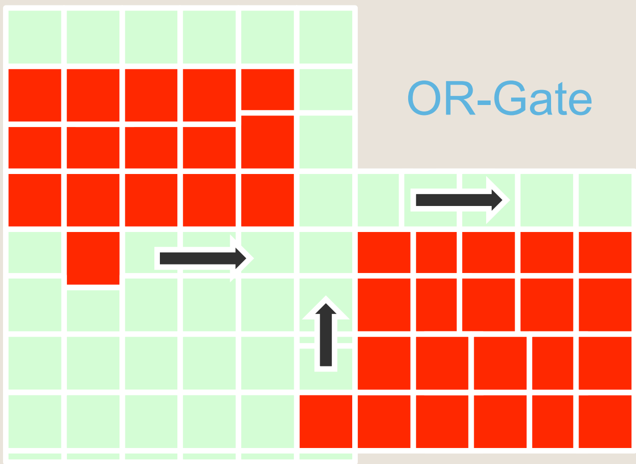
Now we have to see the complexity for $=2$

For $=2$ the segregation problem is P-Complete

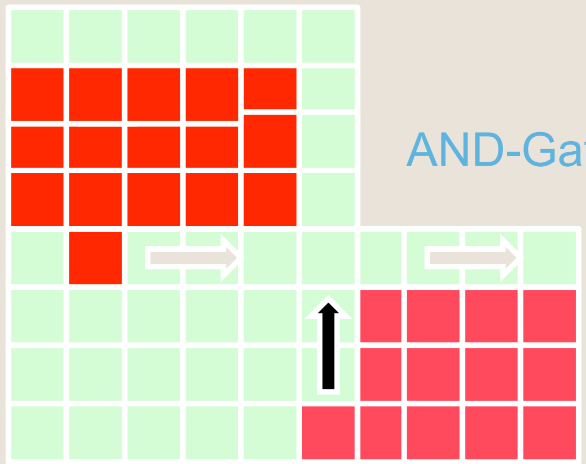
It is in P because we will only accept nearest swaps (*a*,*b*) such that d(*a*,*b*)=1, so it is enough to compute the light-cone associated to the site (0,0)



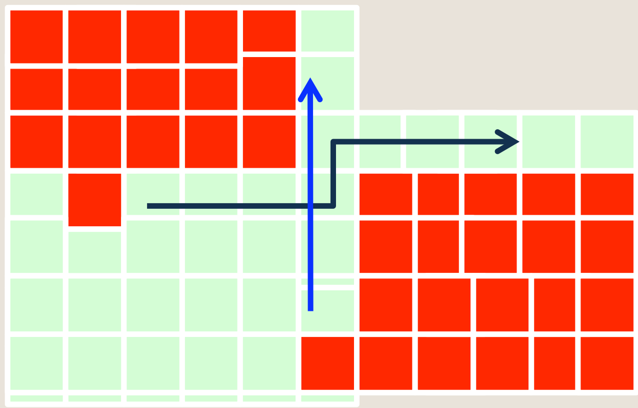
Wire to the rigth



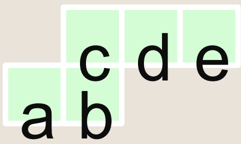
OR-Gate



AND-Gate



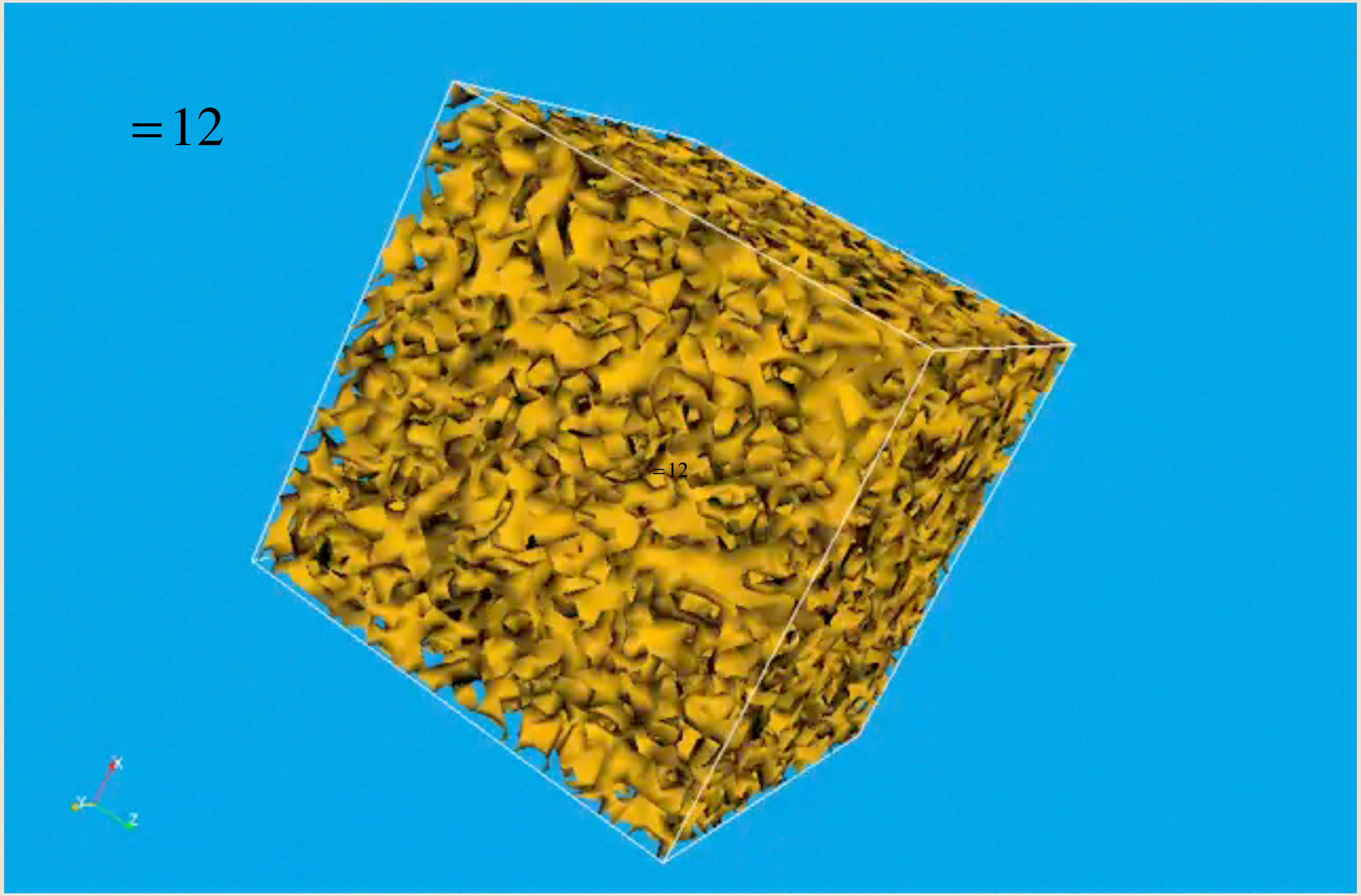
Cross-over



(a,b), (b,c),(c,d),(d,e)

updated mode
For the cross-over

(f,g),(g,h),(h,o),(o,p)

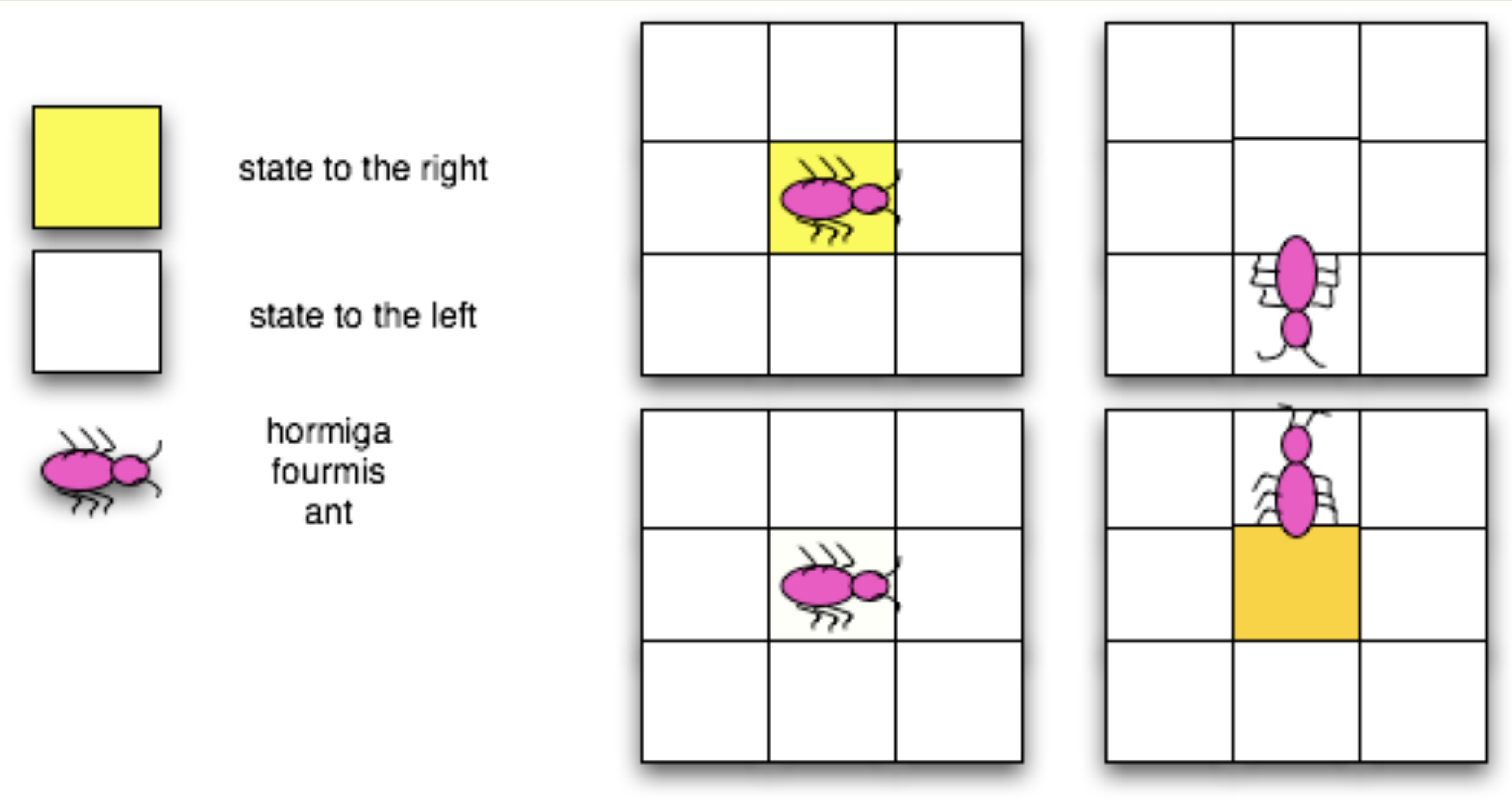

=12
=12

Ants

- Could intelligence be an emergent property???

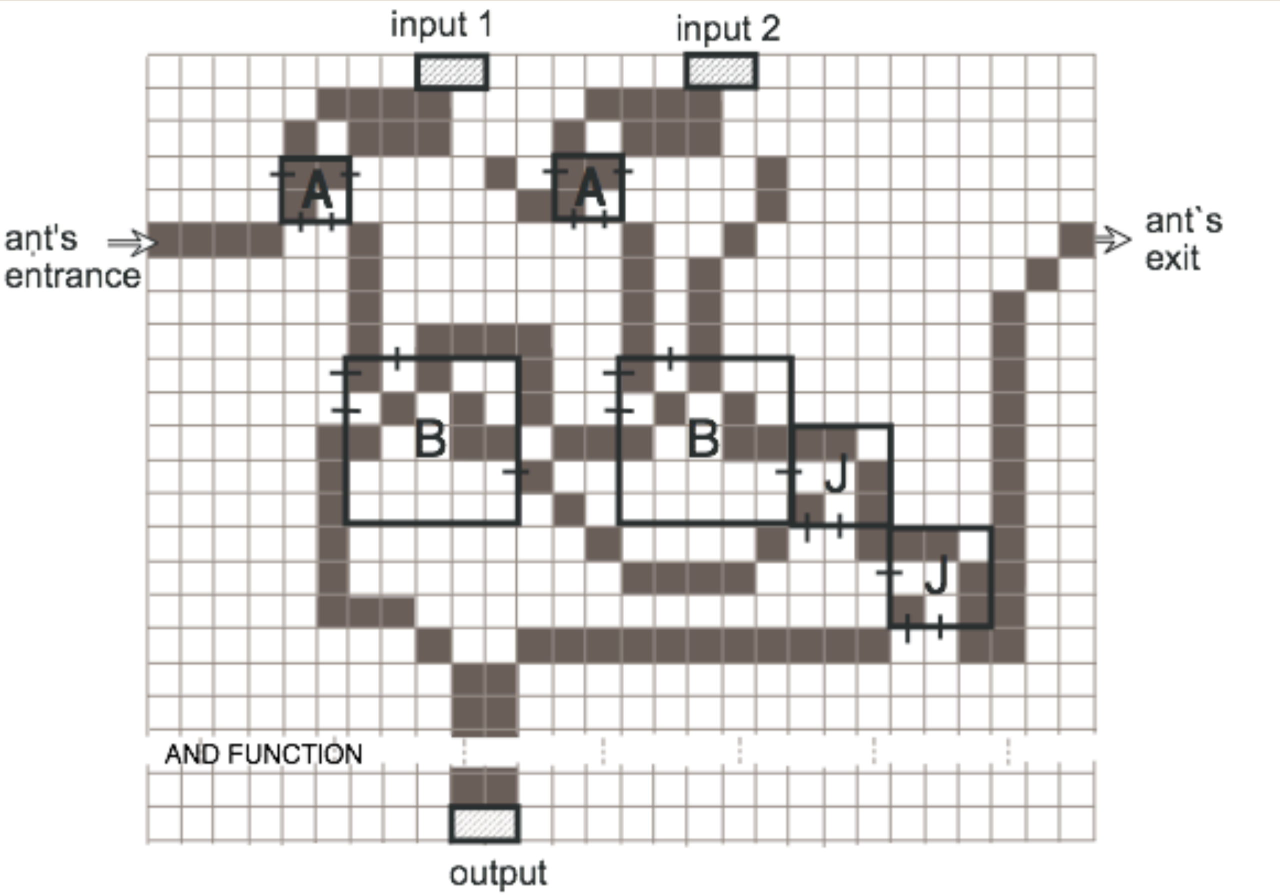
- Au debut de l’été 1982 j’ai reçu une bien curieuse lettre à Grenoble ……

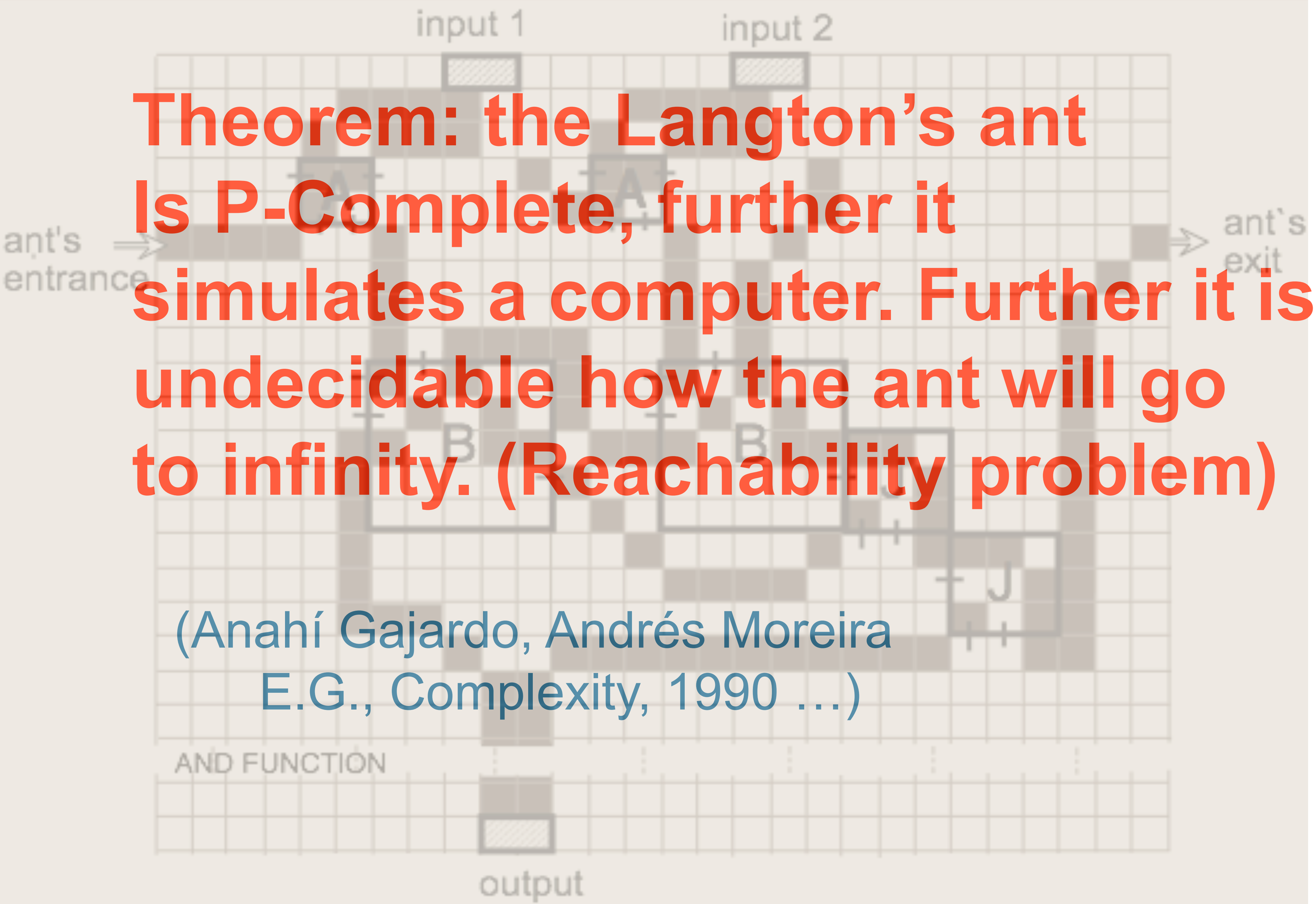
Planar ant model (Langton's ant)



Ant's dynamics

One may design logical gates





Theorem: the Langton's ant Is P-Complete, further it simulates a computer. Further it is undecidable how the ant will go to infinity. (Reachability problem)

(Anahí Gajardo, Andrés Moreira E.G., Complexity, 1990 …)

Some associated models and its dynamics

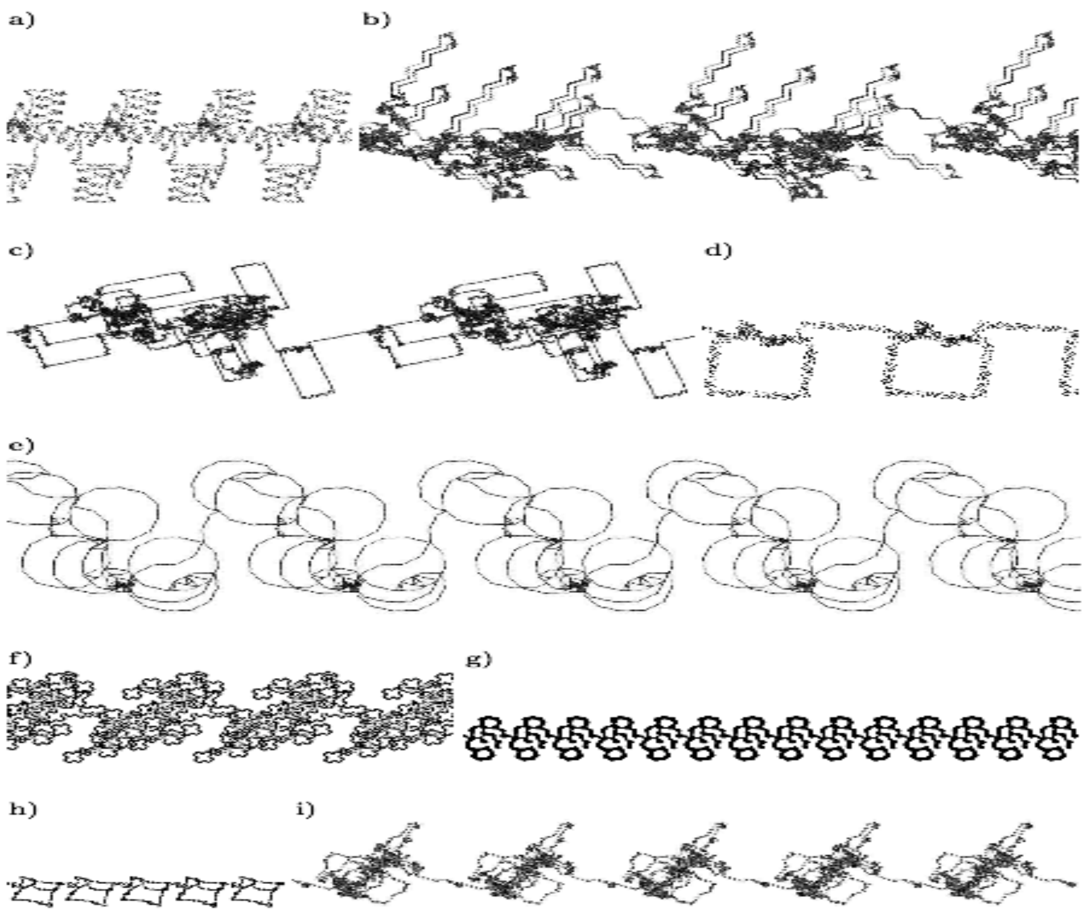


FIGURE 2: Same as Fig. 1, but with a one-bit perturbation in the forcing period.

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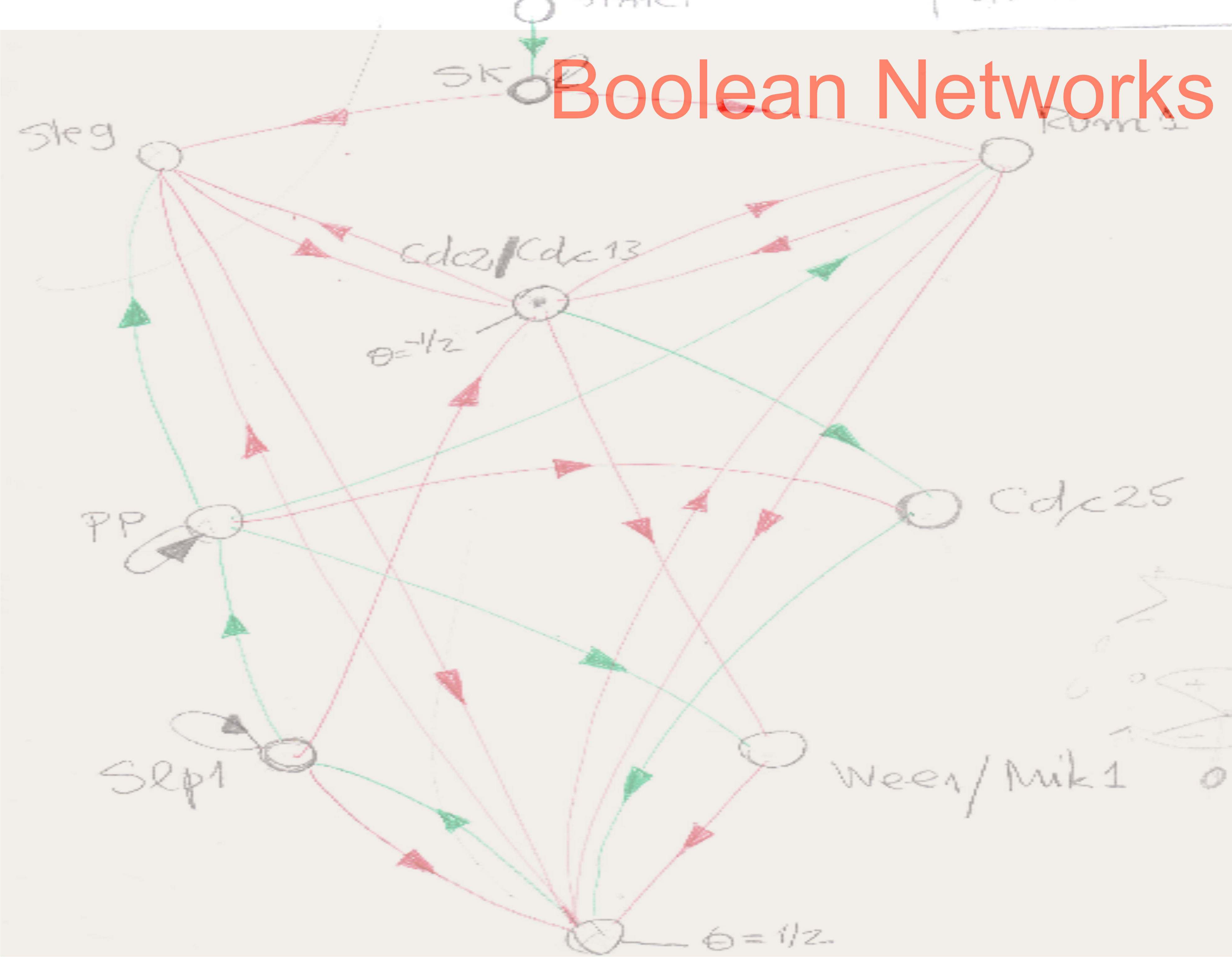
Boolean Networks



History

- Stuart. Kauffman,Metabolic stability and epigenesis in randomly connected nets, J. Of Theor. Biol, 22, 437-67, 1969.

- François Robert, Discrete Iterations, Springer Verlag, 1986.

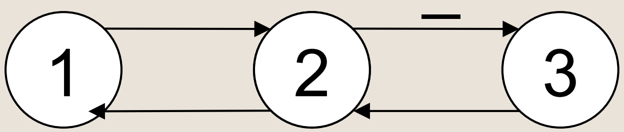
The results presented here were done in collaboration with :

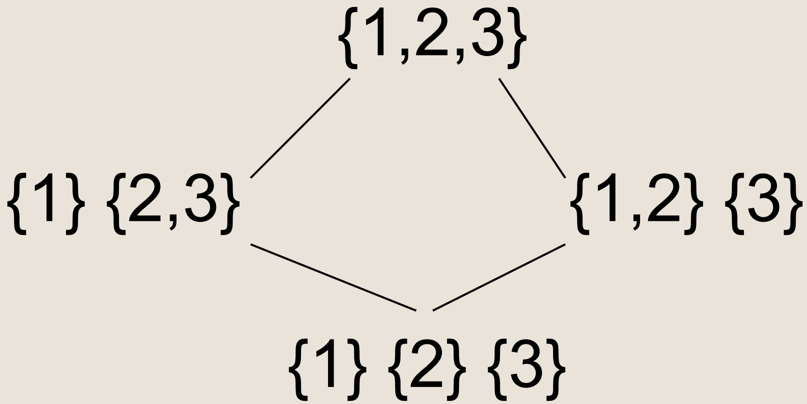
Julio Aracena (Universidad de Concepción, Chile)

Andrés Moreira (Universidad Federico Santa Maria, Valparaíso, Chile)

Lilian Salinas (Universidad de Concepción, Chile)

Gonzalo Ruz (Universidad Adolfo Ibáñez)





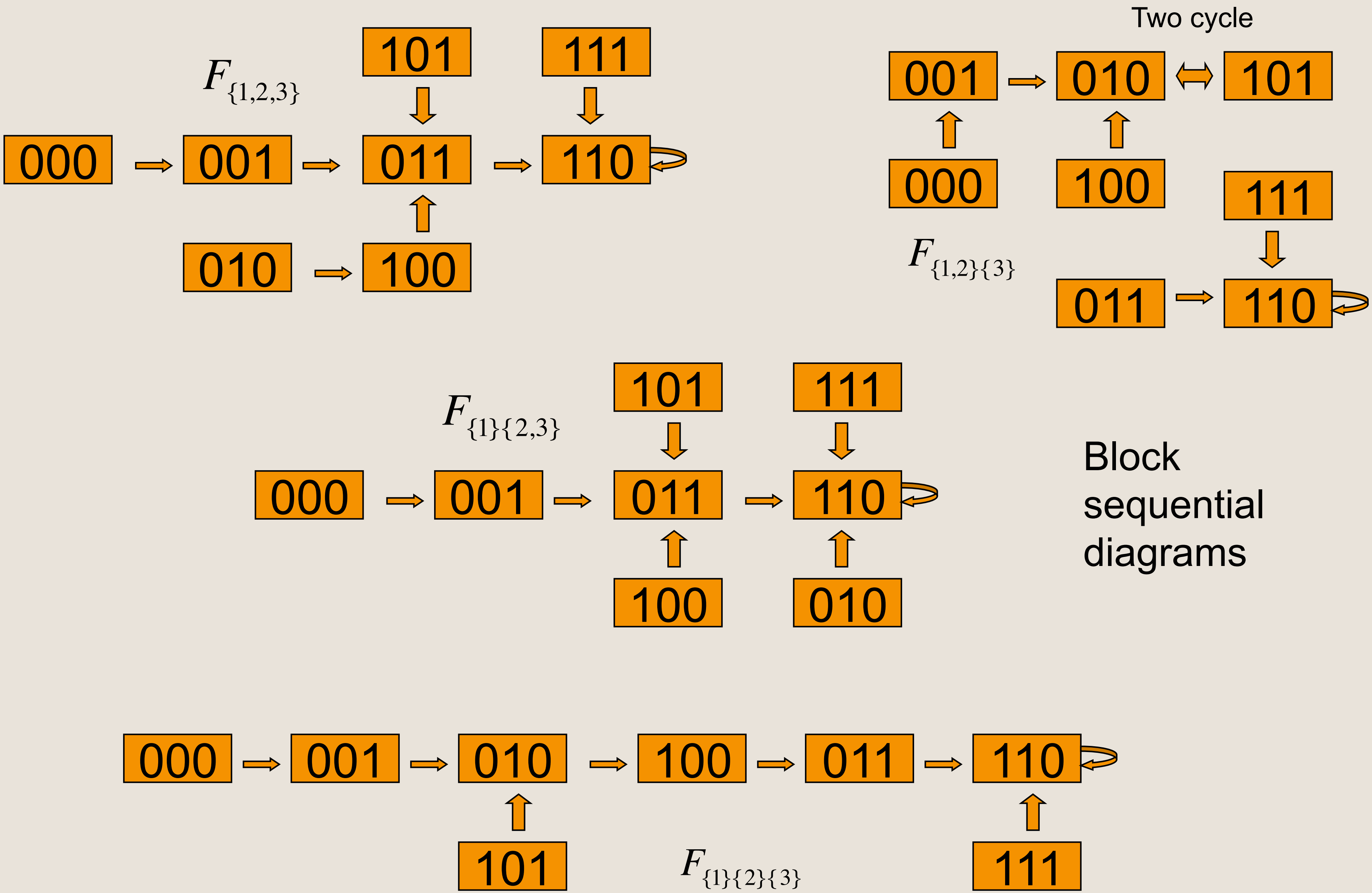
Block Sequential partitions for three elements

$$F_{\{1,2,3\}}(x_1,x_2,x_3) = (x_2, x_1 + x_3,\ \ x_2)$$

$$F_{\{1,2\}\{3\}}(x_1,x_2,x_3) = (x_2, x_1 + x_3, (\ \ x_1)(\ \ x_3))$$

$$F_{\{1\}\{2,3\}}(x_1,x_2,x_3) = (x_2, x_2 + x_3,\ \ x_2)$$

$$F_{\{1\}\{2,3\}}(x_1,x_2,x_3) = (x_2, x_2 + x_3, (\ \ x_2)(\ \ x_3))$$


$F_{\{1,2,3\}}$
000
001
011
110
101
111
010
100
Two cycle
001
010
101
000
100
111
011
110
$F_{\{1,2\}\{3\}}$
$F_{\{1\}\{2,3\}}$
000
001
011
110
101
111
100
010
Block sequential diagrams
000
001
010
100
011
110
101
111
$F_{\{1\}\{2\}\{3\}}$

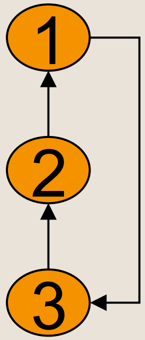
Cycles in synchronous and serial Iterations

$$F:\{0,1\}^3 \qquad \{0,1\}^3$$

$$f_1(x_1,x_2,x_3)=x_2$$

$$f_2(x_1,x_2,x_3)=x_3$$

$$f_3(x_1,x_2,x_3)=x_1$$

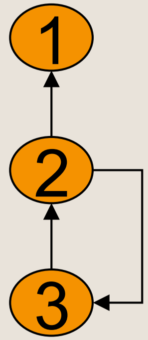


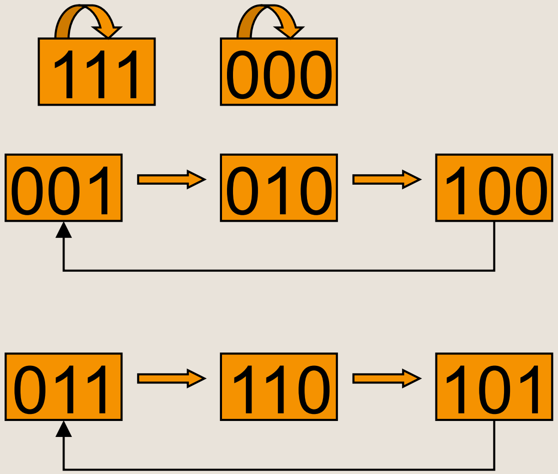
$$G:\{0,1\}^3 \qquad \{0,1\}^3$$

$$g_1(x_1,x_2,x_3)=x_2$$

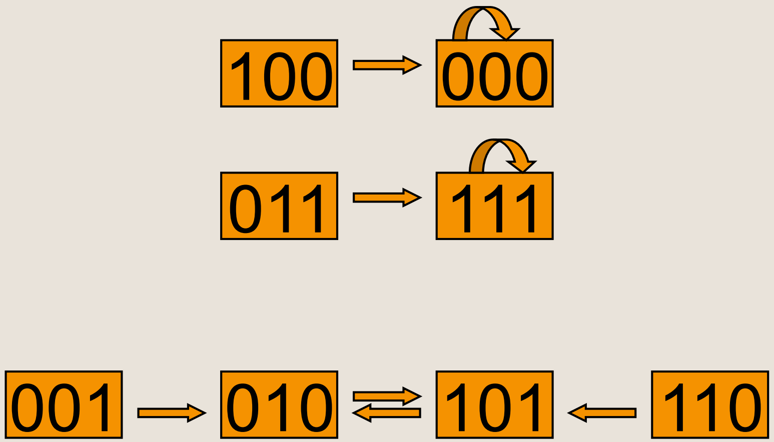
$$g_2(x_1,x_2,x_3)=x_3$$

$$g_3(x_1,x_2,x_3)=x_2$$





Parallel update: 3-cycles



Serial update: 2-cycle

Theorem.

Consider a network with non-negative loops then the cycles with period ≥ 2, if they exists, are different for parallel and serial iteration.

i.e both iterations cannot share non trivial cycles

Comparison between parallel and serial dynamics of Boolean networks
E.Goles, L. Salinas, T.C.S.

There is another way to encode different updates. Consider a network *N* = (*F, s*); where *F* is the set *of n* local boolean functions and *s* is the “order” to update the nodes.

That is to say *s* is a function from the set of nodes on itself.

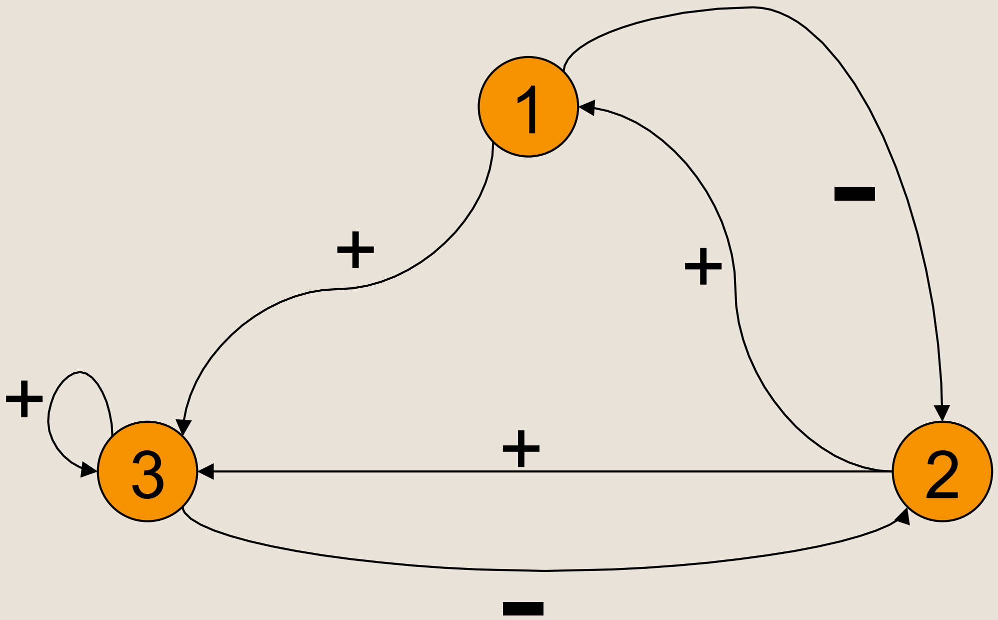
$$s:\{1,\ldots.,n\} \qquad \{1,\ldots..,n\}$$

Such that $s(i) \prec s(j)$ means node *i* is updated before node *j*

From that we may define a signed graph. To the graph *G*, asociated to *F* we define *G*(*s*) as follows:

$\mathrm{sgn}(i,j)$ = + 1 if $s(i) \geq s(j)$

= - 1 if $s(i) \prec s(j)$



$$s(1) = 2$$
$$s(2) = 3$$
$$s(3) = 1$$

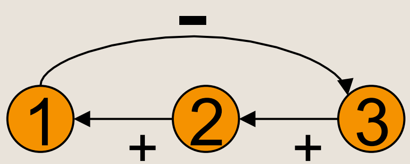
The 3d node is updated first; the 1st is updated the second; and the 2nd is the last to be updated. The iteration corresponds to the serial update (3)(1)(2)

Given two iteration modes on a same boolean function, i.e.

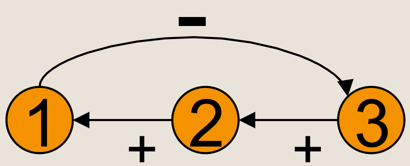
$$(F, s_1) \text{ and } (F, s_2)$$

If they have the same signed graph : $G^F_{s_1} = G^F_{s_2}$

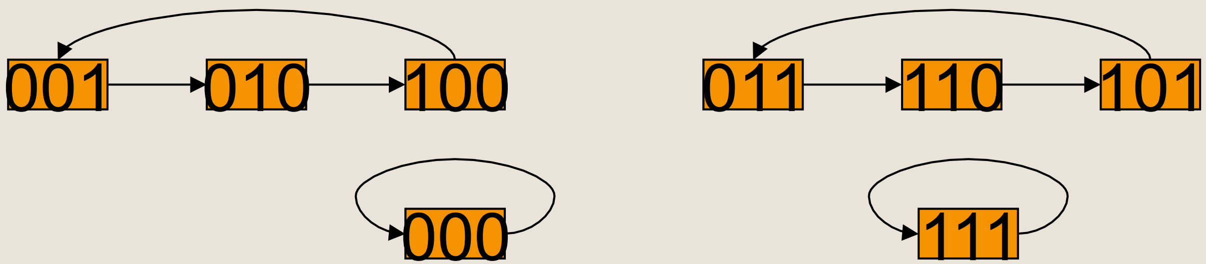
Then they have the same dynamics



(1)(2)(3) or **For *i* =** 1,2,3 $s(i) = i$



(1,2)(3) or $s(1) = s(2) = 1 \quad s(3) = 2$



dynamics

Communication Complexity on Cellular Automata

This work was done in colaboration with:

P.E. Meunier (Ph.D student ENSL- France)

I. Rapaport (DIM, U. De Chile)

G. Theyssier (Univ. de Savoie, CNRS, France)

E.G.

1. Communication Complexity in CA

2. Examples

3. PRED decision problem

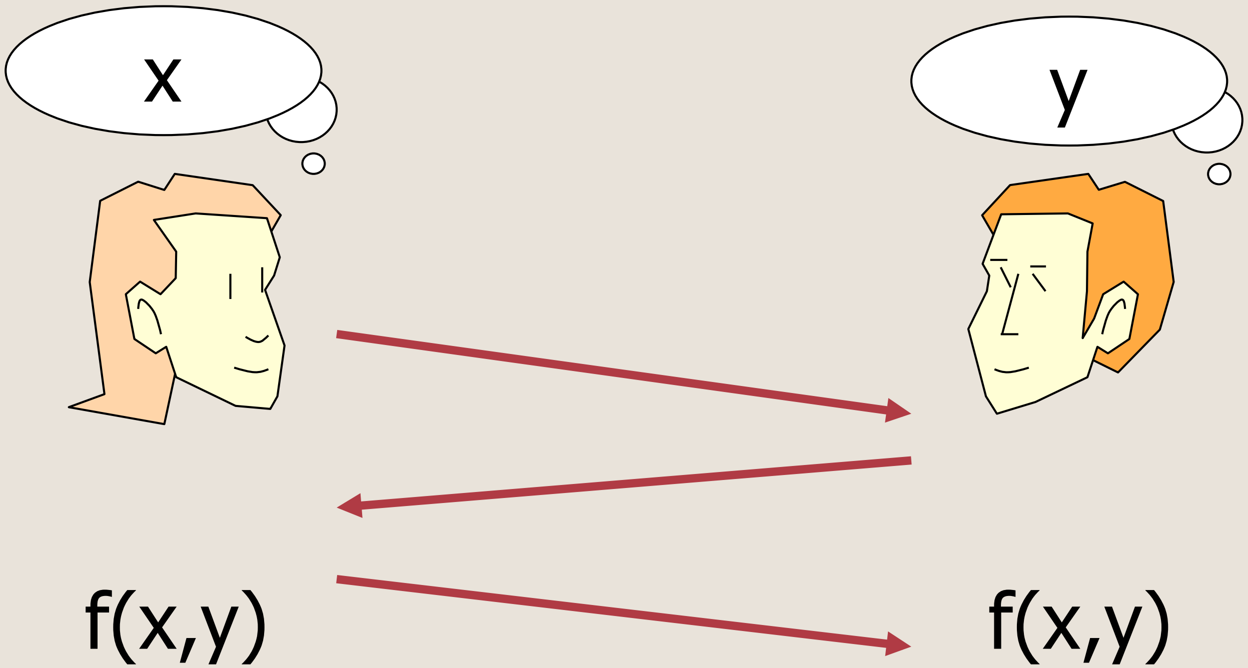
4 Application to rule 218

5 Intrinsic Universality and C.C

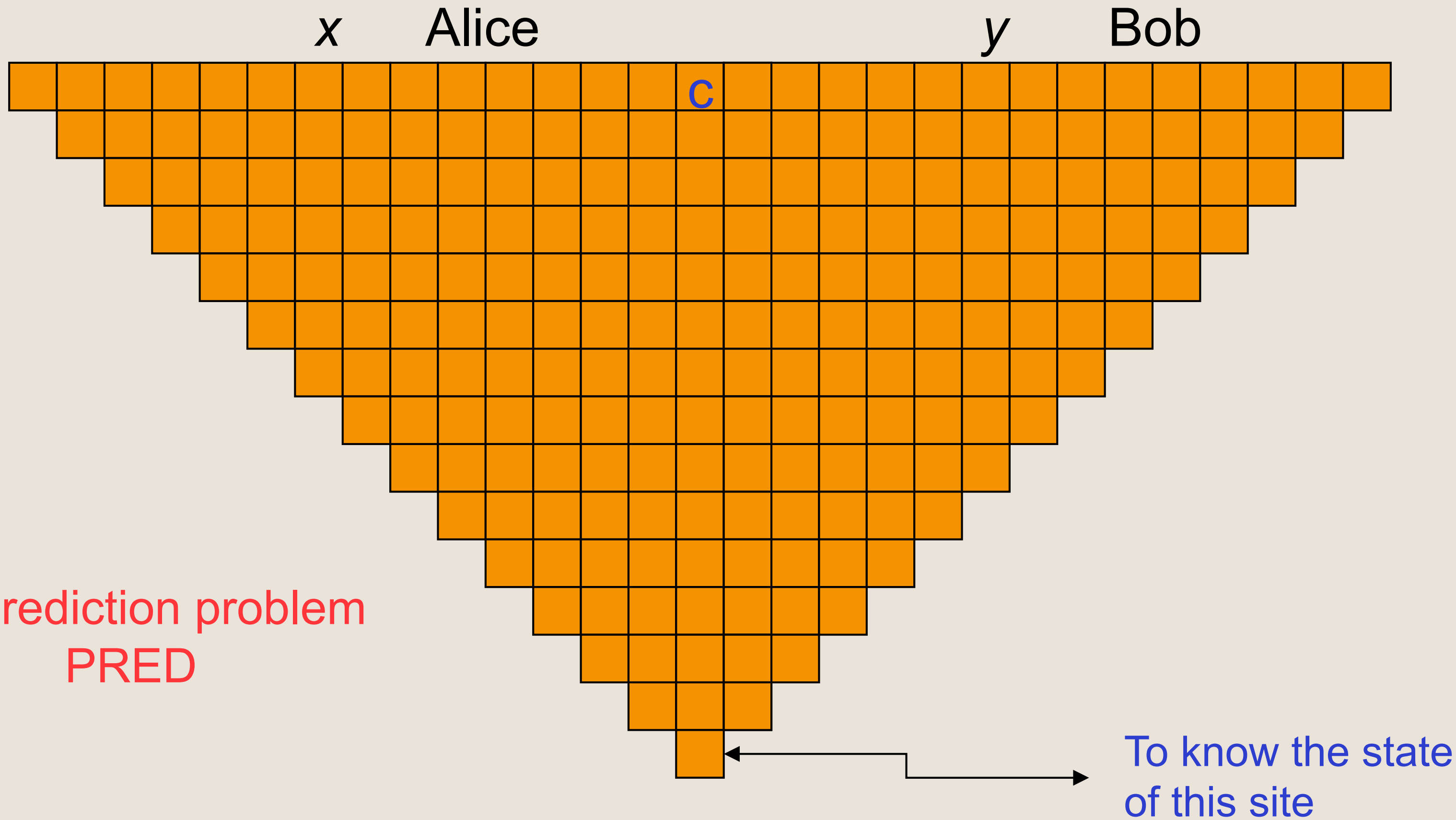
- References:

- C.Durr, I Rapaport, G.Theyssier, C.A and Communication Complexity, TCS 290/3,355-368, 2003
- E. Goles, C. Little,I. Rapaport, Understanding a non-trivial C.A. by finding its simplest underlaying communication protocol, in S.H Hong, H Nagamochi (eds) Oprocc 19th. Int. Symposium in Computer Science (ISAAC2008), LNCS 5369, vol 2380, Springer, 2008.
- E. Goles, P.E. Meunier, I. Rapaport, G. Theyssier, communication complexity and intrinsic universality in cellular automata, to appear in TCS,2009

Def: necessary number of communication bits in order to compute a function when each party knows only part of the input



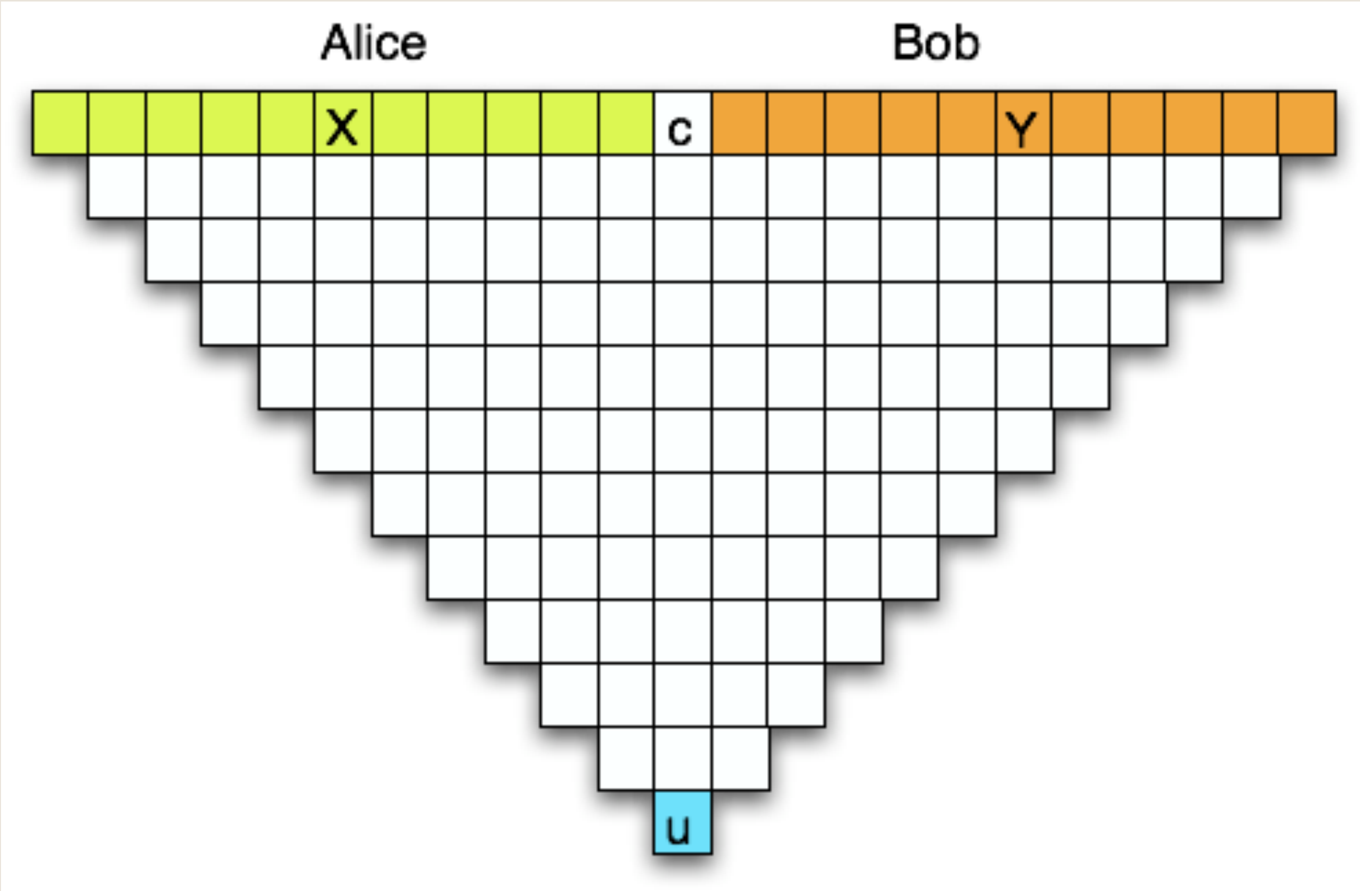
We will present two communication complexity problems related with CA:
The prediction problem (PRED).



The prediction problem
PRED

The PRED problem

Information to be shared by Alice and Bob
X, Y are binary vectors, c is 0 or 1

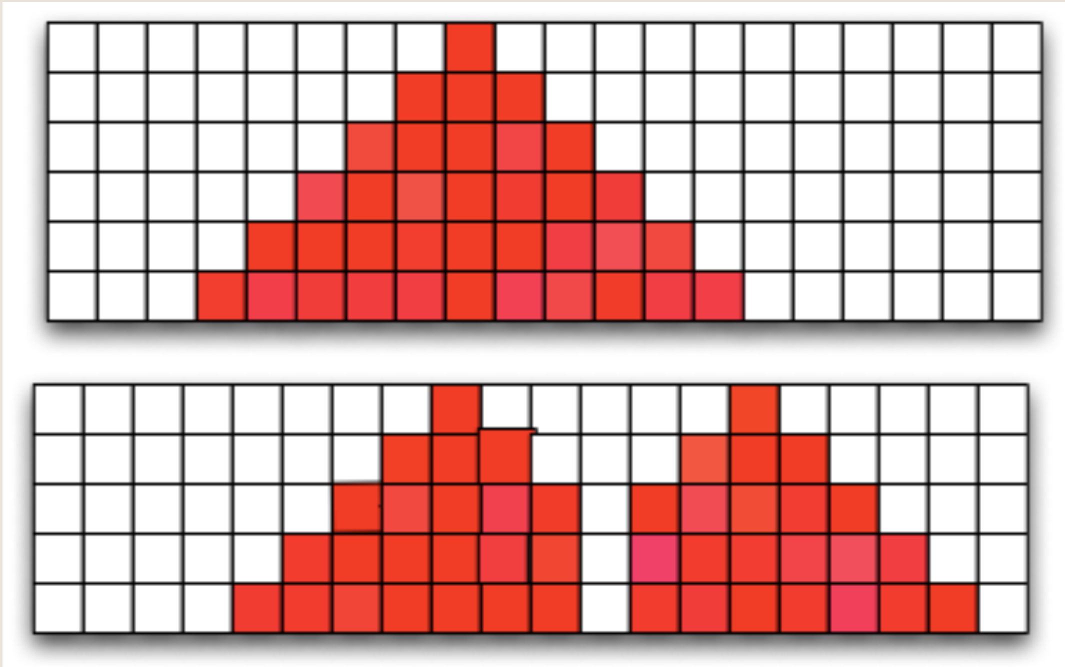


The final bit is the n-th composition of the local rule

f		g	
000	0	000	0
001	1	001	1
010	1	010	1
011	1	011	1
100	1	100	1
101	1	101	0
110	1	110	1
111	1	111	1

f

g



One way protocole for a rule f:
The minimun bit information to be send by A (B) to B (A)

Def: Let $M_n(c)$ the $2^n x 2^n$ matrix

$$(M_n(c)) = \left(f^n(x,c,y)\right)$$

Theorem: the number of different rows or columns is a lower bound for the size of the one way protocol

In Communication Complexity E. Hushilevitz,N. Nisan, Cambridge University Press 1997

The communication complexity for additive rules is O(1)

In fact, given the vector x, y and c

Since f is additive:

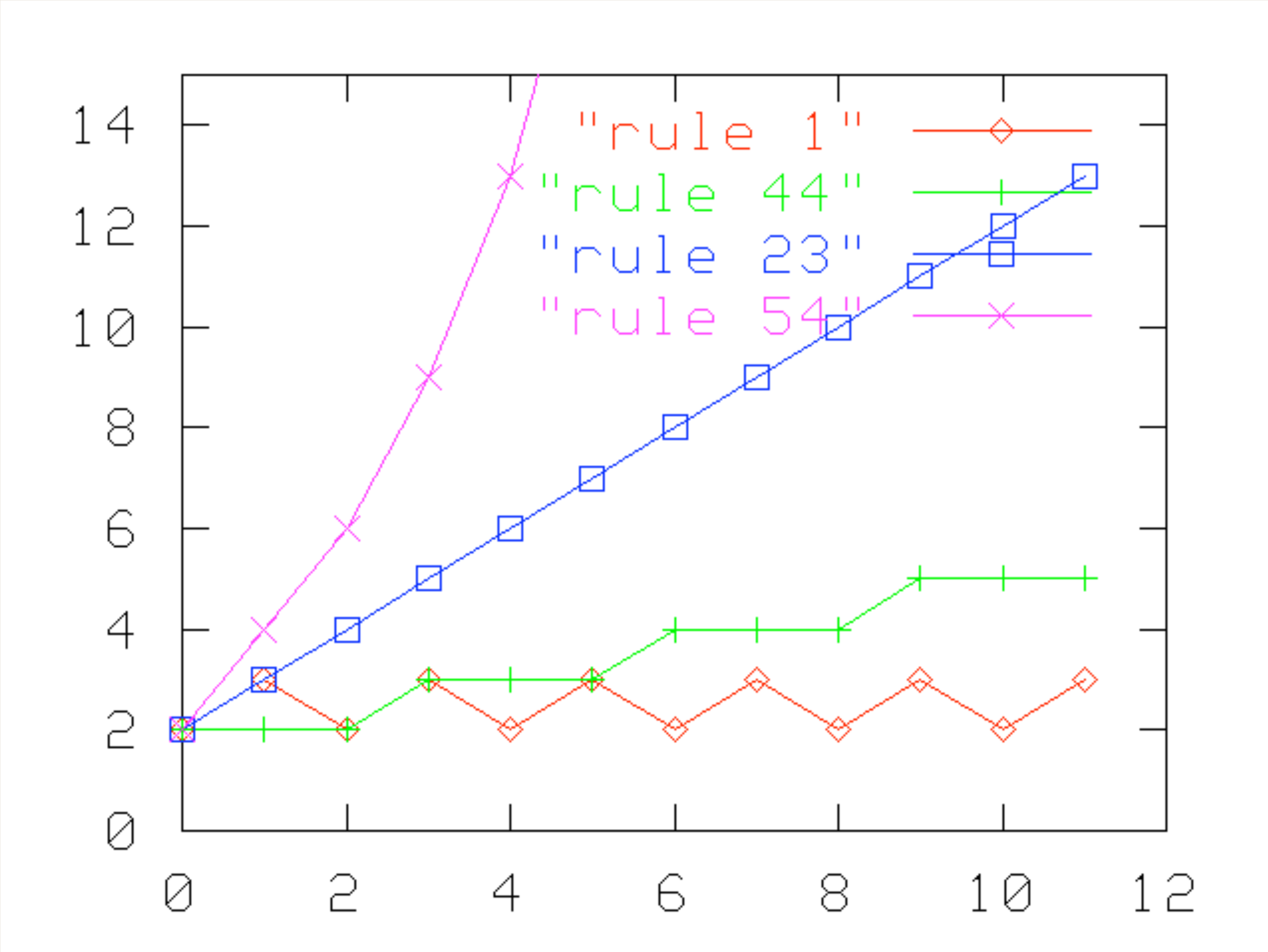
It suffices that Alice send the bit:

$$f^n(x,c,\vec{0})$$

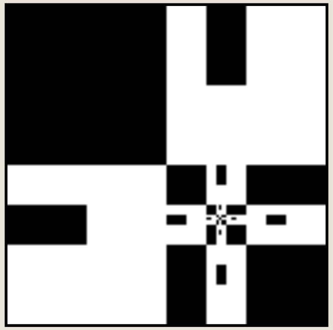
From additivity:

$$f^n(x,c,y) = f^n(x,c,\vec{0}) + f^n(\vec{0},0,y)$$

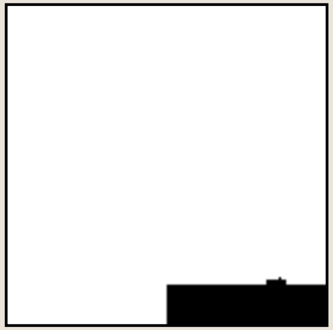
Matrix behavior of different rules



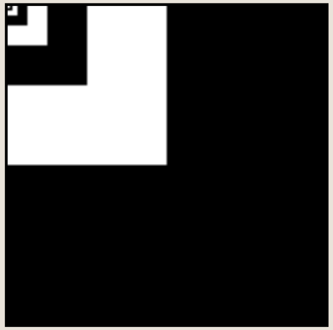
Others examples



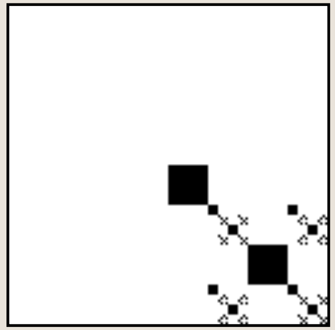
Rule 33



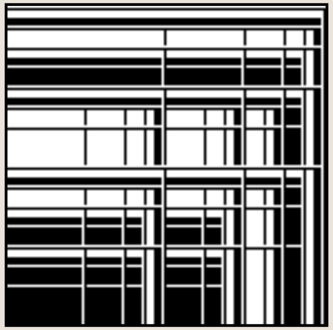
Rule 44



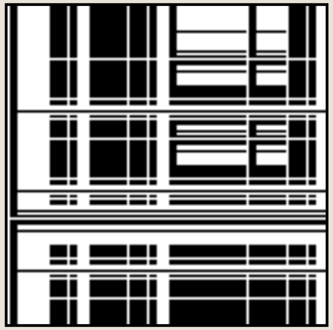
Rule 50



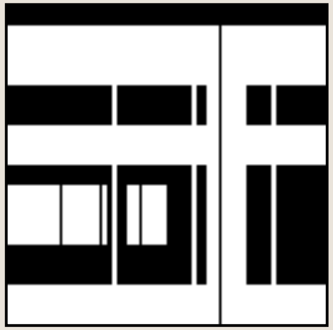
Rule 164



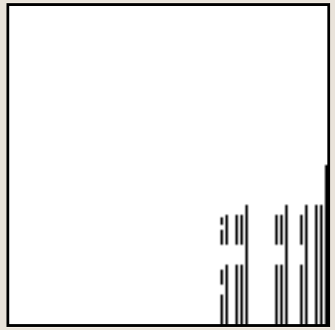
Rule 184



Rule 14



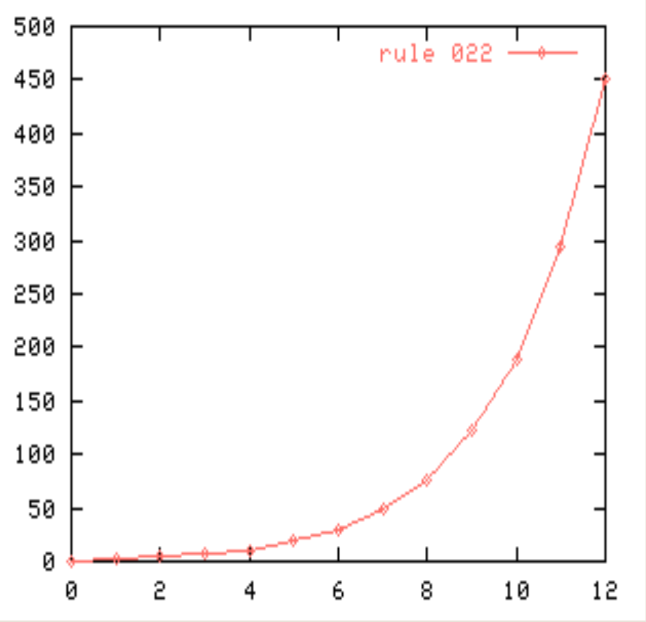
Rule 35

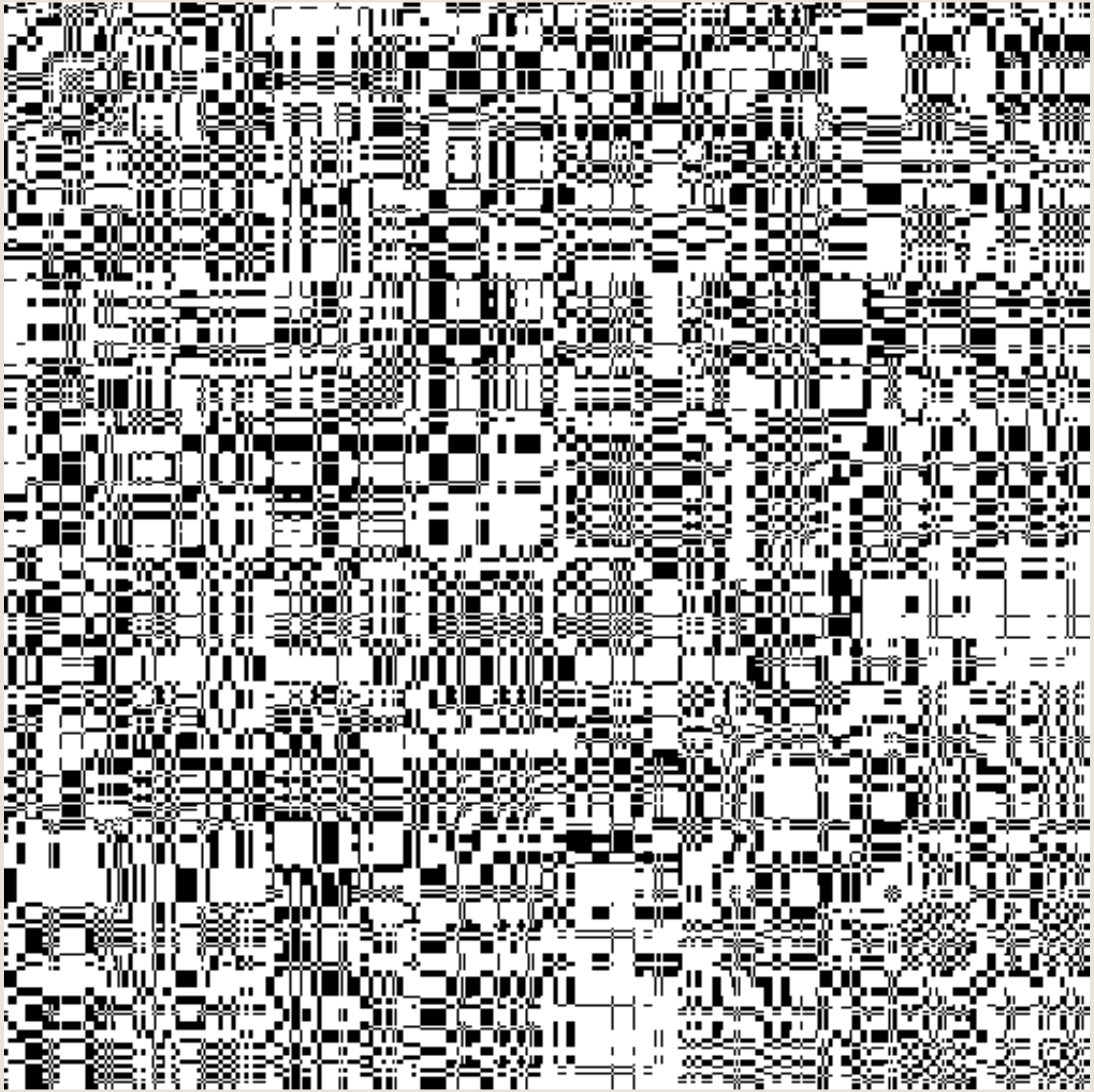


Rule 168

Rule 22

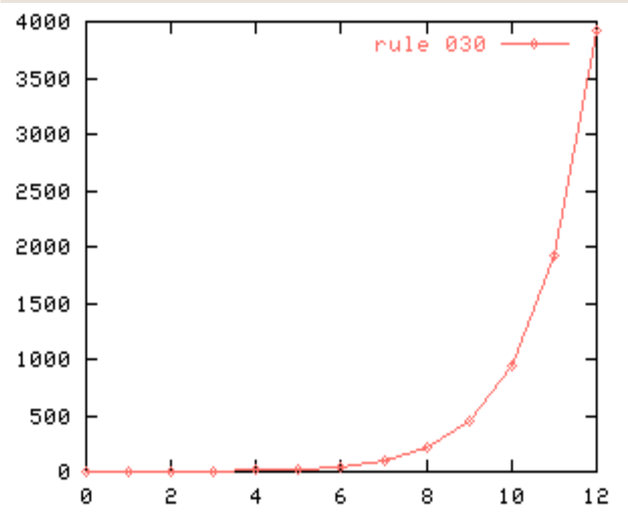
000 0
001 1
010 1
011 0
100 1
101 0
110 0
111 0

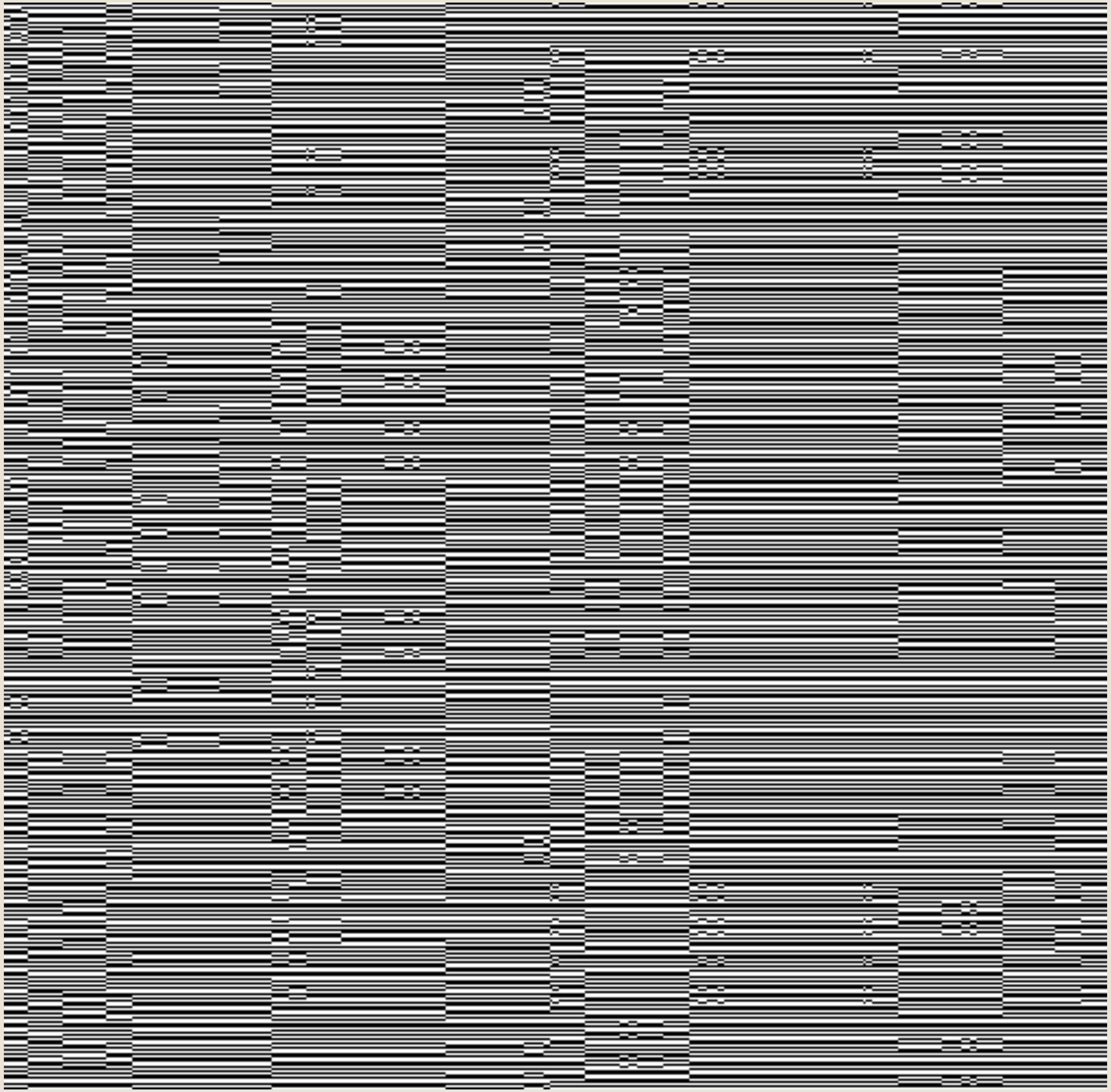




Rule 30

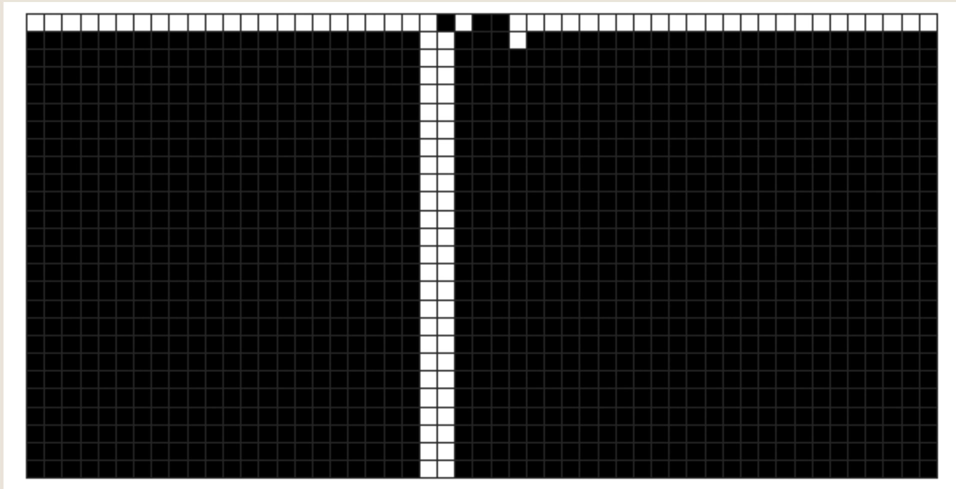
000 0
001 1
010 1
011 1
100 1
101 0
110 0
111 0





RULE 233

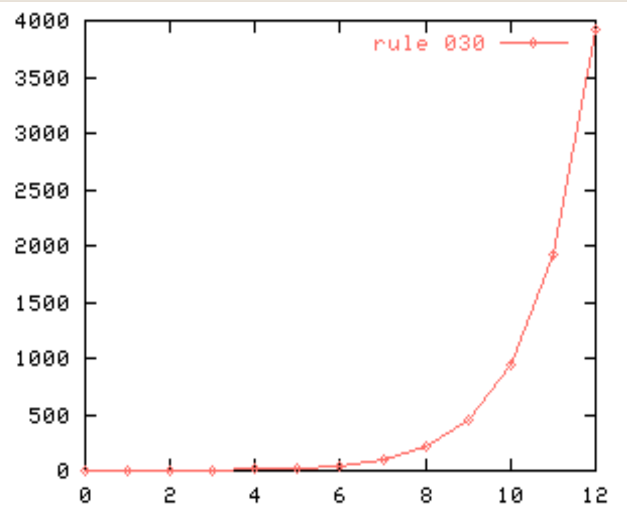
CLASS 3

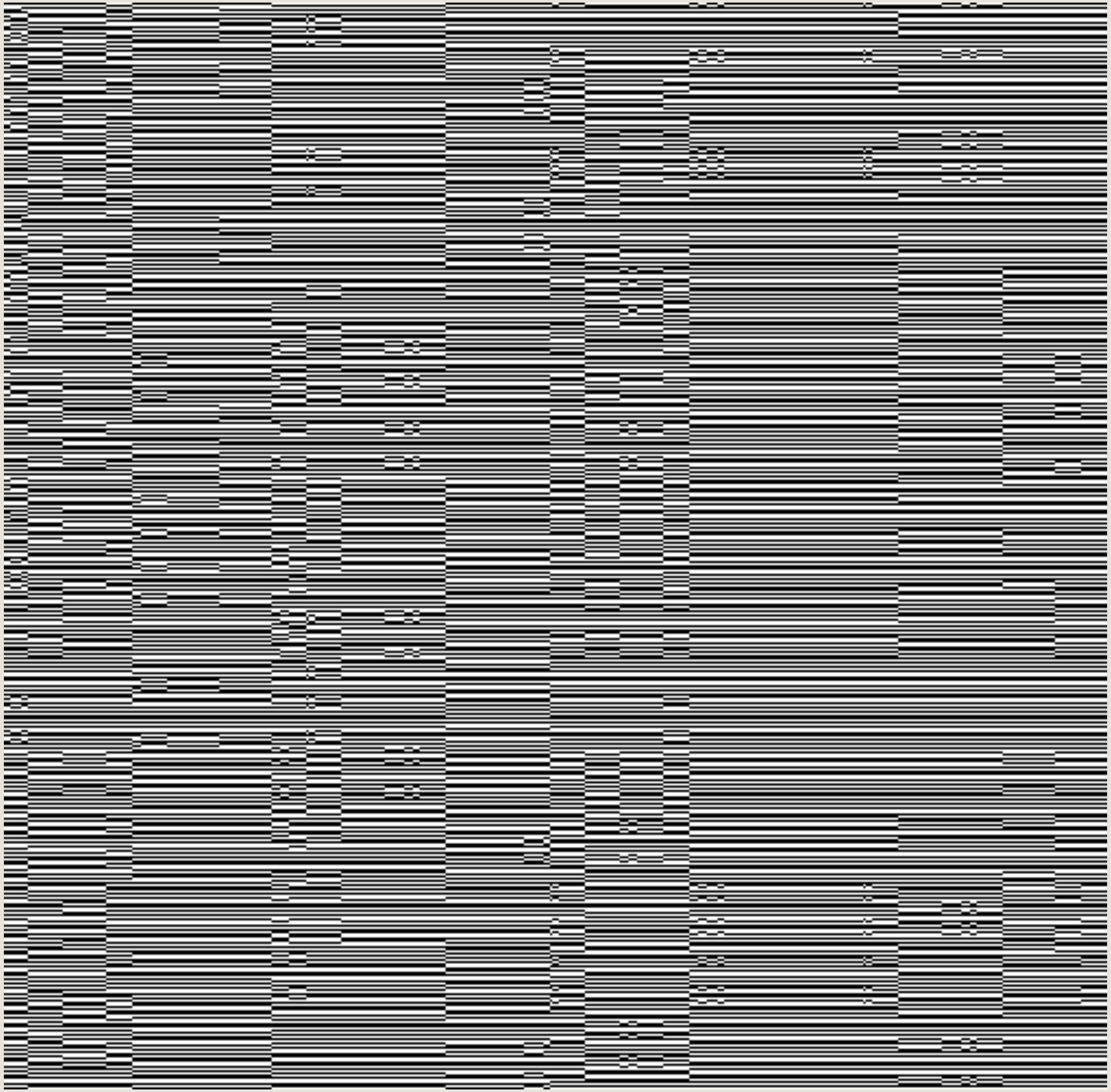


CLASS 1
(Wolfram)

Rule 30

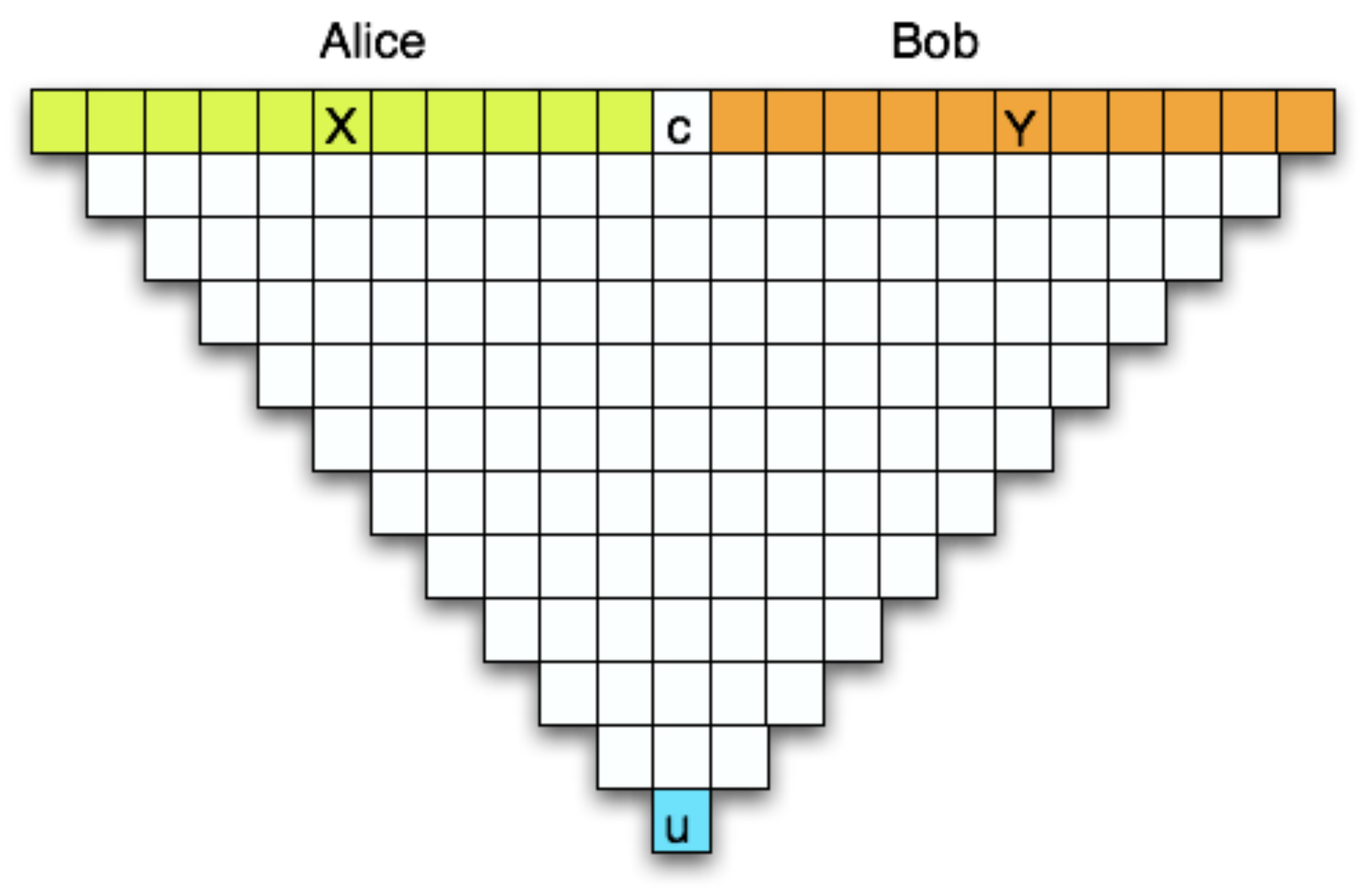
000 0
001 1
010 1
011 1
100 1
101 0
110 0
111 0





Class 4

The prediction problem for rule 218



Minimun information send by Alice such that Bob computes

$$u = f_{218}^{n}(x, c, y)$$

Rule 218

000 0

001 1

010 0

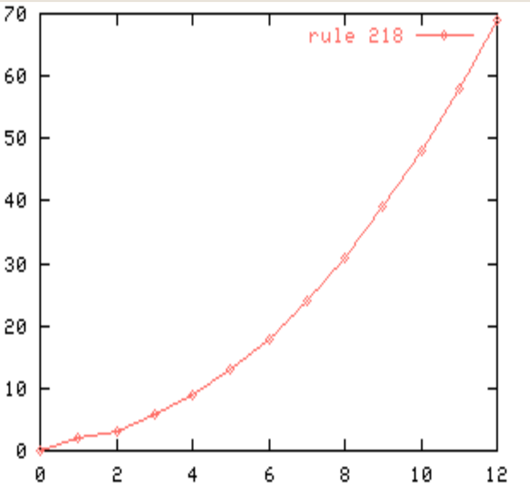
011 1

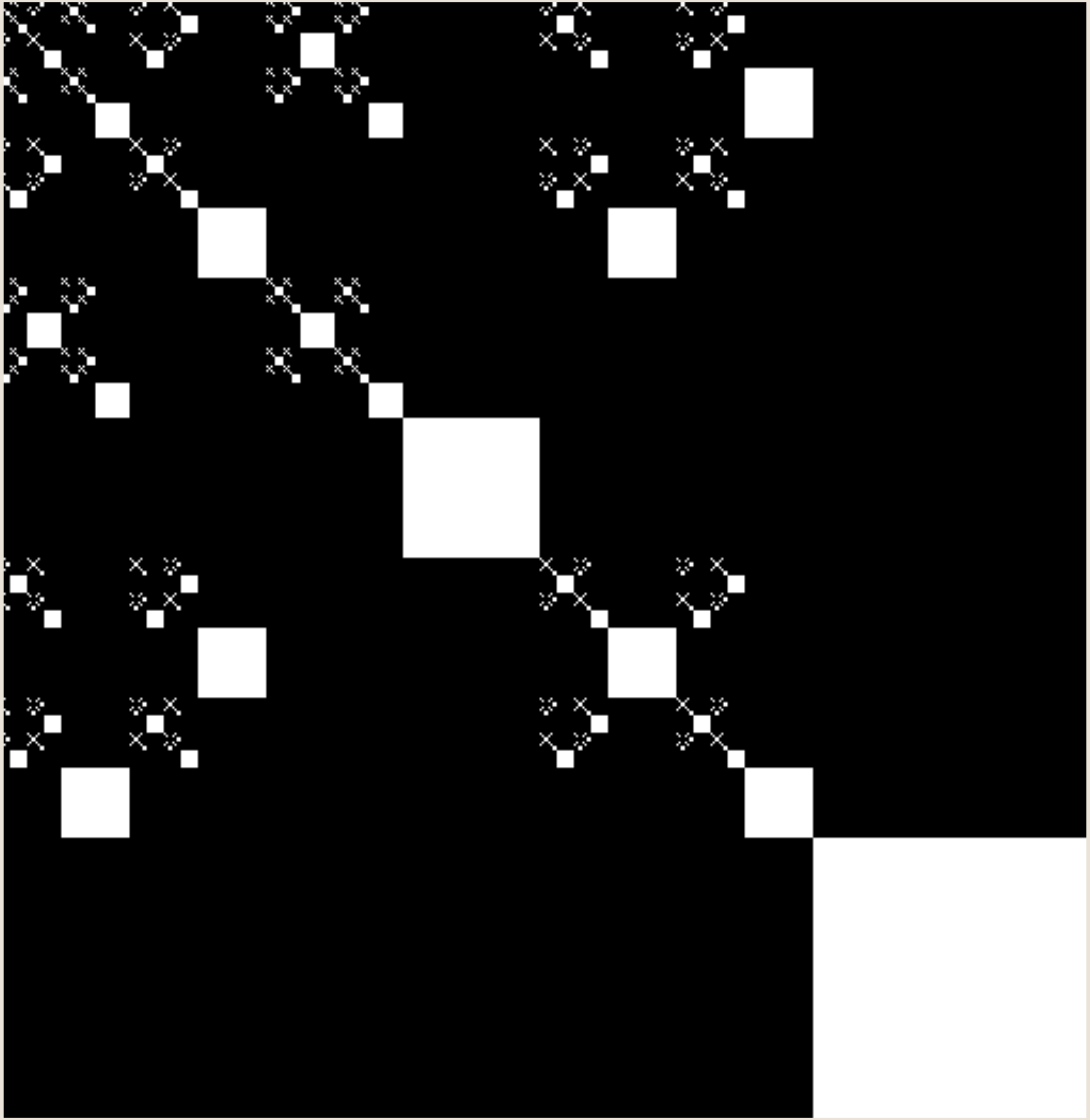
100 1

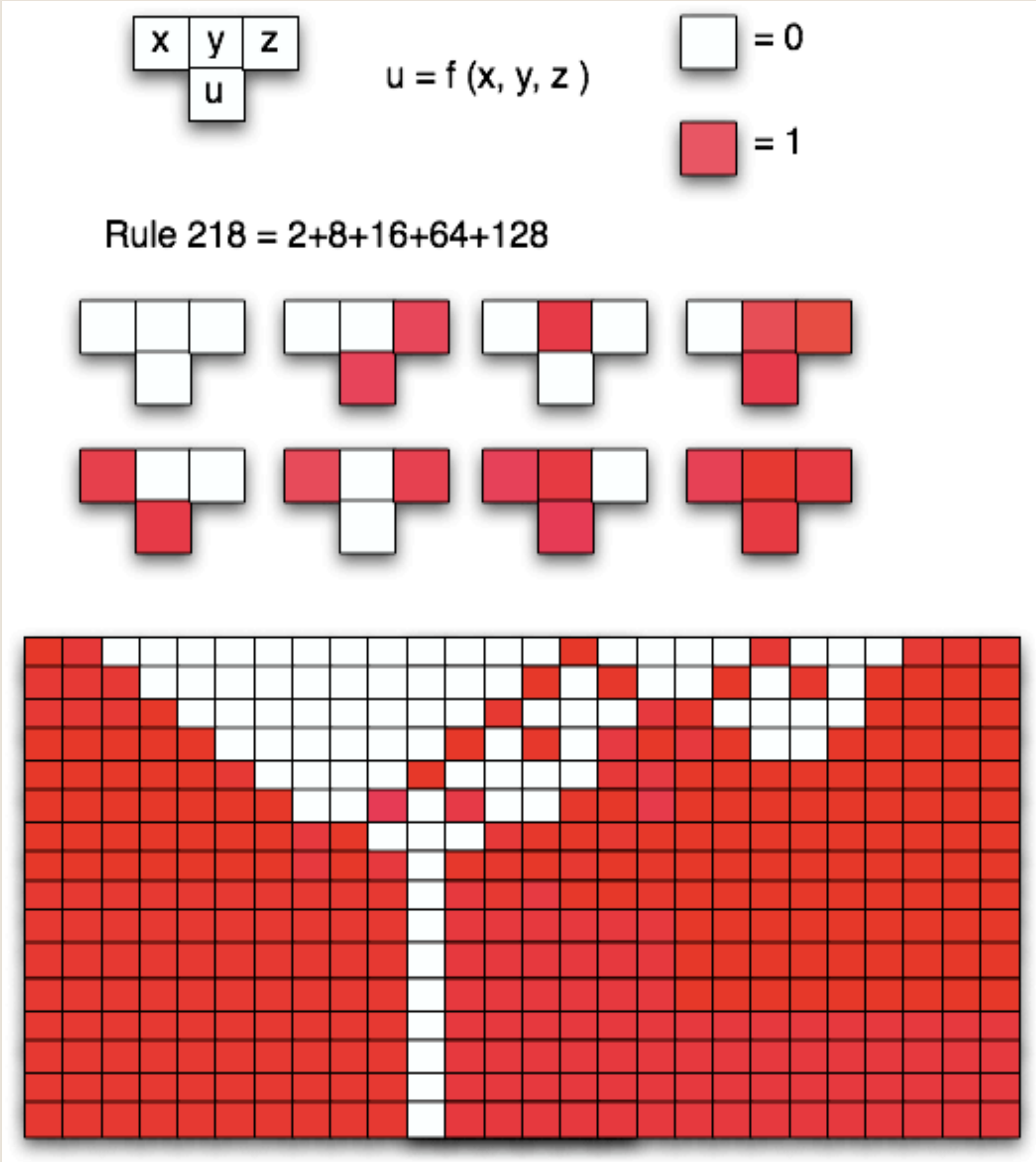
101 0

110 1

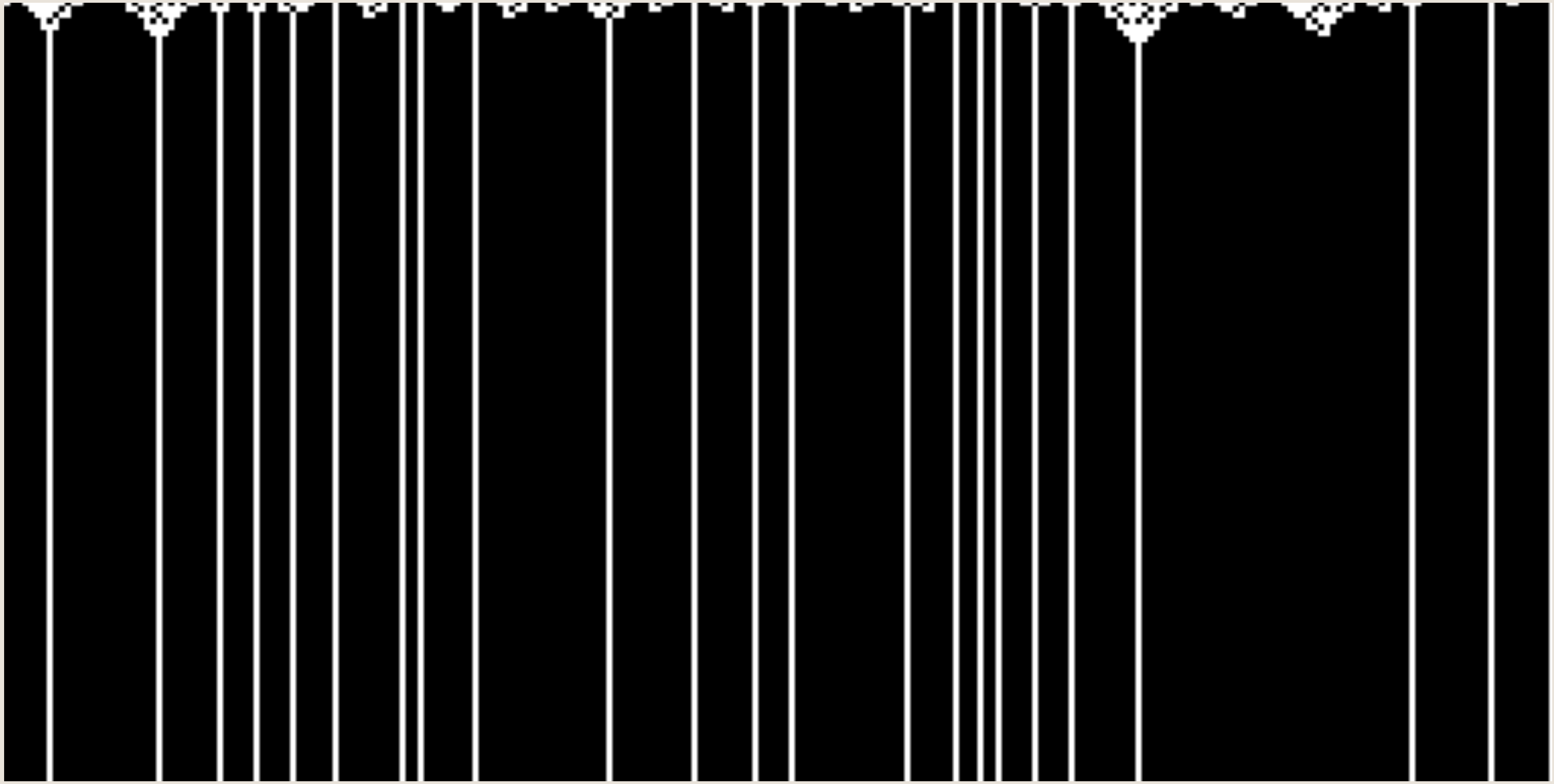
111 1



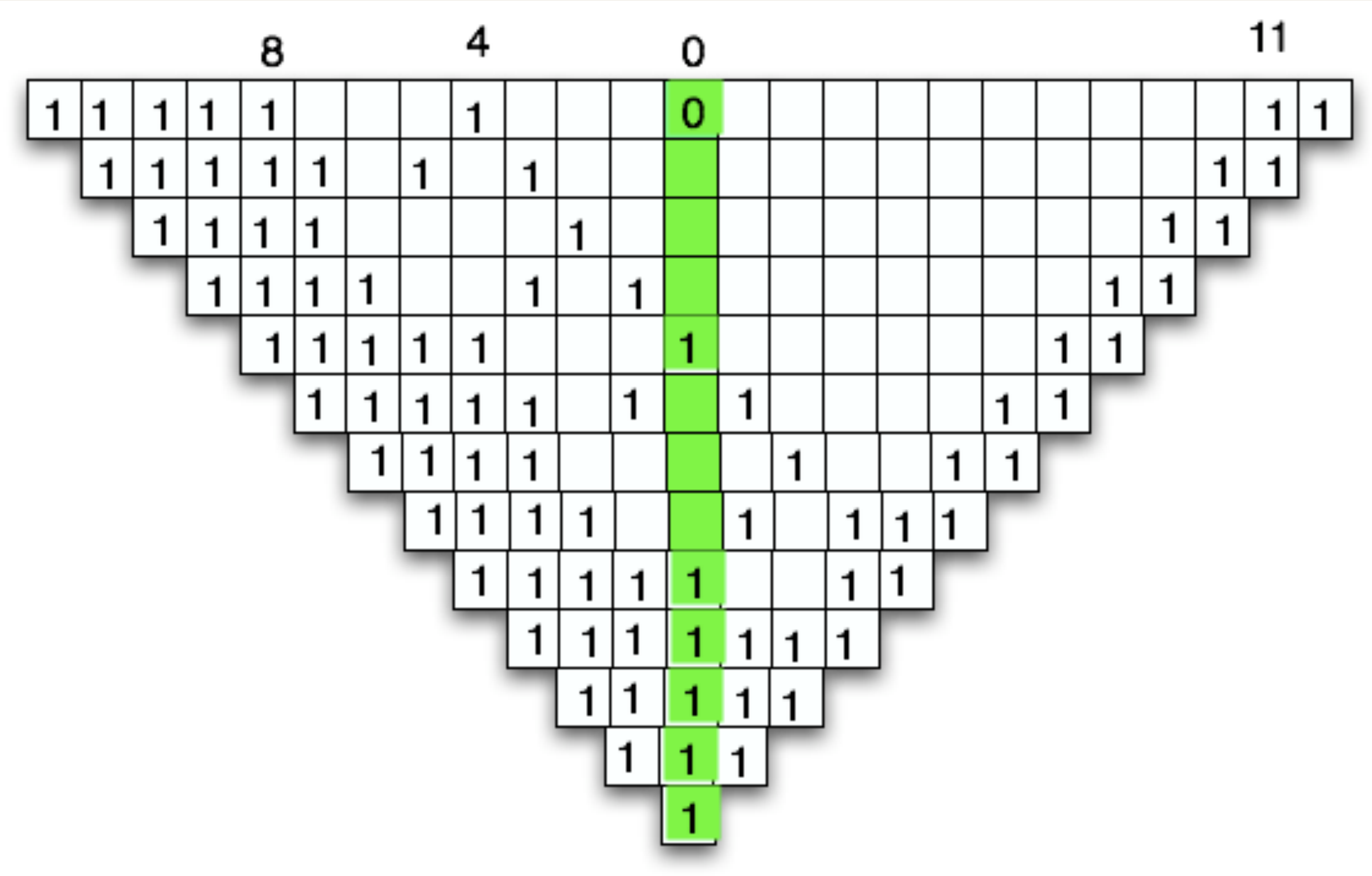



x
y
z
u
u = f (x, y, z)
= 0
= 1
Rule 218 = 2+8+16+64+128

218's dynamics



Example for rule 218



Remarks

If the ones are isolated and every couple is separated
by an odd number of zeros the rule 218 becomes additive.

In this case its behavior is like the rule 90.

$$f\left(a,b,c\right) = f_{90}(a,b,c) = a \approx c$$

For additivity we have

$$f^{n}\left(x,c,y\right) = f^{n}\left(x,c,\vec{0}\right) \approx f^{n}\left(\vec{0},0,y\right)$$

So Alice sends the bit

$$f^{n}(x,c,\vec{O})$$

So the protocol is constant for additive rules

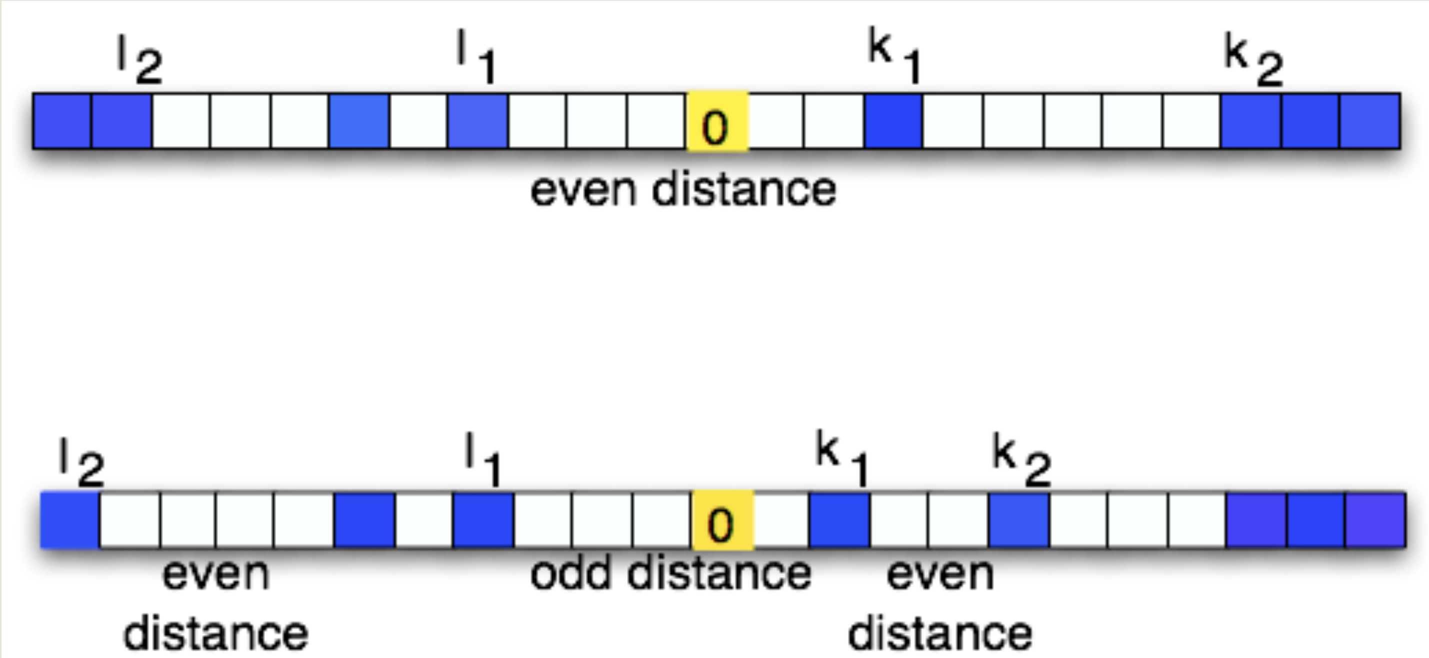
Definition of indexes for the 218 protocol

A crucial observation

Two or more ones remain invariant by the rule application

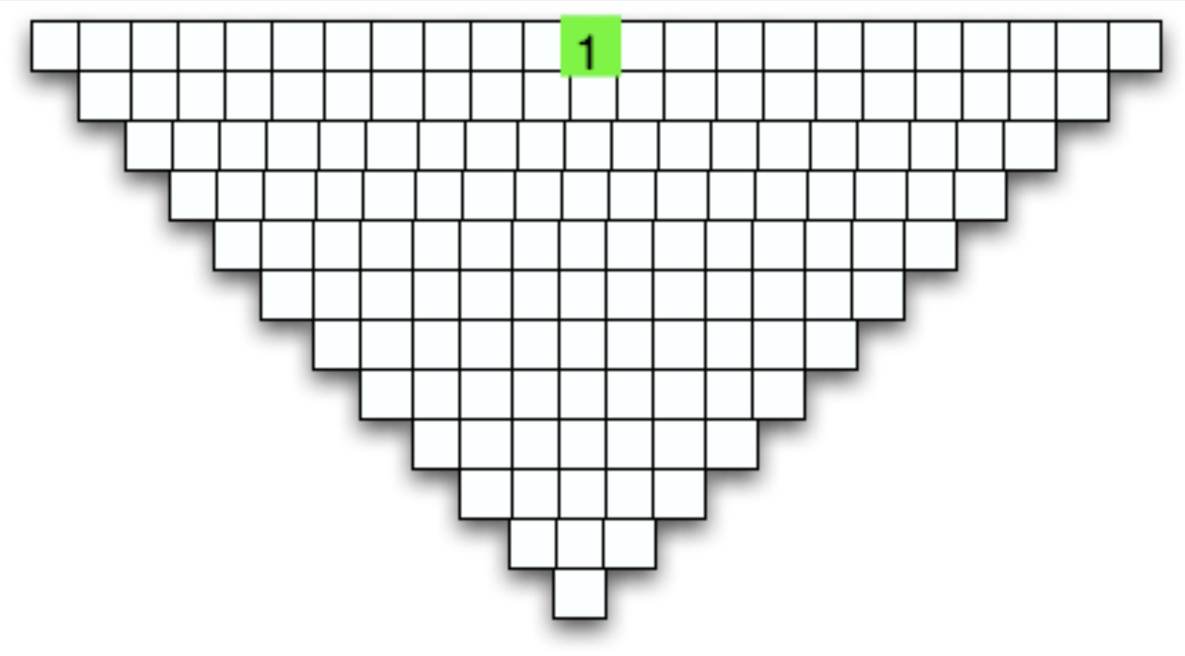
11
11

Definition of indexes for the protocol



l_1 Is the first one from the center cell

l_2 Is the first 1 such that the distance with the previous 1 from the center is even OR the next position is also a 1



Clearly in this situation we do not need the position of the first one in order to send it to Bob.
Since the center is 1 the parity (even or odd) only depens upon each side

So for the protocol it is enough to send only the second index i_2

Also in this case the protocol is optimal. One may exhibit a linear number of different rows in the *M(1,n)* matrix.

So, for the PRED problem $CC(218) = 2\log(n)+Cte$

Rule 218 is the first to have a quadratic number of rows (i.e a 2 indexes protocol) when the center is 0

and a linear one (1 index protocol) when the center is 1.

Intrinsic Universality

A CA which may simulates any other CA

Theorem: F intrinsically Universal implies CC(PRED) is (n)

Some consequences

Additive CA's are not I.U

Positive Expansive Ca's are not I.U

Rule 218 is not I.U

As well as rule 94, 184, 33 etc …..

Et …. Je n’ai pas eu le temps
de vous parler de la cigale (Complexity, 2000)
Et de tant d’autres curiosités
Informatiques …

Il faudra que je revienne vous parler de piles de sable l’été prochain et la complexité associé (Fundamentae Informaticae (2012), Enrico Formenti, Bruno Martin, E.G)

MERCI !!!!
GRACIAS !!!!