Firenze, 2014/09/19

IT CAN BE DONE IN SOFTWARE

Statement attributed to speaker:

"When the physics is completely clear, the only way to improve a device, a measurement, a controller ... is by way of mathematics"

Galileo: "The book of Nature is written using mathematical characters."

Newton: "The Law of Gravitation is a mathematical consequence of Kepler's laws of planetary motion

⋮

Bézout: polynomial equations

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computer
sensors
actuators } a new kind of physical system

Example: KALMAN filter.

KF is not

1. An algorithm
2. A useful procedure
3. A signal processor

KF is

A. An invention
B. A device adapted to a specific environment
C. A dynamical system with precisely controlled behavior

} physical attributes

2A

In order to illustrate the seemingly weak influence that Cauer's work has had on present research in network and systems theory, we shall now quote several authors in this field. In the 13th edition (1975) of Hendrik W. Bode's monograph *Network Analysis and Feedback Amplifier Design* (first edition published 1945) we find the following remarks [17]: "However, special mention should be made of K.W. Wagner and W. Cauer, two Germans whose important contributions were slow to diffuse outside Germany because of the accidental intervention of World Wars I and II. . . . The enlarged second edition, edited by W. Klein and F. Pelz and translated into English by G.E. Knausenberger and J.W. Warfield, appeared in 1958 and Cauer's work became widely known at that time."

In his monograph *Synthesis of Feedback Systems* [18], published in 1963, I. M. Horowitz used many of Cauer's concepts but did not mention Cauer himself.

At a panel discussion on system theory held in 1962 [19], R. E. Kalman, one of the founders of modern control theory, mentioned that "classical work on network synthesis was not too successful from the standpoint of Combinatorial System Theory" and went on to present his own approach to system theory. This is characterized by Bode in the above-mentioned treatise as follows: "The principle reason for working on systems theory is not to advance electrical network theory further but to make the highly developed methods of network theory – and at second hand of communications theory as a whole – available in other fields".

??

Cauer's name is also difficult to find in mathematical literature. Norbert Wiener, who met Cauer in Göttingen in 1927 and later at MIT in 1930/1931, where Cauer was a Rockefeller Fellow, refers to him briefly in his autobiography as 'Richard Cauer'. It is of interest to note in this respect that both Wiener and his Ph.D. Student Y. W. Lee discussed the problem of synthesis of electrical filters around 1930/1932 using Cauer's ideas, among others. Cauer himself, however, explained Wiener's and Lee's approach in his paper and in his book in details. Therefore, in order to gain a clearer idea of the way Cauer's work was regarded between 1928 and 1945, it is worth while to refer to the material contained in his estate.

As already mentioned, Cauer started his scientific career studying under the famous physicist Max von Laue in 1922, his main area of interest at the time being the theory of general relativity. After gaining his diploma in technical physics, Cauer joined an industrial company, where he worked on teletraffic problems. It is not known why he started to work on electrical filters, but in March 21, 1926, he wrote a letter to R. Foster, who had published his famous *Reactance Theorem* paper in 1924 and who was now working as a research scientist at the department of development and research at Bell System in New York. This paper had a considerable influence on Cauer's work. Foster replied fairly promptly with a letter dated April 8, 1926, in which he explained some aspects of their common research interests. For his part, Sidney Darlington mentions in his autobiographical notes that Foster referred to corresponding with Cauer in 1924 and 1926 in a phone call he made with Darlington in 1983, 2C

and Cauer's estate includes a letter written to him by Foster in August 1939, proving that they were in contact until that year.

Cauer had also contacts with mathematicians who worked in his field of interests. For example, the estate contains a postcard from Caratheodory, who was very active in the theory of complex functions. As already mentioned, Hamel and Wagner acted as the academic supervisors for Cauer's 1926 thesis, and a close connection was maintained with Hamel for many years, as the correspondence in Cauer's estate proves. Once Cauer began working at Courant's Institute of Mathematics at Göttingen University, he came into contact with many mathematicians working in related areas. Issai Schur emphasized in a letter to him the significance of the charge to Göttingen for Cauer's career. It was also around this time that his lengthy correspondence with the mathematician G. Herglotz began, ending in late 1944, shortly before Cauer's death. His estate also contains correspondence with the mathematician Georg Pick, whose findings became crucial for Cauer's work, and the graph theorist D. König from Budapest, in which they discussed the relationship between graph theory and network theory. In 1929, Cauer used his contacts to Wiener to ask V. Bush of MIT for help with a Rockefeller grant. At this time, he was interested in mathematical machines for solving (network) determinants, which was one of Bush's main research areas. In 1930 Cauer constructed the above-mentioned electrical linear systems solver that is described in a paper by Petzold [20], and his estate contains several letters concerned with the machine and the photograph shown in Fig. 1

2D

During his stay at MIT, Cauer met many researchers and had strong contacts with G.A. Campbell, O.J. Zobel, Bode, Darlington, and Foster and other members of the research staff at Bell System Laboratories, although it is unclear why Bode does not mention this fact in his bibliographical notes. At MIT, Cauer's collaboration with Otto Brune during 1931, when he acted as Brune's adviser, also proved essential to his work, and it was here that Brune solved, with Cauer's help, the problem of the electrical realization of positive functions. In a footnote on the first page of his celebrated paper [37], Brune acknowledges the stimulus provided by "Dr. W. Cauer who suggested this research", and several letters in Cauer's estate illustrate the close connection between the two men. At the end of his Rockefeller grant, Cauer sold several patents on filter design to Bell System, and with this money was able to provide financial support to his assistant E. Glowatzki in Göttingen.

Back in Germany, Cauer contacted other industrial companies in France and England with regard to his patents, corresponding for example with A.C. Bartlett of the General Electric Company in Wembley, and Roger Julia of Lignes Telegraph Telephone in Paris. The personal nature of these contacts was emphasized by the letter to Cauer from Barlett of March 27, 1933, which is still in existence today and which states: "Many thanks for the reprint on Poisson Integral which you have just sent me. I am afraid I am a very bad correspondent and must also thank you for the New Years Card that you and

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Going back to the beginning:

Main Theorem ~ 1960

A linear finite dimensional (time-invariant = constant coefficient) system is abstractly the same as its behavior (transfer function) if and only if it is generic (minimal dimension, irreducible, ...) that is, completely controllable and completely observable.

⟹ Such a system has a unique state-variable (directly physical) description,

uniqueness means within a nonsingular (& nonphysically interpretable) linear tranformation of the state variables

---

⟹ when 10^x teams construct a simulation of a physical system

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leading to:

PROBLEM OF UNIQUENESS

Do laws of PHYSICS determine "everything"?

Badly posed:

does our interpretation, application, models, .. lead to a unique answer?

(not a question involving statistics or randomness, or indeterminacy)

Does a "theory of everything" lead to explaining everything?

Or nothing?

I will argue:

A theory with strong claims to uniqueness will explain the [pretty] obvious,

a theory that allows diversity is aiming toward illuminating COMPLEXITY

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Network Realization Problem ("Circuit Synthesis")

behavior

Impedance = Z →

black box

(Heaviside, (1850 – 1925)

$\Downarrow$

$$Z = \frac{a(s)}{b(s)}$$

$a(s)$, $b(s)$ polynomials in 1 variable, real coefficients

$Z \sim (a(s), b(s))$ = point in real projective space

R, C, L = real, $\neq \begin{matrix} 0 \\ \infty \end{matrix}$

Graph (circuit diagram) of 1-port network with 2 distinguished vertices $\underset{1}{\circ}$ $\underset{2}{\circ}$

$$\Sigma \longrightarrow Z$$

network ↑ Σ behavior ↑ Z

matrix tree theorem

(Kirchhoff, 1847)

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Is $Z \xrightarrow{?} \Sigma$ (synthesis, inverse of behavior)

a mathematically feasible question?

Yes provided a generic network Σ can be defined.

generic: (intuitive concept)

a network (system) that cannot be changed in any way without an essential modification of behavior

generic networks

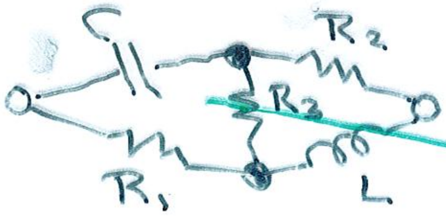
were unknown in circuit theory in 50 years of research, and were discovered by Ronald Foster (1896–1998) around 1948, but not formalized, not even named.

A simple mathematical criterion based on the circuit diagram is currently UNKNOWN. Look at the next trans

Antibes 2014/09/25 DIGRESSION 9 7

generic

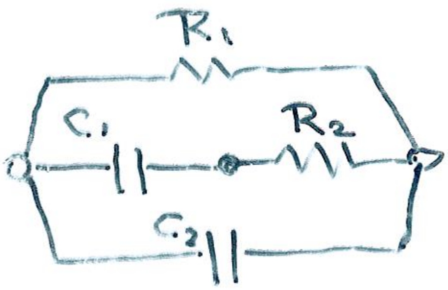
NOT generic

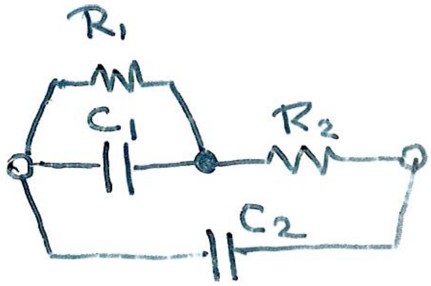


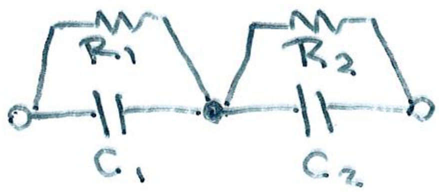
generic

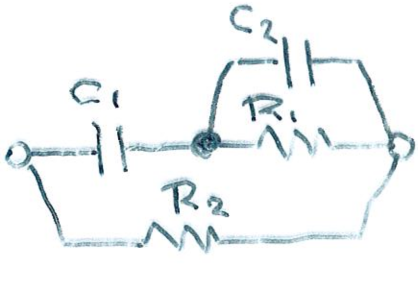
remains generic even if $R_3 = \begin{cases} 0 \\ \infty \end{cases}$

$\boxed{a_2 = 0}$ Equivalent generics









Σ_1, Σ_2 equivalent $:=$ same Z

& a fixed Σ may have n different sets of (R_i, C_j, L_k) yielding the same Z!

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the network realization problem falls directly within the scope of Hilbert's finiteness results in algebraic invariant theory as in his big paper,
"über die vollen Invariantensysteme" (1893)

In the spirit of Hilbert, we have the following BIG theorem:

Every generic network is (fully) characterized by a set of invariants
{ Resultant Z, . . . }
and corresponding conditions $\boxed{>0, <0, =, \neq}$

which suffice for answering all system theoretic questions such as

realizability of a given Z
equivalence
#C and #L and #R
⋮

(and probably others, relating to the network graph, etc.)

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As an example of the process of applying invariant theory to unraveling the structure of the unknown network in the black box from Z (or the network(s) that realize Z)

invariants of network = invariants of Z

(*) #L & #C $\approx$ signature of Bézout(Z)

$\vdots$

The matrix ($n \times n$, real, symmetric)

$$\text{Bézout}(Z) := \text{Bézout}(a(s), b(s))$$

where all entries are sums with positive coefficients of deg 2 invariant

$$[i, j] := a_i b_j - a_j b_i = -[j, i]$$

(*) is perhaps the most amazing result of algebraic system theory that is known today —— discovered by Foster before 1948 and briefly mentioned (without proof, for $n \leq 2$ only) in a survey paper of 1962.

Conclusions :

"System studies" (simulation, data analysis, ...) are meaningless, misleading, or just wrong without taking into account genericness.

The deeper we probe, more the issues complexity come to the fore and the depth of mathematics increases.